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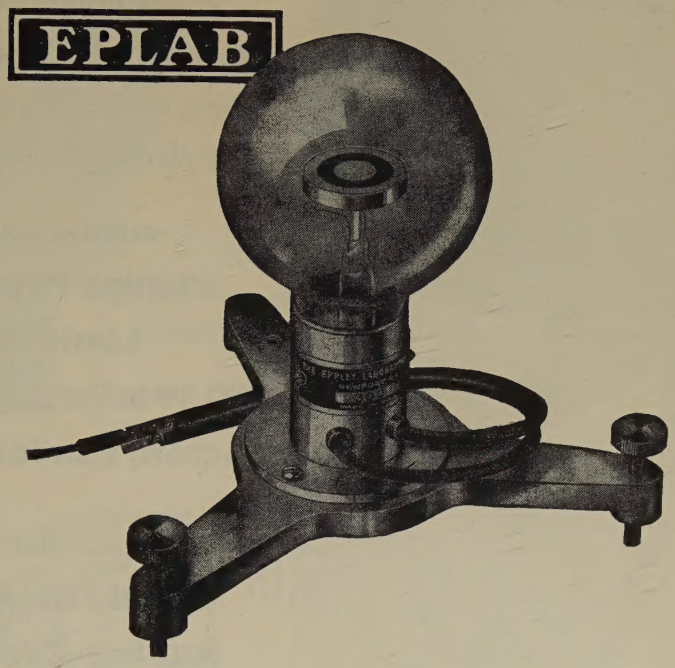

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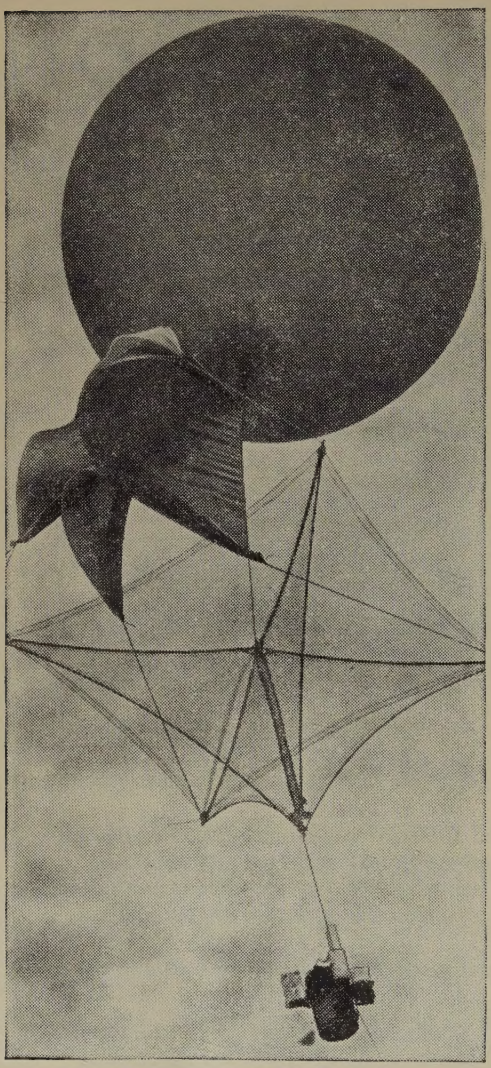

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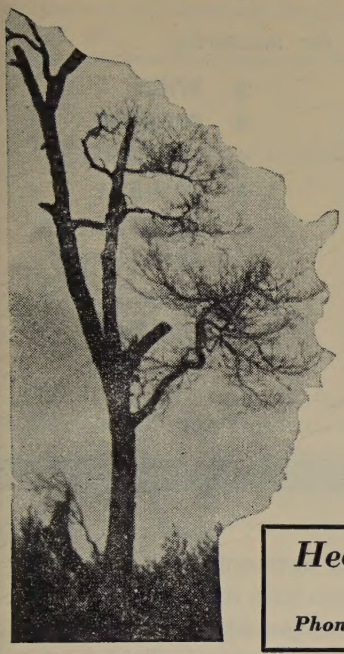

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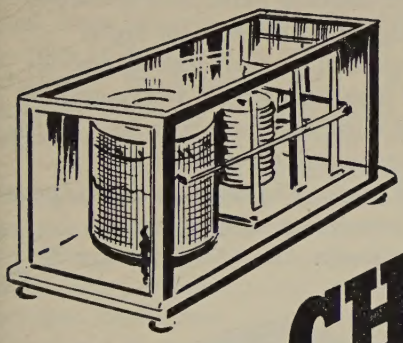

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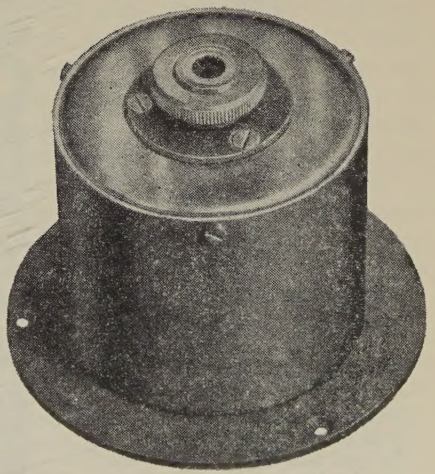

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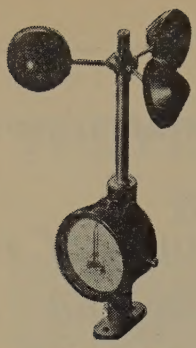

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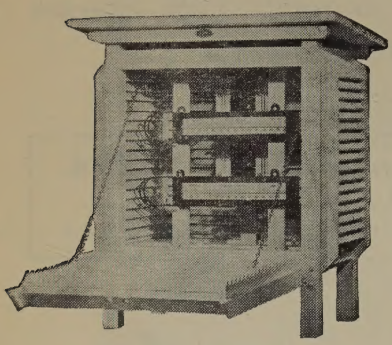

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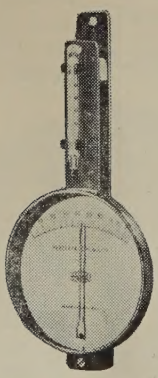

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The summer circulation of the arctic stratosphere below 30 km

By F. KENNETH HARE
McGill University, Montreal

(Manuscript received 29 June 1959)

SUMMARY

The summer circulation of the arctic stratosphere is examined in the light of climatological data and daily synoptic analysis. The Ferrel westerlies are shown to extend to about 20 km (50 mb) immediately above the jet axis. The middle stratospheric easterlies above them descend to 15 km at the pole in midsummer. The easterlies flow round the flanks of a flat-centred anticyclone near the pole, and are almost devoid of perturbations. The lower stratosphere (tropopause to 20 km) has a disturbed thermal regime due to vertical motion associated with waves in the Ferrel westerlies or with cold lows within the westerly ring.

1. INTRODUCTION

The past ten years have seen a progressive improvement in the performance of the rawinsonde, and in the density of the observational network in the remoter parts of the Northern Hemisphere. Synoptic analysis has become feasible at levels as high as 20 and 25 km (50 and 25 mb), and has been attempted at 30 km (10 mb). The stratosphere has thus been brought within the daily range of meteorological practice. A considerable volume of temperature and wind data is now available, and files of analysed synoptic charts are being accumulated. Levels above 30 km remain inaccessible to the synoptic analyst. At lower levels, however, it is already possible to attempt preliminary studies of the characteristic circulation and its perturbations.

Much attention has already been given to the stratospheric circulation of winter, following the realization that the temperature field at high levels is subject to remarkable sudden warmings, first described by Scherhag (1952) and subsequently studied in detail by Warnecke (1956), Teweles (1958) and Godson (1959). These warmings occur at dates between January and late March, and are associated with the instability and subsequent breakdown of the polar-night vortex, the great ring of westerlies (with well-developed jet streams) that encircles the pole at levels above 16 km (100 mb). These very striking events are not related in any simple fashion to the annual radiative rhythm. It is clear, in fact, that dynamical processes, including eddy mixing on the large synoptic scale, are characteristic of the winter stratosphere, whose thermal regime in high latitudes is very far from reflecting the radiative equilibrium once thought typical (Hare 1960).

The much less disturbed circulation of the summer months has attracted little attention. It has been known for many years that the Ferrel westerlies (the term used here for the circumpolar westerlies of the troposphere and lower stratosphere, to distinguish them from the polar-night westerlies)* decrease with height in the lower stratosphere, and are overlain by easterlies at higher levels. This regime lasts from May until mid-August; thereafter the westerlies extend to much greater heights. Whipple (1935) detected the reversal of the circulation above 20 km by inference from acoustical data, christening it the 'stratospheric monsoon.' More recently the reversal has been demonstrated by rawinsonde flights (Scrase 1951) and further acoustical studies (Crary 1953). It is the

* Ferrel appears to have been the first to describe the circumpolar westerlies in modern times. The nomenclature advocated here extends the practice of Professor J. Bjerknes, who refers to the meridional overturning of the westerly belt as the 'Ferrel meridional cell.'

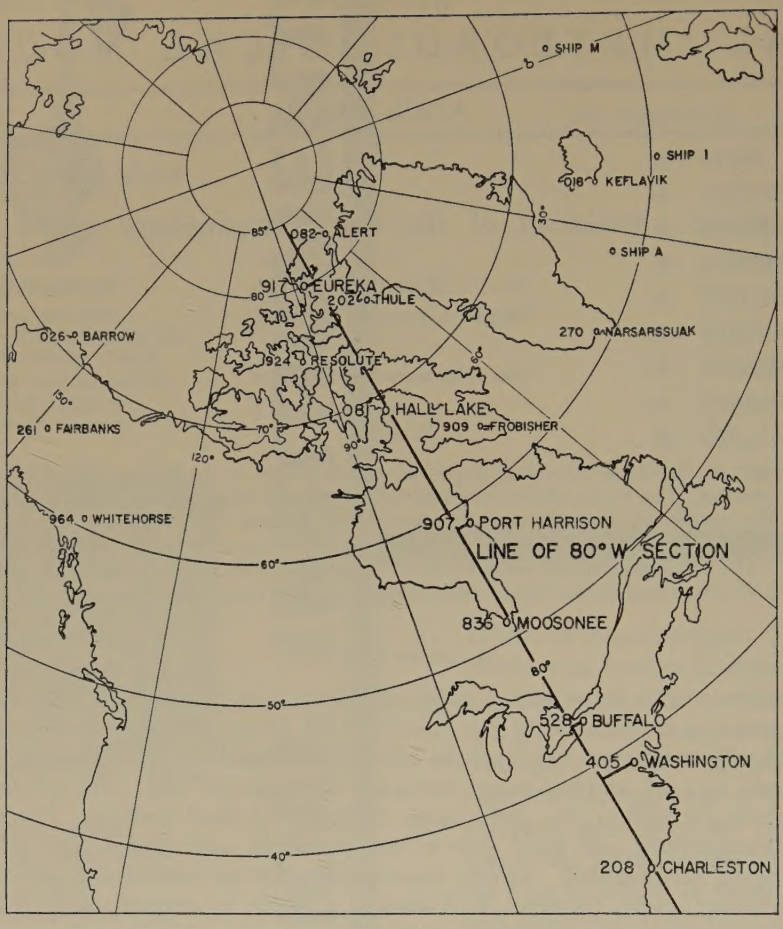

Figure 1. Location and index numbers of stations mentioned in the text. The line of section used in Figs. 8, 11 and 12 is shown by the heavy line.

purpose of the present paper to examine the characteristic structure in greater detail, using climatological and synoptic evidence. In broad outline the results are consistent with the recent synthesis by Murgatroyd (1957). Lack of data made it necessary to confine the study to the American, Atlantic and west European sectors of the hemisphere.

Daily analysis of stratospheric charts has shown that the area north of the axis of the Ferrel westerlies (roughly 50°N in July) is homogeneous in behaviour, and is best treated as a unit. Within this polar cap, called the *arctic stratosphere* in this paper, the entire column above the tropopause is warm, almost isothermal in the vertical, and over wide areas is quasi-barotropic. Hemispheric temperature gradients are directed equatorwards, but are small north of the 60th parallel. The stratosphere south of the Ferrel westerlies displays an entirely different regime; hence the following account is deliberately restricted to the arctic stratosphere. For reasons of convenience, the writer will follow Goody (1954) in referring to the layers above 30 km as the *upper stratosphere* and will subdivide the lower layers into a *middle stratosphere* (20-30 km) containing the ozone peak and the summer easterlies, and a *lower stratosphere* (tropopause to 20 km).

2. Sources of data

Two main sources of data are used in this paper (1) climatological summaries of rawinsonde data, and (2) synoptic charts for various stratospheric levels drawn by the writer and his colleagues at McGill University.

Table 1 gives a list of stations for which rawin data have been made available by the U.S. Air Weather Service and the U.K. Meteorological Office. The table gives coordinates index number, years of observation and the number of ascents reaching 10, 16, 23 and 30 km (300, 200 and 100 mb in the case of U.K. tabulations). Figs. 2-6 are based on these tabulations.

The root-mean-square direct instrumental errors in the rawin measurements are estimated at 2 to 3 m sec^{-1} in the 20-30 km layer (Ference 1951; Chiu 1958). More important sources of error in the mean values presented are selective ascent terminations (due in summer primarily to high winds near the tropopause, defective balloons or careless operation). These probably weight the higher-level parameters towards conditions typical of periods of light winds at tropopause level. Low temperature terminations are not important in summer, but become critical in winter. Table 1 shows that the size of sample decreases greatly at the higher levels, which is a source of error in the profiles of standard vector deviation presented here.

TABLE 1. Upper-wind summaries used in analysis

(a) U.S. Air Weather Service tabulations (summer season)

Station	Index No.	Lat. ° 'N	Long ° 'W	Period	Ascents reaching 10 km	16 km	23 km	30 km
Alert	082	82 30	62 20	1950-56	912	692	288	–
Barrow	026	71 18	156 47	1951-56	774	672	431	–
Eureka	917	80 00	85 56	1951-56	570	373	210	–
Fairbanks	261	64 49	147 52	1951-56	826	611	417	103
Frobisher	909	63 45	68 33	1942-50	195	39	–	–
Keflavik	018	63 59	22 38	1944-56	2,335	1,529	969	309
Narsarssuak	270	61 11	45 25	1943-56	2,486	1,440	887	209
Resolute	924	74 43	94 59	1951-56	585	300	163	48
Thule	202	76 31	68 50	1952-56	1,375	1,120	791	88
Whitehorse	964	60 43	135 04	1945-46 1955-56	262	203	85	–

(b) U.K. Meteorological Office tabulations (July)

Station	Lat. ° ′	Long. ° ′	Period	Ascents reaching 300 mb	200 mb	100 mb
Ship A	62 00N	33 00W	1951-53	122	111	81
Ship I	59 30N	18 30W	1951-55	563	541	349
Ship M	66 00N	02 00E	1949-52	73	67	23

The synoptic materials used in this report were prepared by the Arctic Meteorology Research Group at the Central Analysis Office, Canadian Meteorological Service, Montreal Airport. Case-study analysis based on the *Northern Hemisphere Data Tabulations* (U.S. Weather Bureau) was carried out for selected cases in 1956 and 1957. Since 1 June 1958, charts have been prepared on the following daily bases, in all cases for 0000 GCT :

(a) circumpolar 200 mb charts for areas north of 30°N;

(b) 100 mb charts over the area bounded by the 50°E and 180°W meridians and the 40°N parallel;

(c) 25 mb charts over North America;

(d) cross-sections along two lines, Charleston, South Carolina (208) to Eureka (917) along the 80°W meridian, and Whitehorse, Y.T., (964) to Alert, Ellesmere Island (082) and Nord, Greenland (310).

Synoptic analysis at these levels presents problems arising from poor data coverage, and from unreliability of reported heights and temperatures, due to faulty correction of radiation errors. A separate report on the group's methods is being prepared for publication. Teweles and Finger (1958) have prepared a valuable summary of U.S. Weather Bureau methods at these levels.

3. Mean circulation and thermal structure

Circumpolar 200 mb and 100 mb mean topographies have been available for some time (Heastie 1955, 1956; Namias 1958). Tentative mean charts for higher levels have been drawn for parts of the hemisphere, usually by extrapolation of assumed lapse-rate curves. Thermal structure is well displayed in cross-sections to 30 km by Kochanski (1955), and in the lower stratosphere by sections along several meridians by Goldie, Moore and Austin (1958). There remains considerable uncertainty concerning the radiation corrections that ought to be applied in daylight to duct-type temperature elements so that hydrostatic reconstructions of the topography of pressure surfaces above 100 mb are of only limited accuracy. In the present paper resultant observed winds will be used to give a picture of the mean circulation.

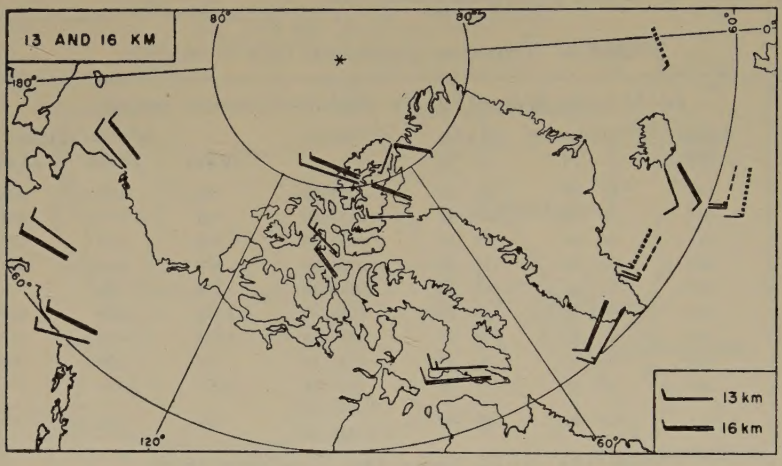

Figure 2. Resultant observed wind vectors in the lower stratosphere, for available period of record, summer season (June-August). The Ferrel vortex centre is close to Alert. A standing ridge lies in 120° to 130°W, and a trough in about 50°W. Speeds at 16 km are little more than half those at 13 km. One full feather (inset) equals 10 kt ($\simeq$ 5 m sec^{-1}).

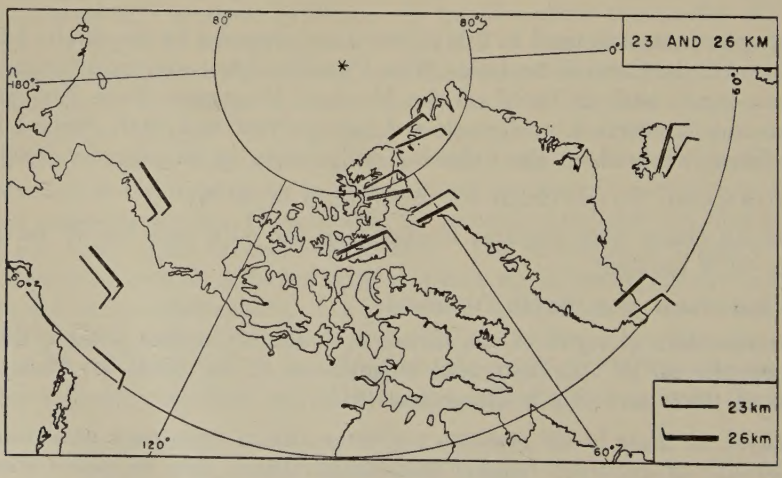

Figure 3. Resultant observed wind vectors in the middle stratosphere, for summer season (June-August). The flow is a light easterly about a centre near the Pole, with speeds slightly greater at 26 km than at 23 km. There is a suggestion of a ridge over western Canada. One full feather (inset) equals 10 kt ($\simeq$ 5 m sec^{-1}).

Figs. 2-3 show resultant wind vectors (V_r) in the lower (13 and 16 km) and middle (23 and 26 km) stratosphere at all available stations north of 60°N, for the standard summer (June to August). Fig. 4 gives vertical profiles of the zonal component of the resultant wind $\bar{u}$ for spring (March to May) and summer at stations in Alaska (Barrow and

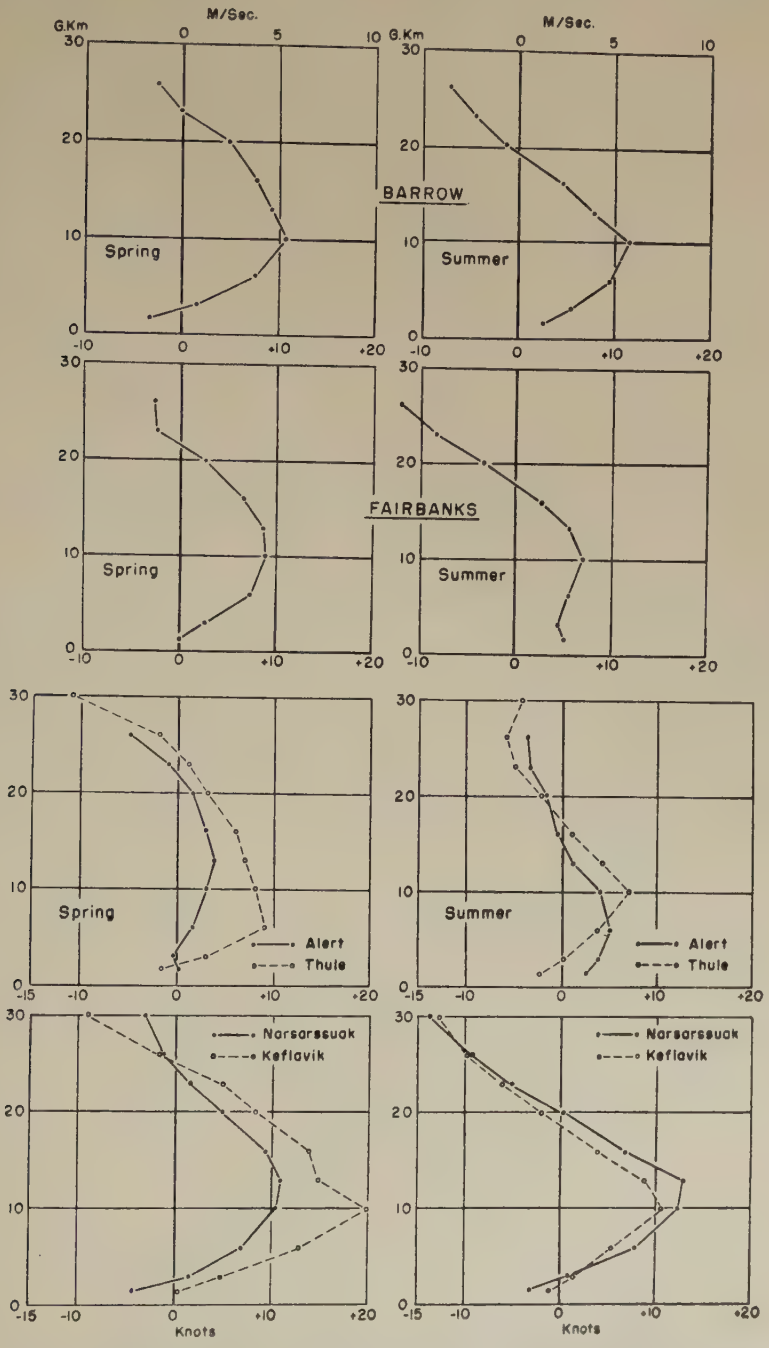

Figure 4. Vertical profiles of the zonal component of resultant observed wind, for spring (March-May) and summer (June-August) for stations in Alaska (Barrow and Fairbanks), Ellesmere-North Greenland (Alert and Thule) and the Atlantic fringe (Narsarssuak and Keflavik). Zonal winds decrease rapidly above tropopause level, in summer in an almost linear fashion.

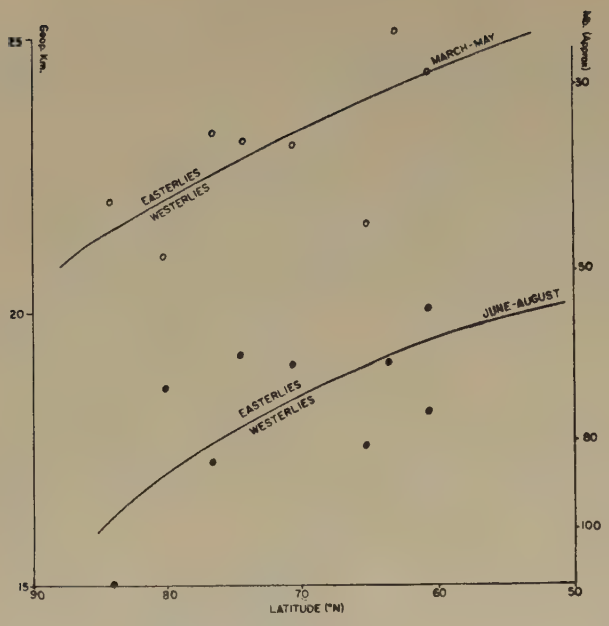

Figure 5. The zero-level of zonal mean flow in spring and summer, based on all available Western Hemisphere stations north of 60°. Circles are values for spring, dots for summer. Curves interpolated by eye

Fairbanks), the Ellesmere-North Greenland area (Alert and Thule) and the Atlantic fringe (Narsarssuak and Keflavik).

The profiles of zonal wind show a steady increase to the 10 km level at most stations; the maximum is actually at the mean tropopause (about 9·5 km at the Pole, 11·0 km in latitude 50°N), so that the maximum on the profiles, which are based on resolutions at 3 km intervals, varies slightly. In the stratosphere the zonal wind decreases almost linearly with height. In spring the easterlies are entered at 23-25 km, but in summer the zero-level for zonal wind is much lower. Fig. 5 shows the zero-level as a function of latitude, based on approximate computations from all available stations in the Western Hemisphere; it may be compared with a similar diagram for the British Isles by Scrase (1951). The base of the easterlies thus rises with distance from the Pole, reaching a maximum over the mid-latitude jet axis. In summer this lies just south of the 50° parallel and near the 50 mb surface. The negative shears of the zonal wind profiles of Fig. 4 correspond to very small

TABLE 2. MEAN VERTICAL SHEARS ($\Delta\bar{u}/\Delta z$) OF ZONAL WIND COMPONENTS, WITH CORRESPONDING LAYER-MEAN COMPUTED TEMPERATURE GRADIENTS ($\partial\bar{T}/\partial y$)

Station	Lat. °N	13 to 20 km		20 to 26 km	
		$\Delta\bar{u}/\Delta z$** (m sec^{-1} km^{-1})	$\partial\bar{T}/\partial y$* (deg C (10^3 km)$^{-1}$)	$\Delta\bar{u}/\Delta z$** (m sec^{-1} km^{-1})	$\partial\bar{T}/\partial y$* (deg C (10^3 km)$^{-1}$)
Alert	82½	− 0·15	+ 0·53	− 0·15	+ 0·52
Thule	76½	− 0·51	+ 1·71	− 0·27	+ 0·96
Barrow	71½	− 0·69	+ 2·21	− 0·49	+ 1·72
Fairbanks	65	− 0·67	+ 2·27	− 0·75	+ 2·33
Keflavik	64	− 0·82	+ 2·51	− 0·62	+ 1·90
Narsarssuak	61	− 0·96	+ 2·84	− 0·97	+ 2·88

* positive for equatorward gradients

** negative for decrease of W to E component with height

layer-mean temperature gradients, as shown in Table 2. The equatorward temperature gradient seems to be largely independent of height, and is everywhere less than 3°C/1,000 km. It increases outward from the Pole. In the lower stratosphere south of 60°N it rises to much higher values ($\geqslant$ 6°C/1,000 km) north of the tropical tropopause cold zone at 16 km.

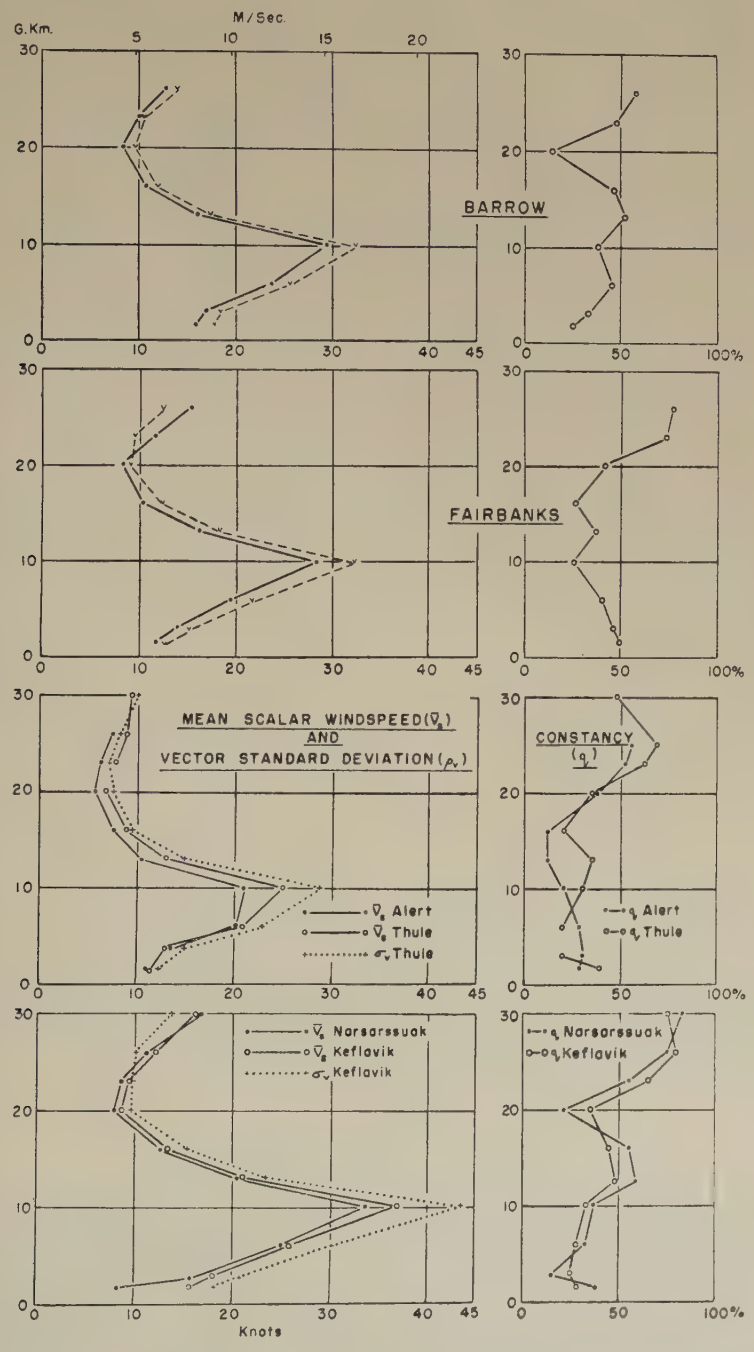

Figure 6. Vertical profiles of scalar mean windspeed V_s, and standard vector deviation (σ_1), for summer (June to August) at the same stations as in Fig. 4. Profiles of constancy 100 $V_s V_s^{-1}$ are given on the right.

The spring and summer profiles are very similar, and are strikingly different from those of autumn and winter, when zonal components remain constant or even increase above the tropopause north of about 65°N, as the polar-night vortex is entered (Hare and Orvig 1958). This implies that the breakdown of the polar-night vortex must take place early in spring : otherwise it would be impossible for spring statistics to show resultant easterlies. In actual practice it has been shown (Godson 1959; Hare 1960) that the breakdown over North America normally occurs before mid-March, but may be delayed over Europe and Asia, towards which the dying vortex is often displaced.

The maps for the summer season (Figs. 2-3) further illustrate the reversal with height. Fig. 2, for the lower stratosphere, clearly indicates that the Ferrel vortex is centred north of the Canadian Islands or over Ellesmere Island at both levels, though Heastie's (1955) reconstruction for 100 mb (16 km) implies that it drifts away from the Pole near its upper limit. The resultant flow is sluggish, and the discrepancy is of little importance. The vectors show the Alaskan ridge in 120°-130°W and the trough in about 50°W. These standing perturbations are, of course, present at upper tropospheric levels also. The middle stratospheric map (Fig. 3) shows the easterlies to be almost zonal; an appreciable north component at Resolute indicates a possible ridge in about 120°W, but otherwise there is little evidence for anything but straight easterly flow. Daily analysis, however, suggests a minor trough in the easterlies in about 50°W, directly above the westerly trough below; it also confirms the existence of the western Canadian ridge. Speeds in the easterlies increase from 23 to 26 km, and are undoubtedly higher at 30 km and above (Crary 1953).

Fig. 6 presents profiles for summer, at the same stations, of mean scalar wind speed $\overline{V}_s$, standard vector deviation σ_v, and constancy $100\ \overline{V}_v\ \overline{V}_s^{-1}$. There is a strong maximum of speed at the tropopause, followed by a rapid, non-linear decrease to a sharp minimum at 20 km (50 mb) at all stations. The 50 mb map is thus usually one of indefinite circulation in summer, and is hardly worthy of analysis. Constancy is also a minimum at this level, or just below it. This association between the minimum of scalar wind speed and the 20 km level holds good even at stations where the zero-level for zonal flow (Fig. 5) is well below this height.

The seasonal rhythm of mean temperature is the remaining climatological feature of interest. Fig. 7 shows the annual course of mean temperature at Alert (82½°N), for the years 1950-53 (Canada, Department of Transport, Meteorological Branch, 1955) for various isobaric surfaces. The stratospheric levels (200, 100 and 50 mb) are strictly in phase, and the curves are very much alike, though the amplitude increases upwards. On the other hand, the 500 mb curve is quite different, and is in phase with that for the surface; both the tropospheric curves lag two to three weeks behind those for the stratosphere. The tendency for the stratosphere to react thermally as a unit right down to the tropopause is also true of the impulsive temperature changes of the polar-night vortex (Hare 1960). The ' shelf ' in February, March and April is a remarkable feature of spring on the American side of the Arctic. It is related to the sudden warming phenomena that usually begin in February. It is absent from the contemporary records of stations in Russia, and is barely detectable over Alaska. It is too early to say whether a longer period of record will remove the anomaly.

4. Synoptic illustrations

The mean structure just discussed is a fair representation of the stratospheric flow from May until mid-August. Though there are wide daily variations in the circulation, a high degree of homogeneity reveals itself when a long series of daily analyses is examined. The characteristic structure will be illustrated here by means of charts and cross-sections drawn from the summer of 1958, during which the writer personally maintained the historical chart series being accumulated by the Arctic Meteorology Research Group.

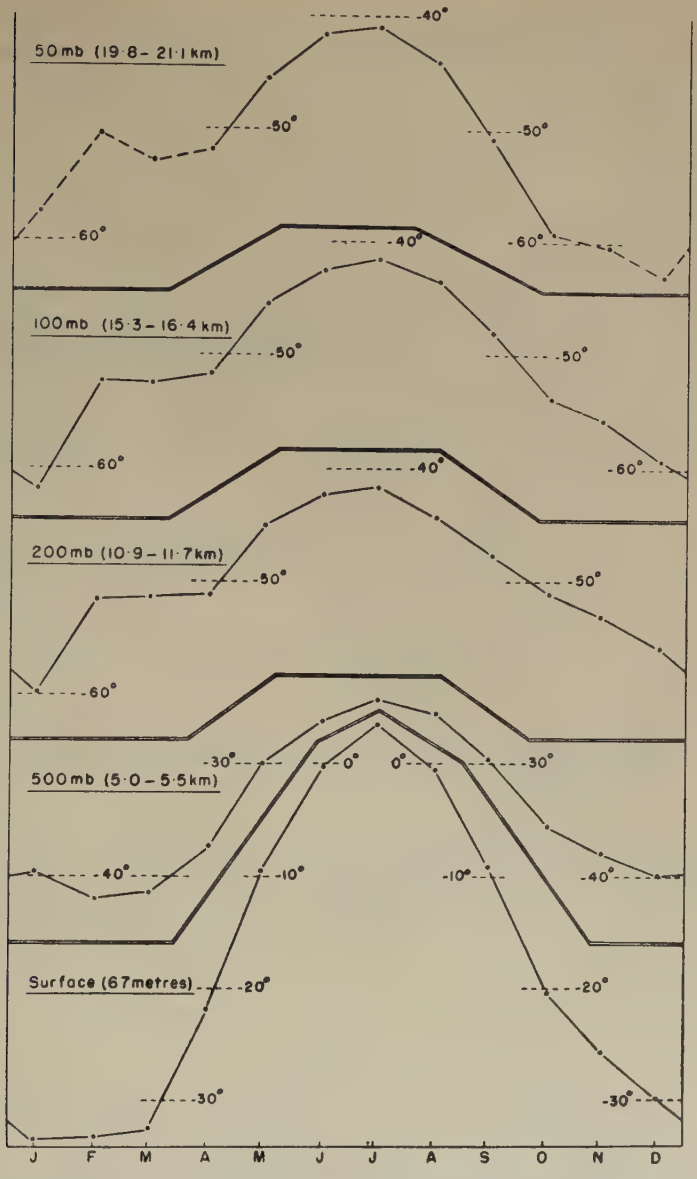

Figure 7. The annual course of mean temperature (°C) at various standard levels, Alert (82½°N). The stratospheric levels are strictly in phase, and all display a conspicuous ' shelf ' in spring associated with the sudden warming phenomenon. The 50 mb profile is weighted towards high temperatures by balloon losses in winter months.

(*a*) *The lower stratosphere* (*tropopause to 20 km*)

The lower stratospheric circulation is dominated throughout the summer by the Ferrel westerly vortex, whose upper half is within the stratosphere. The jet core of the vortex, associated with the break between polar and tropical tropopauses, is located at about 225 mb on the 80°W meridian (Fig. 8), and is not far from this level in any longitude. It oscillates through about 15 degrees of latitude. The centre of the vortex is, of course, close to the Pole.

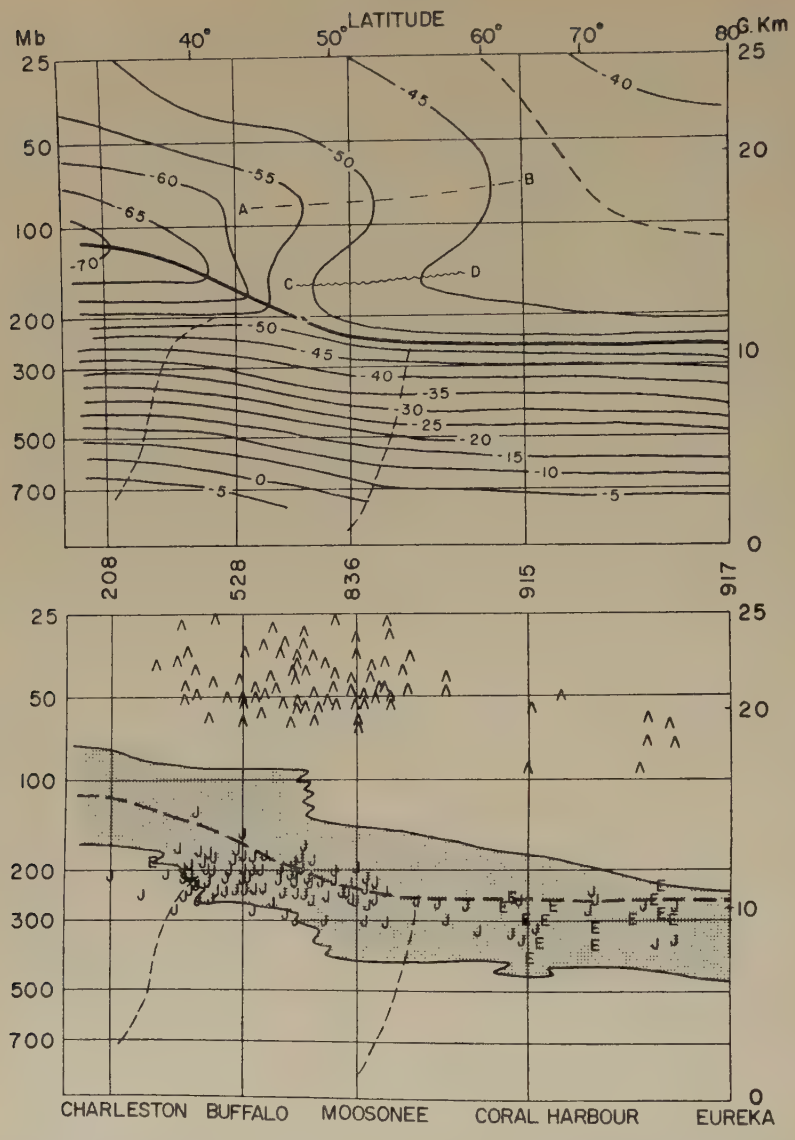

Figure 8. Mean cross-sections along 80°W from Charleston (208) through Buffalo (528), Moosonee (836), Coral Harbour (915) to Eureka (917), 1 June to 14 August 1958. Upper diagram gives mean temperature: heavy curve is locus of minimum temperature. A-B is cold belt extending tropical tropopause layer; C-D is warm belt above Ferrel jet. Lower diagram gives site of all westerly (J) and easterly (E) jet cores with speeds $\geqslant 40$ kt ($\simeq 20$ m sec^{-1}) from daily cross-sections; V's show the crest of all westerly jets (zero isotachs). Stippled area is zone of tropopause fluctuations.

Fig. 8 presents a summary of conditions along the 80°W meridian between 1 June and 14 August 1958. This period was chosen because no detectable seasonal variation of temperature occurred at levels above 500 mb, and statistical analysis was therefore easy. Daily cross-sections were prepared along the meridian up to 25 km, and were averaged in overlapping 10-day periods. Fig. 8 is the final average for the entire 75-day period. A selection of the rawinsonde stations available is shown for location purposes.

The upper diagram of Fig. 8 shows the mean thermal field. The heavy black curve is the level of minimum temperature. It corresponds to the mean arctic tropopause north of Moosonee (836) and to the mean tropical tropopause south of Charleston (208). In

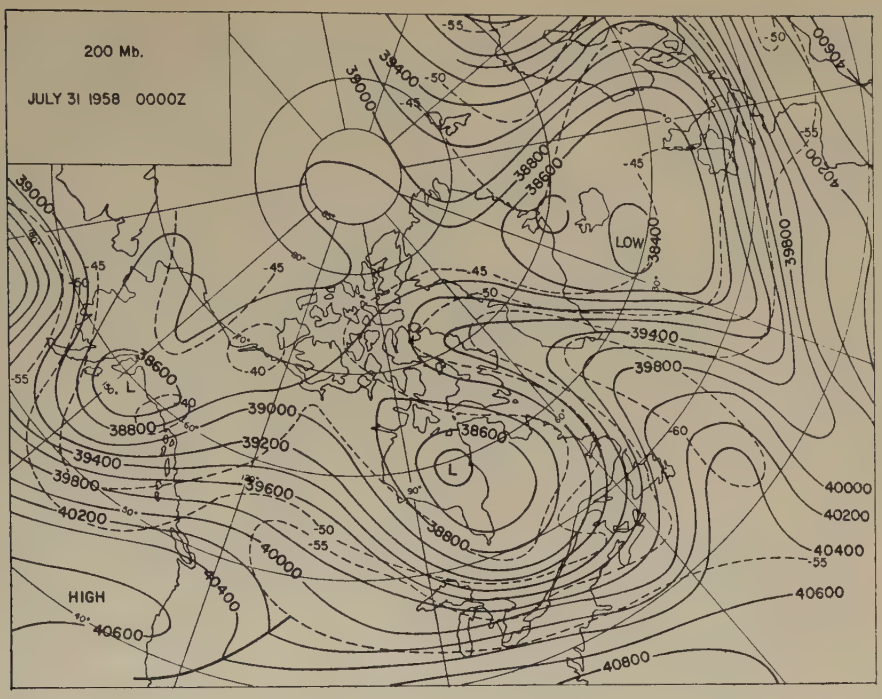

Figure 9. 200 mb map for 0000 GCT, 31 July 1948. (Heights in geopotential feet, at 200 ft contour intervals). Temperatures (°C), 5° intervals. Note that the height (i.e., motion) and thermal fields are in antiphase.

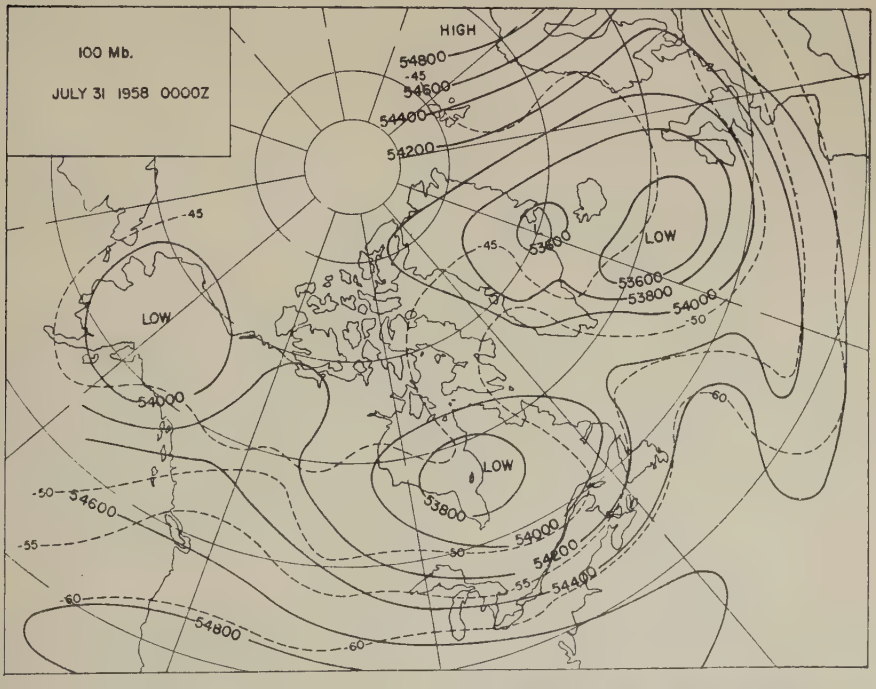

Figure 10. 100 mb map for 0000 GCT, 31 July 1958, uniform with Fig. 9. Note reduced amplitudes and stream velocities of Ferrel waves, as compared with the 200 mb flow.

between, where tropopause behaviour is complex, it is a composite curve. The baroclinicity of the troposphere is concentrated into about 20 degrees of latitude (outlined by dashed lines) centred on latitude 46°N. The characteristic structures of the stratosphere are the warm belt C-D, associated with the vertical circulations above the jet cores, and the cold belt A-B at 70-85 mb, which continues poleward the cold tropical tropopause layer, and which is detectable in nearly all daily cross-sections to about 65°N. The warmth of the upper levels in high latitudes, and the uniform equatorward temperature gradient above 100mb, are both quasi-permanent features of the structure.

On the lower diagram of Fig. 8 are superimposed (a) the position of all westerly and easterly jet cores with zonal speed $\geqslant 40$ kt ($\simeq 20$ m sec^{-1}); (b) the crest of the zero isotach of all such westerly jets; (c) envelope curves showing the upper and lower limits of tropopause fluctuations. Because of the presence of multiple tropopauses, these envelope curves locally intersect the vertical more than once. For convenience, the minimum temperature level and the limits of the tropospheric baroclinicity are reproduced from the upper diagram. The association of westerly jets with the baroclinic zone is, of course, inevitable. Much more striking is the clustering of jet crests (zero isotachs) between 18·5 and 25·5 km. On this meridian, the following generalizations hold good for 1958 :

(i) no organized motion system was observed within the lower stratosphere beyond those that extended upwards from velocity maxima near the tropopause;

(ii) the Ferrel westerlies extended upwards to a level averaging 21 km, the centre of gravity of the crests being about two degrees north of the mean jet core;

(iii) the vertical extent of the Ferrel westerlies was not only conservative in time, but seemed largely independent of both the strength of the jet core and the width of the current.

These statements are based on detailed analysis for a single summer and a single meridian, but they are believed generally valid for the Western Hemisphere in most summers. Evidence is presented later, however, that the depth of the westerlies varies slightly with longitude. In particular these generalizations are unlikely to be valid over central and eastern Asia, where the warmth of the Tibetan plateau forces the isobaric surfaces upwards to a unique extent (Flohn 1957).

The jet cores and crests shown north of the baroclinic zone represent for the most part cold-lows or obtuse anticyclonic waves of the type shown in Fig. 9. These systems vanish well below 20 km in most instances.

Figs. 9-10 give 200 mb and 100 mb charts for 0000 GCT, 31 July 1958. A cross-section for the 80°W meridian for the same time forms Fig. 11. The two charts are closely similar, though the 100 mb map (Fig. 10) has gradients only about half those at 200 mb. The decrease in intensity is associated with the strong baroclinicity of the well-developed Ferrel waves. At both levels, but especially at 200 mb, the ridges are cold and troughs warm, the thermal field being in virtual antiphase with the topography. The eastward progression of these strong thermal systems is governed by the wave motion; the wind blows through the isotherms. It is well known (Kochanski 1954; Sawyer 1951) that vertical motion – descent in the troughs, ascent over the ridges – is responsible for them. The motion of these perturbations produces a high variability of temperature in the lower stratosphere. Fig. 12 shows, for the period 1 June-14 August 1958 (corresponding to Fig. 8) the standard deviation of daily temperature on the 80°W meridian. The high values immediately above the arctic tropopause are very striking, and follow directly from the motion of the Ferrel systems.

The cross-section (Fig. 11) for 31 July 1958 is very typical. The baroclinicity of the troposphere is concentrated into a single frontal zone extending to the tropopause, with an associated jet at 225 mb of about 115 kt ($\simeq 55$ m sec^{-1}). The crest of the westerlies is at about 30 mb (23 km). The back-bent cold ridge (Fig. 9) has strongly supergeostrophic flow on either side of Hall Lake (081) extending to about 110 mb. A noteworthy feature

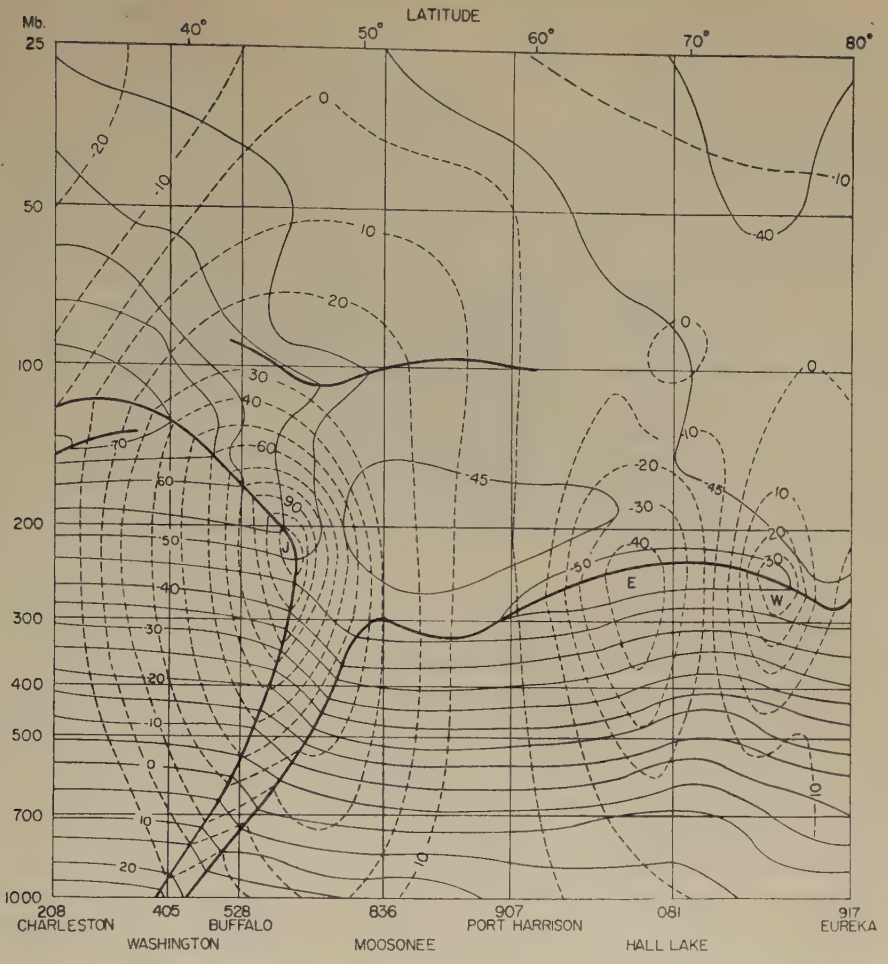

Figure 11. Cross-section along 80°W from Charleston (208) to Alert (082), 0000 GCT, 31 July 1958, corresponding to Figs. 9 and 10. Tropopauses and outer surfaces of frontal zone in heavy lines. Isotherms (°C) thin continuous lines. Isotachs (based on observed winds extrapolated where necessary by thermal wind equation) dashed (at 10 kt intervals). Note crest of zero-isotach of Ferrel westerlies at about 30 mb betweeen 528 and 836.

of Fig. 11 is the coldness of the arctic tropopause. This is largely the consequence of the anticyclonic flow over Hall Lake, such flow being normally accompanied by a high cold tropopause. It is normally true, however, that the arctic tropopause, especially when circulation is sluggish, is about 10°C colder than the basal stratospheric temperatures, an effect also reported from Russian data by Gaigerov (1957).

(*b*) *The middle stratosphere (20 to 30 km)*

The 50 mb map in summer is always weak and disorganized, since it is close to the zero-level of zonal circulation (Fig. 5). From May until late August, however, the 25 mb chart (the highest for which analysis on a routine basis is feasible) lies fully within the middle stratospheric easterlies. Hence this level has been made standard for daily analysis.

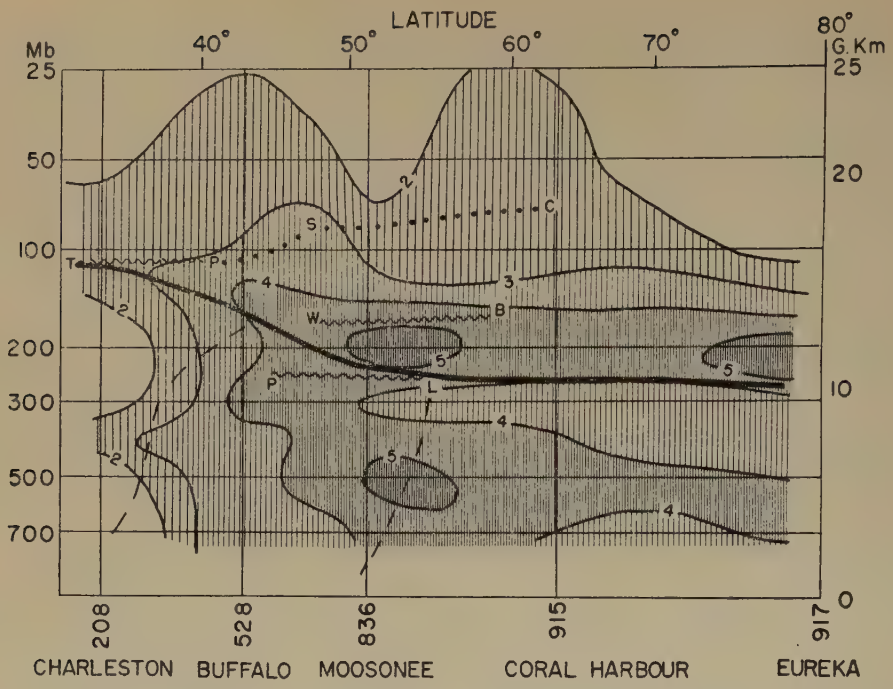

Figure 12. Standard deviation of temperatures (°C) along 80°W between Charleston (208) and Eureka (917), 1 June to 14 August 1958. Locus of temperature minimum and limits of tropospheric baroclinic belt transferred from Fig. 8. T-P is mean tropical tropopause; P-S-C is axis of cold belt extending towards the Pole. P-L is the mean arctic tropopause in zone of overlapping tropopauses. W-B is warm belt associated with jet circulation.

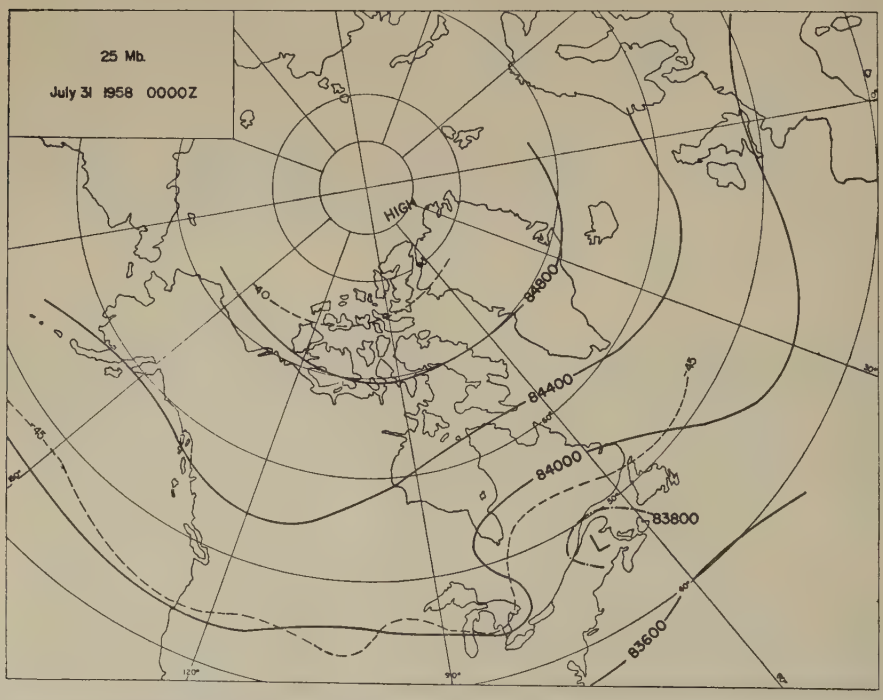

Figure 13. 25 mb map for 0000 GCT for 31 July 1958 (compare Figs. 9-11). Contour interval reduced to 400 ft because of reduced certainty of analysis. Note weak cyclonic flow near the Gulf of St. Lawrence.

Fig. 13 shows the 25 mb map for 31 July 1958, 0000 GCT, corresponding to the 200 mb and 100 mb maps discussed above. It shows slack, easterly flow round the flank of a large anticyclone near the Pole. Every map for the summer of 1958 closely resembled Fig. 13. Rawin ascents north of 60°N rarely show a west component in the summer months; at Keflavik (64°N), for example, of the 785 available ascents at 26 km between 1944 and 1956, only 8 per cent had a west component. Even Thule (77°N), much nearer the Pole, had only 14 per cent among 609 ascents. Climatology thus reinforces the suggestion that there exists (1) almost continuous control of the circulation by an anticyclonic centre that rarely oscillates far from the Pole, and (2) a virtual absence of closed cyclonic centres, which necessarily require west components on their southern perimeters. No trace could be found of the closed highs and lows described for these levels by Kochanski and Wasko (1956). The quasi-barotropic nature of the flow is consistent with the absence of disturbances. The extremely small dispersion of observed temperature at these levels (Fig. 12) confirms that vertical motion is also very small.

These generalizations hold good for regions north of 60°N. Further south the picture is more complex. Immediately above the Ferrel westerlies there is usually a wide area of indeterminate flow separating the arctic easterlies from the more vigorous easterlies of the sub-tropics (Kochanski 1955; see also Fig. 11 of this paper). In certain longitudes, the Ferrel westerlies occasionally extend upwards so that they rise above the 25 mb surface. Such displacements appear as feeble closed lows on the charts for that level An example appears over Eastern Canada on Fig. 13.

These upward displacements of the Ferrel westerlies are common over eastern Labrador, Newfoundland and the Labrador Sea, east of the mean trough in the Ferrel system. Daily analysis suggests, as was the case on Fig. 13, that such oscillations tend to occur east of velocity maxima on the Ferrel jet stream. The equation of horizontal motion requires cross-contour flow towards the higher pressure in such situations, i.e, towards the right. Such was the case at 200 mb and 100 mb on 31 July 1958, when the observed winds at Goose Bay (816), Stephenville (815) and Weather Ship Baker (56½°N, 52°W) were all directed up the isobaric slope at angles of 20° to 30° to the contour. Apparently such ageostrophic flow tends to be associated with upward oscillations of the Ferrel crest.

5. The end of summer

The structure described above collapses annually in late August. Cooling of the middle stratosphere begins early in that month, and the cooling wave spreads rapidly downwards. Tropospheric cooling lags by two or three weeks. The two cooling waves overlap near the tropopause, where, however, the annual temperature range is a minimum, of the order of 15°C (Wexler 1958). Their combined effect is to lower the heights of all stratospheric isobaric surfaces, with maximum falls near the Pole. The easterlies are driven upwards; the zero-level passes through the 25 mb level near the end of August, when the map is completely featureless. By mid-September the greater part of the stratosphere, except for the cold tropical tropopause layer near 16 km, is isothermal and barotropic. Hence the Ferrel westerlies extend upwards throughout the layer below 30 km, appearing on the 25 mb map in high latitudes in early September and spreading rapidly equatorwards. This monsoon-like reversal repeats itself annually, with far less interannual variability than that which attends the start of summer.

6. Conclusions

This paper has been directed at a description of the characteristic circulation of that part of the stratosphere lying over and north of the Ferrel westerlies in summer. Limitations of observational methods and analytical techniques made it necessary to confine the treatment to the layers below 30 km. The approach has been empirical, through the media of climatological and synoptic analysis.

The lower stratosphere's behaviour appears to be wholly governed by the wave systems of the Ferrel westerly vortex or by closed systems near the Pole, wholly within the vortex. Both the basic westerly current and the wave trains are progressively weakened with height above the tropopause by the baroclinic structure, in which temperature and contour height are in antiphase. Most of the energy of the travelling waves is gone at 15 km, and the entire vortex is eliminated below a zero-level that slopes upwards from the Pole (15 km in summer) to about latitude 50°N (20 km). Vertical motion associated with the Ferrel waves (and independent systems within the Ferrel ring) cause a highly disturbed temperature regime in the layers below 16 km, with maximum variability just above the arctic tropopause. On the charts and cross-sections analysed by the writer there is not a trace of any wholly stratospheric motion system below the easterlies.

The middle stratospheric easterlies flow almost without disturbance round an anticyclone near the Pole. At the 25 mb level (25 km) closed cyclonic systems are confined to the belt above the Ferrel westerlies, whose crest may oscillate upwards through the surface, especially east of the trough over eastern North America. Temperatures are very steady, pointing to an absence of vertical motion. The easterly regime lasts from May until mid-August.

This account has been based on a restricted sector of the hemisphere, owing to lack of data over the Soviet Arctic, and it cannot safely be assumed that identical conditions hold beyond the Pole. Kochanski (1958) has recently suggested that impulsive temperature changes may affect the stratosphere in summer over Siberia, akin to those common in winter over North America. On the other hand Gaigerov's (1957) account of conditions over the polar pack is fully consistent with the present results, and it is difficult to visualize powerful disturbances that have escaped the observational network in the Western Hemisphere so completely as to vanish from the climatological summaries. Circumpolar analysis at 25 or 20 mb will be necessary to solve the question.

Acknowledgments

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Mean meridional distributions of ozone in different seasons calculated from *umkehr* observations and probable vertical transport mechanisms*

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(Manuscript received 21 September 1959; in revised form 22 December 1959)

SUMMARY

The paper contains a study of the mean meridional distributions of ozone in different seasons. The vertical distributions of ozone at a number of stations at different latitudes in the Northern Hemisphere were worked out from the intensities of light scattered from the zenith sky on individual clear days using method B, i.e., the method of curve fitting, with suitable correction for secondary scattering. Diagrams of vertical distribution corresponding (1) to high ozone amounts (frequent in winter and spring) and (2) to low ozone amounts (frequent in summer and autumn) are shown. Mean distribution diagrams are also given for the months of March, July and November. An important feature of the distribution is the strong increase in ozone amount in March below 18 km when the latitude increases beyond 30°. The results are discussed in relation to the idea of a poleward flow of air from the lower stratosphere and the neighbourhood of the equatorial tropopause, as suggested by Dobson and Brewer from frost-point measurements over the United Kingdom. The existence of a stratospheric warm air pool over the middle latitude in winter and spring as revealed by sounding-balloon ascents is confirmatory evidence of this. The influence of the pool of cold air which exists in winter above 20 km in the stratosphere of the dark polar night is also considered. It is suggested that in winter and spring there is an ozone-regenerating cycle due to meridional circulation in the stratosphere of tropical and middle latitudes, which carries air from the lower equatorial stratosphere to lower levels in the stratosphere of middle latitudes and puts back some of this accumulated air to higher levels in the equatorial stratosphere above 25 km. The more vigorous this circulation, the greater will be the rate of ozone storage in the lower stratosphere of extra-tropical latitudes.

1. INTRODUCTION

In a paper by Kulkarni, Angreji and Ramanathan (1959) on measurements of atmospheric ozone in Kashmir, the data of vertical distributions calculated from observations on the *umkehr* effect, dividing the atmosphere into 6 km layers and allowing for secondary scattering, have been given. In this paper, the study is extended to higher latitudes, using *umkehr* data of individual days with varying amounts of ozone, from Tateno in Japan, from Arosa and Tromsö in Europe, and Alaska in North America. The data of all these stations together with those of Indian stations are integrated to give pictures of the mean vertical meridional distributions of ozone in March, July and November. March and November are the months when the total ozone amount is maximum and minimum respectively in the Northern Hemisphere. In the light of these distributions, a stratospheric circulation which can lead to the annual regeneration of ozone in the stratosphere and its destruction in the troposphere is suggested.

NOTE : All ozone amounts in this paper are expressed in terms of Nye and Choong's values of ozone absorption coefficients. To express them in terms of Vigroux's coefficients of absorption, they should by multiplied by 1·33.

2. VERTICAL OZONE DISTRIBUTION DATA FROM INDIAN STATIONS

Vertical distributions of ozone by the *umkehr* method are available from observations over Kodaikanal, Poona, Abu and Delhi (Karandikar 1952; Ramanathan and Kulkarni 1953; Degaonkar 1955). In particular, a large number of observations have been made over Abu, many of them on a series of consecutive days. The analysis of these observations has shown that as the ozone amount increases, most of the increase takes place in the 18-27 km layer, a conclusion arrived at many years ago by Karandikar and Ramanathan (1949).

* Presented at the International Ozone Symposium held at Oxford in July 1959.

In his address on atmospheric ozone at the Rome Assembly of the International Association of Meteorology (1956), Ramanathan drew attention to the sharp increase in atmospheric ozone at about 30°N as one moved poleward from tropical to middle latitudes and suggested that the increase might be connected with the tropopause break associated with the sub-tropical jet stream.

The starting of an ozone station in Kashmir at about 34°N in 1955, gave an opportunity for the study of the vertical distributions associated with changes of ozone in that region. Vertical distributions have been calculated from *umkehr* observations at Gulmarg or Srinagar on days with ozone amounts varying from 0·160 cm to 0·230 cm, dividing the atmosphere into 6 km layers and allowing for secondary scattering as described by Ramanathan and Dave (1957). The following table shows the percentages of ozone amounts in successive 6 km layers for typical small and large ozone amounts.

TABLE 1. PERCENTAGES OF OZONE AMOUNTS IN SUCCESSIVE 6 KM LAYERS OVER KASHMIR FOR TYPICAL SMALL AND LARGE OZONE AMOUNTS

		0-6	6-12	12-18	18-24	24-30	30-36	above 36 km
Small amount O_3	0·160 cm	4	4	13	14	35	20	9 per cent
Large amount O_3	0·230 cm	5	**12**	11	**22**	34	**10**	7 per cent

The Kashmir distributions show :

(1). Above 36 km, there is no significant change in ozone content.

(2). With increase in total ozone amount, there is in general an increase in ozone in the 6-12 and 18-24 km layers and a decrease in the 30-36 km layer. When the ozone amount exceeds 0·190 cm, the increase in ozone content in the 6-12 km layer becomes very pronounced. The amount in the 6-12 km layer is sometimes even larger than in 12-18 km.

The occasional appearance of more ozone in the 6-12 km layer than in the 12-18 km layer over Srinagar is significant. Srinagar being situated in a region where the occurrence of double tropopause is frequent in winter and spring, it may be expected that occasionally the cold air adjacent to the equatorial tropopause will spread out into the stratosphere of middle latitudes, and the ozone-rich air of the lower stratosphere of middle latitudes be drawn into the troposphere of the lower latitudes, roughly following the isentropes.

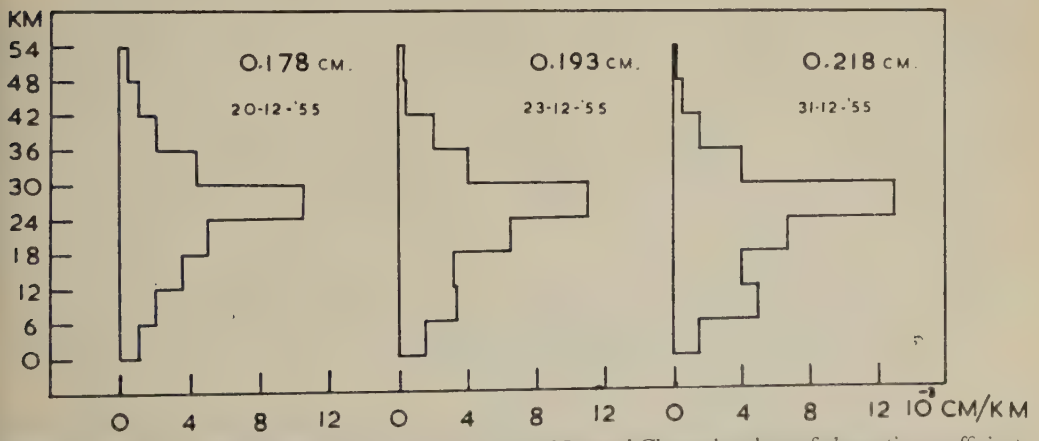

Figure 1. Srinagar – vertical distribution of ozone, using Nye and Choong's values of absorption coefficients of O_3.

3. Vertical distributions of ozone over Tateno (35°·7 N, 139°·7 E)

Owing to the kindness of the Japan Meteorological Agency (1958/9) we have been receiving their upper-air and ozone data, and have calculated from their *umkehr* observations the vertical distributions over Tateno on a number of days in 1958. The latitude of Tateno near Tokyo is only 2° higher than that of Srinagar, yet the ozone amounts over Tateno vary over a much larger range of values, from about 0·180 cm to over 0·360 cm. When the ozone amounts exceed those usually found in India, we have used the following corrections ΔN to the values of N to allow for multiple scattering.

TABLE 2. Correction for multiple scattering to observed values of N on C wavelengths

	Zenith angle of sun							
Total ozone amount	60°	70°	75°	80°	84°	86·5°	88°	90°
0·230 cm	0	1·0	2·0	4·0	6·0	6·0	6·0	6·0
0·230-0·310 cm	0	3·0	4·5	6·0	6·0	6·0	6·0	6·0
0·310 cm	0	3·5	5·0	7·0	6·5	6·0	6·0	6·0

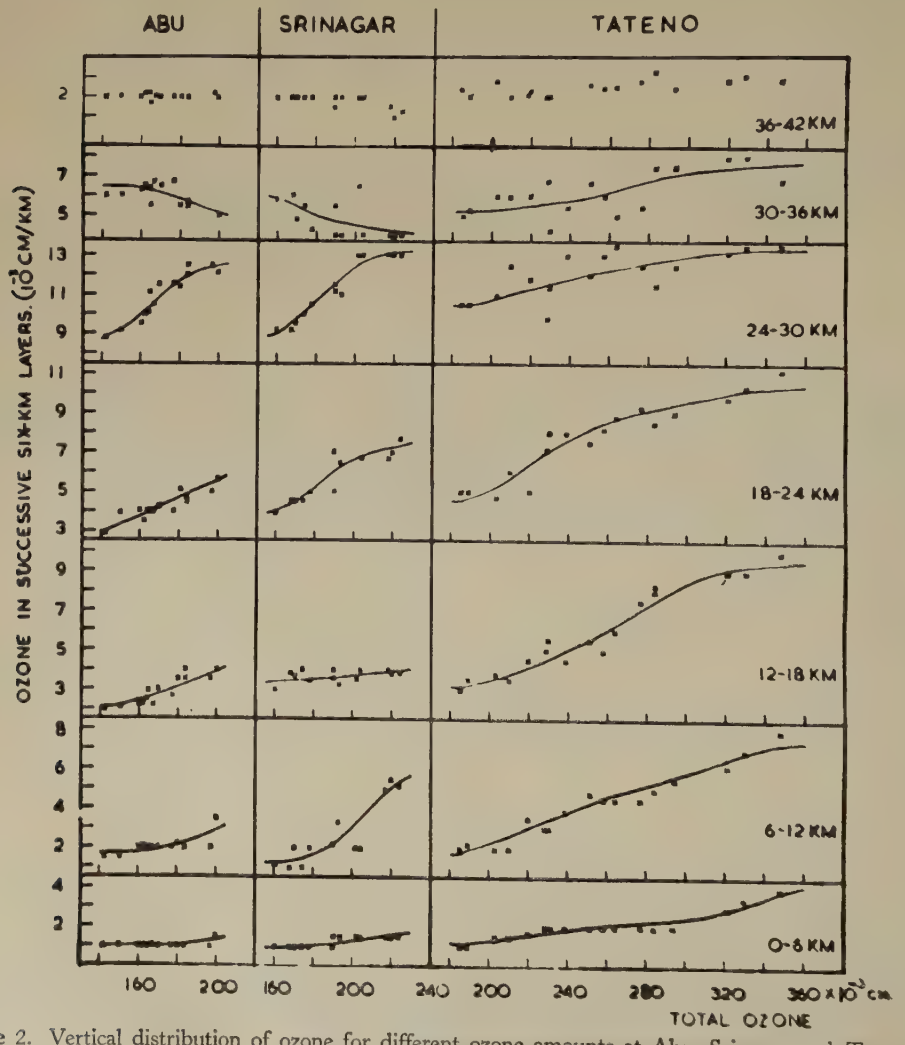

Figure 2. Vertical distribution of ozone for different ozone amounts at Abu, Srinagar and Tateno (with Choong's α).

The corrections were estimated from Dr. Walton's calculations (1953) of secondary scattering and empirical values based on *umkehr* observations on different wave-lengths made in India. Compared with the vertical distributions found over India, the distributions found over Tateno show the following features :

(1). In all layers up to 36 km, there is, in general, an increase of ozone with increasing ozone amount. This may be compared with observations over Srinagar where, with increase of ozone amount, there is a decrease of ozone in the 30-36 km layer and only a small change in 0-6 and 12-18 km layers. There is very little variation above 36 km.

(2). The ozone amount in 12-18 km increases rapidly when the total ozone increases from 0·220 cm to about 0·300 cm.

(3). When the total ozone exceeds 0·300 cm, the amounts at levels above 12 km tend to reach saturation, but the increase continues at lower levels.

(4). At Tateno in winter and spring, double and multiple tropopauses are frequent. On occasions when the ozone amount is high, the vertical distributions over Tateno correspond to those of higher latitudes with the same high ozone amount. Perhaps owing to its location to the south-east of the Siberian high-level low-pressure area in winter, the winter character of a higher latitude is sometimes imported to as low a latitude as 36°N.

A further study of the variations in the 30-36 km layer by analysing the observational data of other stations is desirable in order to elucidate the reason for the difference between Tateno and the Indian stations.

4. Meridional distribution of ozone in spring and autumn

For the understanding of the transport mechanism of ozone, one would like to study the varying distributions of ozone from day to day. If synoptic distributions could be obtained, even over limited areas, from the I.G.Y. observations, they would be of great value. In anticipation of such analysis, it was felt desirable to compute vertical distributions from the available *umkehr* observations made at different places, but following an identical procedure for computation. Vertical distributions have thus been calculated from observations on individual days at the following places :

Kodaikanal (10°·2 N, 77°·5 E),	Poona (18°·5 N, 73°·9 E),	Mt. Abu (24°·6 N, 72°·7 E),
Delhi (28°·5 N, 77°·2 E),	Srinagar (34°·1 N, 74°·9 E),	Tateno (35°·7 N, 139°·7 E),
Arosa (46°·8 N, 9°·7 E),	Tromsö (69°·7 N, 18°·9 E),	Alaska (64°·8 N, 147°·9 W).

In computing the vertical distributions, the tables given in the *I.G.Y. Instruction Manual* have been used. When the ozone amounts were higher than 0·230 cm, two different corrections for multiple scattering were used for ozone amounts 0·230 to 0·310 cm and for amounts greater than 0·310 cm respectively.

Dr. Dütsch in a paper on ' Vertical distributions from *umkehr* observations ' (*Arch. Met. Geophys. und Biokl.*, **11**, p. 240, 1959) has given the distributions of ozone calculated by him on a large number of days from the Arosa *umkehr* data. Two of these days, 9 August 1956 and 13 February 1956, are common to our Tables. Dr. Dütsch's results differ from ours on 13 February 1956 with significantly larger ozone amounts in the layers 6-24 km. The difference is less marked on 9 August 1956. The differences arise from two causes :

(1) Dr. Dütsch has not used the observed value of total ozone but has obtained it by integrating the amounts in different layers as calculated from the *umkehr* observations. There is a difference between the observed total ozone (0·278 cm) and the integrated total ozone (0·298 cm). We have, on the other hand, used in our calculations the observed total ozone with the *umkehr* curve.

(2) The corrections for multiple scattering used by Dr. Dütsch are smaller than the corrections used by us.

The differences between the results of our calculations and those of Dr. Dütsch do not affect the main conclusions of the present paper. A further examination of the effect of multiple scattering on the *umkehr* curve is in progress.

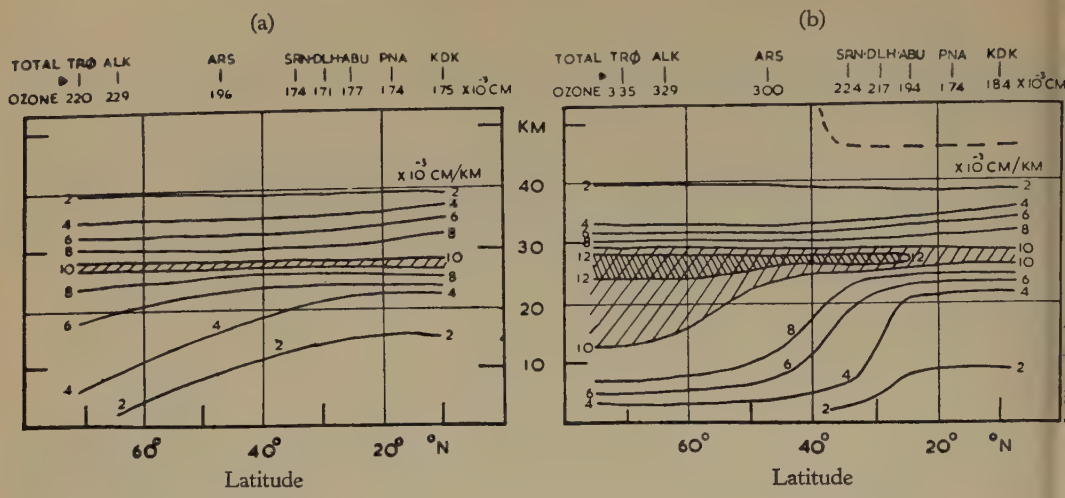

Figure 3. Vertical distribution of ozone (with Nye and Choong's α). (a) when ozone amounts are low, (b) when ozone amounts are high.

Although the same value for the total ozone amount does not mean that the vertical distribution is the same, it is found that there are certain general changes in the vertical distribution which go with changes in the total ozone. Taking into consideration the fact that in middle and high latitudes the ozone amount is generally high in January to April, and low in September to November, mean meridional distributions of ozone were prepared (a) when the ozone amounts are low and (b) when ozone amounts are high.

Figure 3 shows vertical distributions corresponding to winter-spring and late-summer and autumn and the following points are of interest.

(1) Throughout the year, there is more ozone in the lower layers in latitudes higher than 30°N than in lower latitudes.

(2) The increase of ozone in the lower levels with increase in latitude is gradual in summer, whereas in winter, the rise is steep at 30°N. This amplifies Ramanathan's picture (1956) of the latitudinal distribution of total ozone with its steep rise north of 30°N.

(3) There is a tendency for the ozone in the 30-36 km layer to decrease with latitude, and more rapidly when the ozone amounts are high. At 40 km, there is little change in ozone content with latitude or season. There is also a general tendency for the ozone amounts above 42 km in middle and high latitudes to be larger than in India. We are not sure whether this is real or due to the inaccuracy of the multiple scattering corrections for high values of ozone. These corrections for higher values of ozone amount require further examination.

Mean meridional distributions of ozone have also been calculated for each of the months March, July and November (Fig. 4), using the appropriate ozone amounts from the diagram of annual variation published by Normand (1956). Two steps of ozone variation with latitude are seen in the March diagram; presumably they correspond to the tropopause breaks associated with the sub-tropical and polar jet streams.

As the ozone amount increases from November to March, the increase takes place mainly at levels below 30 km. In March at 15 km, the ozone content changes from 4×10^{-3} cm km^{-1} at the Equator to 10×10^{-3} cm km^{-1} at 70°. At 27 km, the change from the Equator (10×10^{-3} cm km^{-1}) to 70° (12×10^{-3} cm km^{-1}) is small in all the three months. At 30-36 km in March, the ozone content decreases from the Equator (7×10^{-3} cm km^{-1}) to higher latitudes (4×10^{-3} cm km^{-1}). (See Table 3).

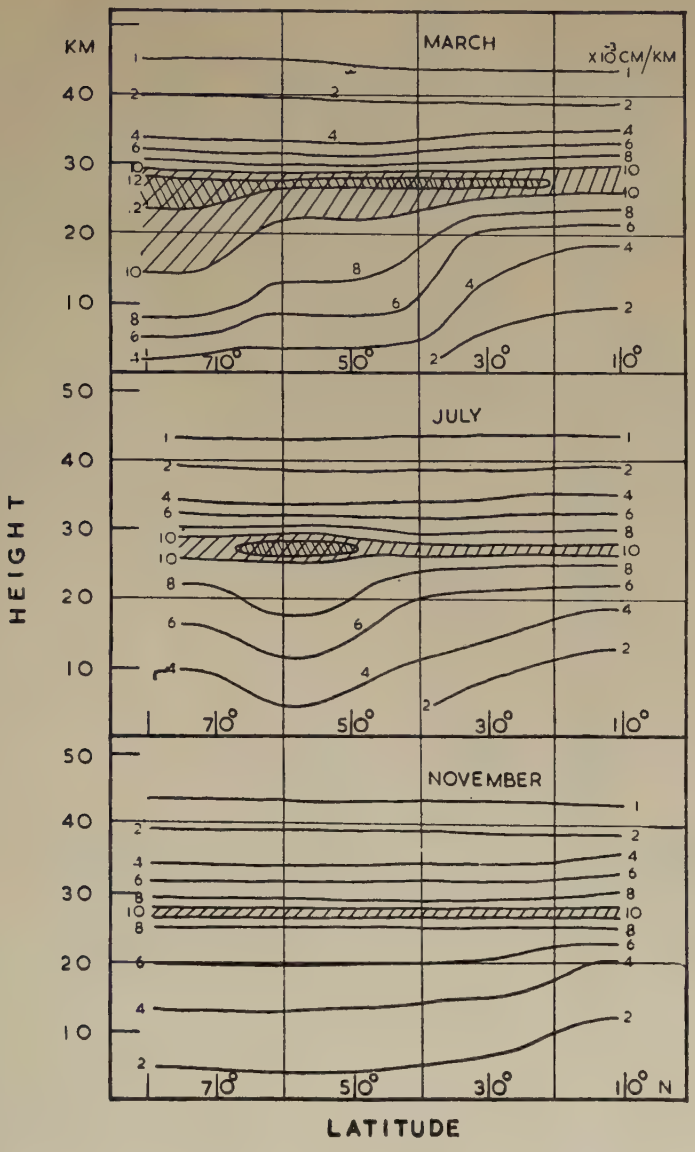

Figure 4. Mean vertical distribution of ozone in March, July and November (10^3 cm km^{-1} with Nye and Choong's α).

A mean meridional distribution is a greatly simplified picture. Not only does the distribution change from month to month; there is evidence that, in the same month, there are significant differences of distribution over different meridians. Still, even the simplified picture adds to our understanding.

5. Discussion

How is the large increase in ozone in the lower stratosphere from November to March brought about ? The rate of production of ozone by photo-chemical processes is, according to Dütsch (1956), at maximum at about 25 km for vertical incidence of sunlight, and the ozone mixing ratio increases with height up to 30-35 km and decreases thereafter.

TABLE 3. VERTICAL DISTRIBUTIONS OF ATMOSPHERIC OZONE

(Amounts calculated with Nye and Choons's values of α)

Date	Total ozone (cm)	Mean ozone amount (10^{-3} cm km^{-1}) 0-6	6-12	12-18	18-24	24-30	30-36	36-42	42-48	48-54
KODAIKANAL (10°N)										
15 February 1958	0·167	1·0	1·5	2·0	3·0	9·9	7·5	2·0	0·5	0·4
10 March 1949	0·175	1·0	1·5	2·0	3·3	10·5	8·0	2·0	0·5	0·3
16 March 1949	0·184	1·0	2·0	2·0	4·0	11 2	7·7	2·0	0·5	0·3
18 April 1958	0·187	1·0	2·0	2·0	4·2	11·5	7·5	2·0	0·6	0·4
POONA (18½°N)										
15 March 1948	0·174	1·0	2·0	2 0	3·5	10·2	7·0	2·0	0·8	0·4
MT. ABU (24½°N)										
27 November 1957	0·152	1·0	1·5	2·0	2·9	8·8	6·0	2·0	0·7	0·5
25 November 1953	0·160	1·0	1·5	2·1	3·9	9·2	6·0	2·0	0·5	0·4
23 November 1953	0·170	1·0	2·0	2·2	4·0	9·5	6·3	2·0	0·6	0·2
8 January 1958	0·172	1·0	2·0	2·2	3·5	10·0	6·5	2·2	0·8	0·4
21 March 1954	0·174	1·0	2·0	2·5	4·0	10·1	6·4	2·2	0·5	0·4
6 March 1955	0·175	1·0	2·0	2·9	4·0	11·1	5·5	1·7	0·5	0·4
3 July 1957	0·177	1·0	2·0	2·2	4·0	10·5	6·7	2·0	0·6	0·5
18 June 1957	0·180	1·0	2·0	3·0	4·2	11·5	6·5	2·0	0·6	0·6
26 January 1958	0·187	1·0	2·0	2·7	4·0	11·5	6·8	2·0	0·8	0·4
12 March 1958	0·190	1·0	2·2	3·5	5·1	11·4	5·5	2·1	0·5	0·4
26 February 1954	0·194	1·0	2·0	3·5	4·5	12·5	5·7	2·0	0·6	0·5
31 March 1955	0·194	1·0	2·0	4·0	4·7	12·0	5·5	2·0	0·6	0·5
1 April 1955	0·207	1·0	2·0	3·5	5·0	12·5	7·0	2·2	0·5	0·3
2 March 1955	0·210	1·5	3·5	4·0	5·6	12·1	5·0	2·0	0·7	0·5
NEW DELHI (28½°N)										
10 March 1955	0·170	1·0	2·0	2·9	5·0	9·0	5·7	1·8	0·5	0·4
18 March 1955	0·171	1·0	2·0	3·0	5·0	9·1	5·7	1·8	0·5	0·4
17 March 1955	0·190	1·5	2·0	3·7	5·1	10·6	5·5	2·0	0·8	0·5
4 March 1955	0·209	1·5	2·0	4·0	5·5	12·5	6·0	2·0	0·8	0·6
20 January 1947	0·217	1·5	3·0	4·5	5·5	13·0	6·0	2·0	0·5	0·2
SRINAGAR (S) OR GULMARG (G) (34°N)										
G. 23 October 1955	0·160	1·0	1·2	3·0	3·9	9·2	5·8	2·0	0·3	0·2
G. 20 October 1955	0·168	1·0	1·0	3·8	4·5	9·2	6·0	2·0	0·3	0·2
S. 11 December 1955	0·170	1·0	2·0	3·7	4·5	9·5	4·8	2·0	0·6	0·2
G. 25 October 1955	0·174	1·0	1·0	4·0	4·5	10·0	5·5	2·0	0·6	0·4
S. 20 December 1955	0·178	1·0	2·0	3·5	5·0	10·5	4·3	2·0	1·0	0·4
G. 29 October 1955	0·190	1·0	2·2	4·0	5·0	11·5	5·5	2·0	0·3	0·2
S. 23 December 1955	0·193	1·5	3·3	3·2	6·5	11·0	4·0	2·0	0·3	0·2
S. 7 February 1956	0·195	1·5	3·3	3·6	7·0	11·2	4·0	1·5	0·6	0·3
S. 5 August 1957	0·202	1·5	2·0	3·5	4·2	13·0	6·5	2·0	0·5	0·4
S. 26 December 1955	0·204	1·5	2·0	4·0	6·7	13·0	4·0	2·0	0·5	0·2
S. 31 December 1955	0·218	1·5	5·0	4·0	6·7	13·0	4·0	1·5	0·5	0·2
S. 5 February 1956	0·220	1·5	5·5	3·8	7·0	13·0	4·0	1·0	0·5	0·4
S. 16 February 1956	0·224	1·5	5·2	3·8	7·7	13·0	4·0	1·3	0·5	0·4
TATENO (36°N)										
4 November 1958	0·185	1·0	2·0	3·0	5·0	10·5	5·0	2·4	1·0	0·9
26 November 1958	0·189	1·0	2·0	3·5	5·0	10·5	5·0	2·0	1·1	0·9
29 October 1958	0·203	1·5	2·0	3·8	4·7	11·0	6·0	2·8	1·0	0·9
10 January 1958	0·210	1·5	2·0	3·5	6·0	12·5	6·0	2·0	0·8	0·6
8 October 1958	0·220	1·7	3·5	4·5	5·0	11·8	6·0	2·3	1·0	0·9
11 August 1958	0·229	2·0	3·0	5·0	7·2	9·8	6·8	2·0	1·4	1·0
19 November 1958	0·230	2·0	3·0	5·5	8·0	11·4	4·2	2·0	1·4	1·0
6 January 1958	0·239	2·0	3·9	4·5	8·0	13·0	5·4	2·1	1·0	0·8

TABLE 3 (continued)

Date	Total ozone (cm)	Mean ozone amount (10^{-3} cm km^{-1}) 0-6	6-12	12-18	18-24	24-30	30-36	36-42	42-48	48-54
		TATENO (36°N)								
21 June 1958	0·251	2·0	4·8	5·5	6·5	12·0	6·7	2·6	1·0	0·9
19 March 1958	0·258	2·0	4·5	5·0	8·2	13·0	6·0	2·5	1·0	0·9
4 January 1958	0·264	2·0	4·5	6·0	8·8	13·5	5·0	2·5	1·0	0·8
18 January 1958	0·277	2·0	4·5	7·5	9·3	12·5	5·5	2·8	1·1	0·9
31 March 1958	0·284	2·0	5·0	8·0	8·5	11·5	7·5	3·3	0·8	0·7
31 January 1959	0·294	2·0	5·5	8·3	9·0	12·3	7·5	2·4	1·0	0·9
3 March 1958	0·321	3·0	6·2	9·0	9·8	13·2	7·0	2·8	0·8	0·6
4 March 1958	0·330	3·5	7·0	9·0	10·3	13·5	7·0	3·0	0·9	0·8
30 March 1958	0·348	4·0	8·0	10·0	11·2	13·5	6·8	2·8	0·8	0·7
		AROSA (47°N)								
9 August 1956	0·196	1·5	2·5	3·6	6·0	10·5	5·0	2·0	0·8	0·8
1933 (average)	0·240	2·0	4·0	5·5	8·0	13·2	4·0	2·0	0·8	0·6
14 February 1933	0·253	2·0	4·5	5·4	8·3	13·5	5·0	2·0	0·8	0·7
3 June 1933	0·263	2·0	4·5	5·5	8·8	13·5	5·5	2·5	1·0	0·9
13 February 1956	0·278	3·0	6·0	6·1	8·5	13·3	5·0	2·5	1·0	0·9
1933 (average)	0·300	3·5	7·0	9·0	9·0	12·0	5·5	2·3	1·0	0·9
		ALASKA (65°N)								
25 October 1953	0·229	2·0	4·5	4·7	7·2	11·5	4·5	2·0	1·0	0·8
24 March 1954	0·329	4·5	8·6	9·5	11·5	12·5	4·2	2·3	0·9	0·8
		TROMSÖ (69·5°N)								
25 March 1941	0·335	3·0	9·1	11·5	11·5	12·5	4·5	2·5	0·9	0·8

The mixing of air from different levels below 30 km will, in general, increase the ozone content in the lower levels and decrease it in the higher. In the lower levels of the stratosphere of middle and high latitudes, ozone can be effectively conserved for a period of many months as the air is cold, dry, and stably stratified and most of the ozone-destroying radiation from the sun has been filtered-out by the higher layers. Dobson, Brewer and Cwilong (1946) and Brewer (1949), from measurements of frost-points in the stratosphere over England, produced evidence for a large-scale flux of dry air into the stratosphere over the United Kingdom. It was suggested that this came from near the equatorial tropopause. The large amount of data collected by Murgatroyd, Goldsmith and Hollings (1955); Helliwell, Mackenzie and Kerley (1956) show that the dryness at 14-15 km is not occasional but general. The occurrence of a warm-air pool in the stratosphere over middle latitudes, its extreme dryness corresponding to equatorial tropopause temperatures, and accumulating ozone amount during winter, all support a poleward transport of subsiding air from near the equatorial tropopause. How is the circulation completed? For the present, we can only make reasonable guesses at the character of the circulation. It will take many years before we shall have enough wind data in the stratosphere to calculate the magnitude of the meridional mass fluxes there.

Fig. 5 shows diagrams of distribution of temperature (T°C) and potential temperature (θ°K) over a sample meridian in the months of January, April, July and October. These have been put together from the work of Kochanski (1955) and Murgatroyd (1957). The January diagram shows a pool of warm air (-50 to -55°C) in the stratosphere lying between 45° and 65° and extending from the tropopause to a height of over 24 km. Considering its position and the trend of the lines of potential temperature, it appears likely that this warm pool is formed mainly by the subsidence of originally cold air from near the equatorial tropopause, and to a smaller extent from the upper cold pool in the Arctic stratosphere which is located at 25-30 km. There is much more volume of relatively cold air near the tropopause over equatorial regions.

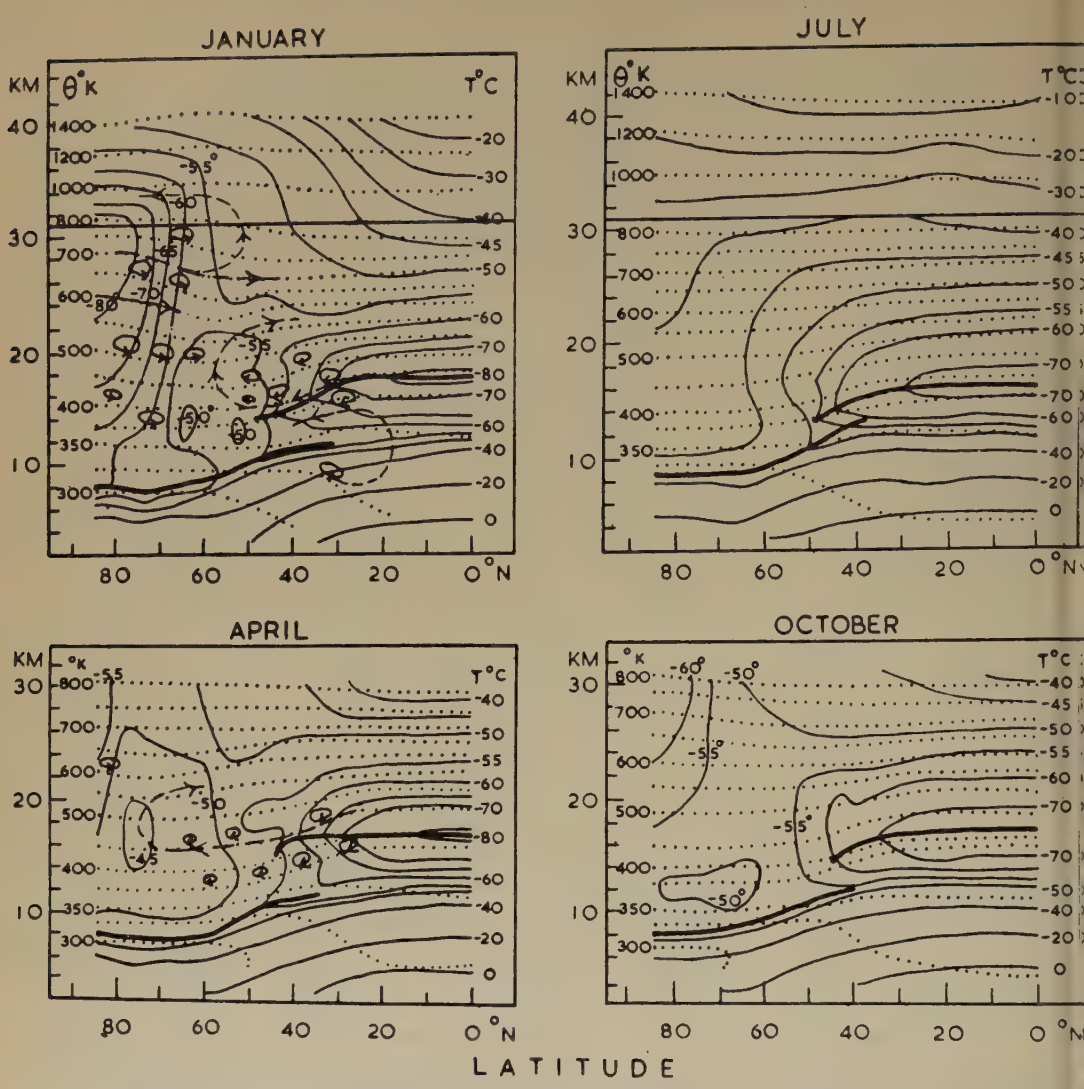

Figure 5. Meridional cross-sections of temperatures and potential temperatures θ°K (after Kochanski and and Murgatroyd).

The air collected in the mid-latitude stratosphere could be removed (a) by a return circulation at higher levels in the stratosphere towards the Equator; (b) by a similar circulation towards the pole, and (c) by entry into the troposphere at favourable places and times. Of these, the circulation involving the return of air towards the Equator may be expected to produce in its upper arm an increase in the ozone content, because it would be travelling towards a region where ozone-producing radiation is more abundant. During its subsidence and travel towards the Pole, the ozone will be conserved and stored in the lower stratosphere. Part of the stored ozone will periodically leak into the troposphere, particularly in the region of sub-tropical jets, and get lost by chemical decomposition or wash-out by rain. Whenever this circulation accelerates, there will be an increase of ozone in the lower stratosphere and whenever it weakens, the ozone will decrease or remain steady depending on its rate of leakage into the troposphere. It is unlikely that the exchange of air is purely meridional. It is more likely that the contemplated circulation is associated with zonal waves in the upper baroclinic region at levels below and above the

equatorial tropopause between latitudes 20° and 40°. As is well known, the mid-latitude tropopause reaches its lowest level in March-April.

The formation of the upper cold pool in the stratosphere of the Arctic during winter is an important contributory cause for intensifying the warm pool in the stratosphere of middle latitudes and pushing it towards the Equator. The increased gradient of temperature between the tropics and middle latitudes at, and near, the level of the equatorial tropopause will strengthen the stratospheric circulation between the middle and equatorial latitudes, and increase the rate of storage of ozone in the stratospheric reservoir. The mixing associated with the upper cold pool over the Arctic at 50-25 mb can only lead to a small overall averaging of ozone. The distributions of temperature and potential temperature in the stratosphere in January, April and July make it clear that the ozone-regenerating circulation will be weaker in April than in January and weakest in July. It again begins to be active in October. It may thus be expected that the period January to April will be the period of maximum ozone storage in the Northern Hemisphere.

Entry of ozone from the stratosphere into the troposphere can take place by small-scale turbulence associated with wind shears. Perhaps this is the only mechanism by which ozone passes through the tropopause over the tropics. Large-scale entry can occur through (1) the sub-tropical tropopause breaks and the associated jet streams and (2) polar front breaks and jet streams. The sub-tropical breaks extend over a greater depth of the atmosphere and are longer and are therefore of greater significance for the seasonal variations of ozone. The strength and number of jet streams associated with the breaks vary with the season. Occasionally, and over certain areas such as Korea and Japan, the sub-tropical and polar jet streams approach each other in winter and spring, permitting the entry of large masses of stratospheric air into the troposphere and vice versa.

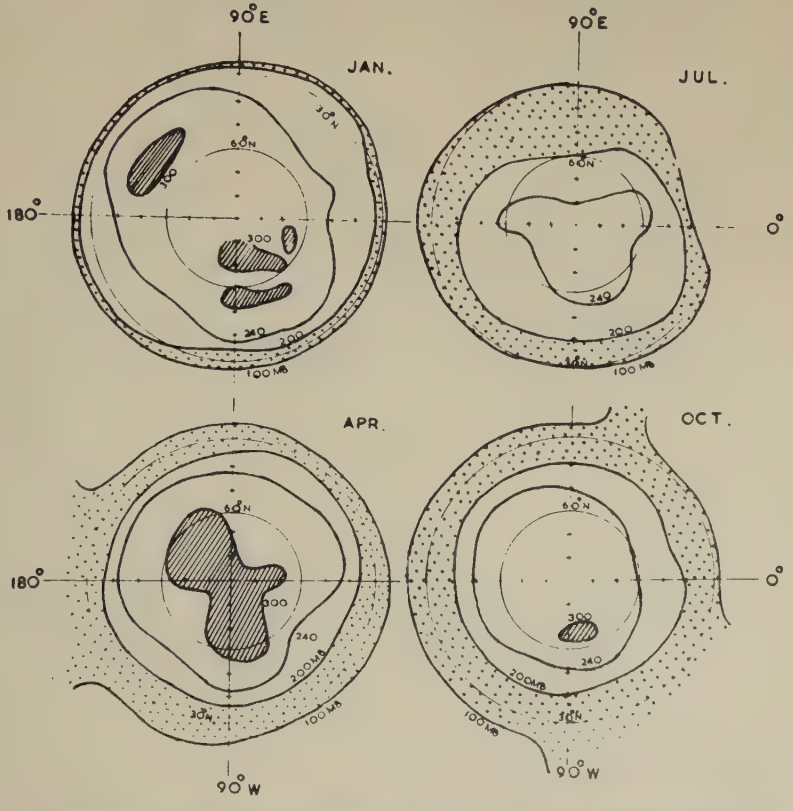

Figure 6. Average isopleths of tropopause pressure levels in January, April, July and October (100, 200, 240 and 300 mb); (after Goldie, Moore and Austin).

6. Conclusions

The following significant facts regarding ozone variations have been established. Over equatorial regions, the day-to-day variations are small, but there is a seasonal variation of ozone with maximum in June-August and minimum in December-January. In middle and high latitudes, the maximum occurs in March-April and the minimum in October-November. There is high correlation between ozone amount and tropopause pressure in winter and spring, and when increase of ozone takes place, it is mainly in the atmosphere below 18 km.

In view of these facts, it is relevant to link up the study of ozone problems with the study of the time and space variations of tropopause levels, of the discontinuities occurring in them and of the causes leading to such discontinuities.

Fig. 6 gives the isopleths of average tropopause pressure levels (100, 200, 240 and 300 mb) over the Northern Hemisphere in January, April, July and October according to Goldie, Moore and Austin (1958).

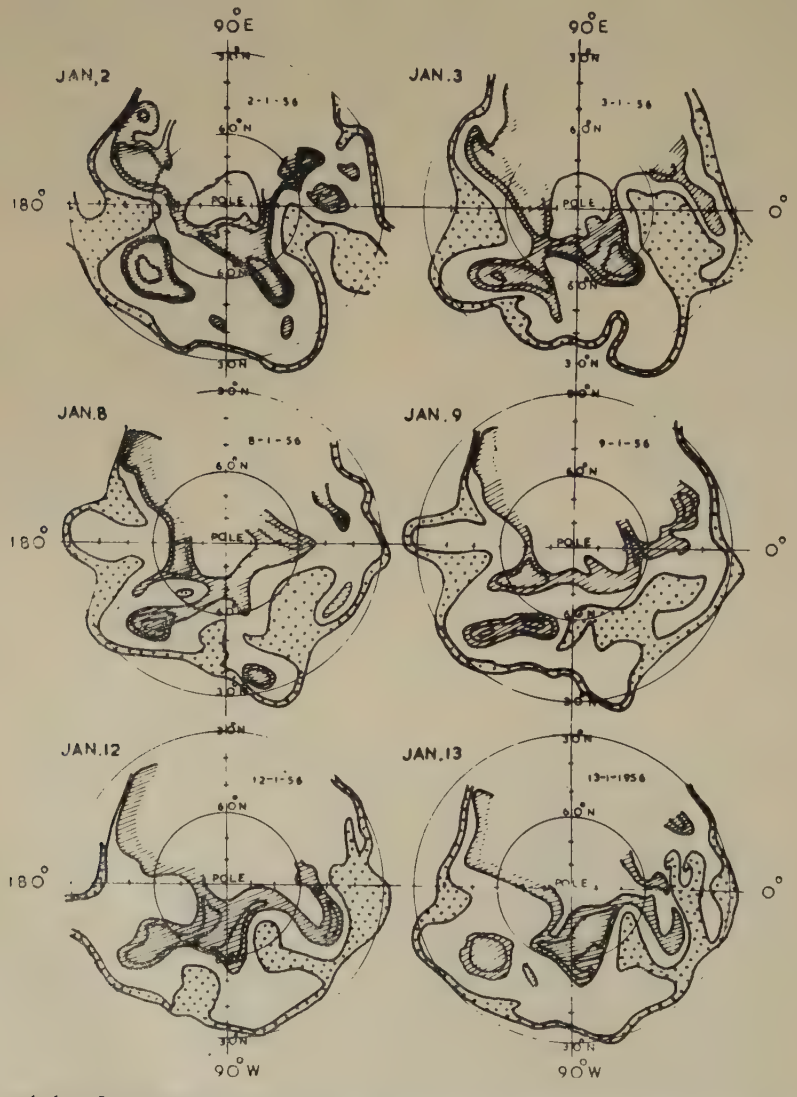

Figure 7. Isopleths of tropopause levels (100, 200, 300 and 350 mb) on individual days in January 1956 (after Fr. Defant and H. Taba and printed by courtesy of *Tellus*).

The closeness of the 100 and 200 mb-level tropopauses and the occurrence of the lowest tropopauses, not close to, but some distance away from the Pole are noteworthy features of the January diagram. In April, low-level tropopauses at pressures higher than 300 mb occur over a wide area round the Pole. The separation between the 100 mb and 200 mb-level tropopauses is widest in October and they are nearly symmetrical round the Pole. October-November are the months when the ozone amounts are smallest in extra-tropical latitudes. The day-to-day variations of ozone are largest in January to March and are least in June to October.

The January diagram of average tropopause levels may be compared with the day-to-day tropopause isopleths of the first half of January 1956 recently published by Defant and Taba (1958) (see Fig. 7). While the 100 mb level remains near 30°, the 200 mb isopleth takes deep excursions towards the Pole. Similarly, tropopauses at 300 or 350 mb move down to lower latitudes. There seem to be preferred regions for such excursions. Occasionally the 200, 300 and even 350 mb-level isopleths come close to each other. The pattern changes from day to day with a general slow W-E movement. We should expect corresponding complex changes in ozone amounts with regional preferences for high or low ozone amounts. We look forward to corresponding isopleth diagrams of ozone amounts.

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Interhemispherical features of the general circulation of the atmosphere

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Summary

Mean meridional wind components in the troposphere are computed at five stations along the Equator. Their significance is discussed in regard to mass exchange between the hemispheres. The eddy transport of heat across the Equator is found to be negligible. The possibility of seasonal exchange of heat between the hemispheres through meridional circulation across the Equator is discussed.

1. Introduction

The belt of the atmosphere between the latitudes 30-35° N and S absorbs more solar radiation than it radiates out, taking the year as a whole. In the rest of the atmosphere the reverse is the case. Similarly the tropics with easterly surface winds gain westerly angular momentum from the earth while the higher latitudes with westerly surface winds lose angular momentum to the earth. For maintaining balance the excess of heat and angular momentum must be transported from low to high latitudes. The investigations of Priestley (1951) and Starr and White (1951) have been concerned with finding evidence of such transport from wind, temperature and humidity data. It has been established that substantial amounts of heat and angular momentum are transported by the large-scale eddies in the atmosphere. However, the magnitude of the meridional currents and their relative role in transporting heat and angular momentum to higher latitudes is still under study. All these investigations are concerned with the meridional transport processes in each hemisphere and not between the hemispheres. The present study is concerned with the seasonal transport of mass and heat from one hemisphere to the other.

2. Statement of problem

Sir Napier Shaw showed that the mass of air in the Northern Hemisphere is a maximum at midwinter and a minimum at midsummer. Compared to the total amount of air in a hemisphere of $2{\cdot}7 \times 10^{15}$ metric tons, the annual range of variation is about 10^{13} metric tons. Gordon (1953) computed the changes in mean pressure from one month to the next for each 10° square and converted them into units of relative mass and integrated around the globe. From his figures of the changes in mass for 10° belts of the atmosphere, the monthly changes in the total mass of the Northern Hemisphere have been worked out and are given in Table 1. These amounts of mass have to be carried across the Equator. Even the maximum rate of this mass flow of $1{\cdot}86 \times 10^{12}$ metric tons which is from May to June is equivalent only to a mean meridional flow of 0·003 kt.

TABLE 1. Monthly change in the mass of Northern Hemisphere

Period	Dec. to Jan.	Jan. to Feb.	Feb. to Mar.	Mar. to Apr.	Apr. to May	May to June	June to July	July to Aug.	Aug. to Sept.	Sept. to Oct.	Oct. to Nov.	Nov. to Dec.
Change in total mass	+	−	−	−	−	−	−	+	+	+	+	+
(10^{12} metric tons)	0·46	0·14	0·49	1·35	1·79	1·86	0·70	0·34	1·05	1·44	1·42	0·98
Rate of mass transport across Equator	+	−	−	−	−	−	−	+	+	+	+	+
(10^2 metric tons cm^{-1} $month^{-1}$)	1·14	0·35	1·22	3·35	4·44	4·62	1·74	0·84	2·60	3·56	3·51	2·41

Baur and Philipps (1935) showed that there should be marked seasonal transport of heat across the Equator representing the difference between the net heat gain and heat storage in each hemisphere. From May to October the transport is from the Northern Hemisphere to the Southern Hemisphere and in the other months the reverse. The rate of heat flow across the Equator should be $2{\cdot}9 \times 10^7$ cal cm^{-1} min^{-1} in January, $0{\cdot}4 \times 10^7$ in April, $-2{\cdot}2 \times 10^7$ in July and $-0{\cdot}8 \times 10^7$ in October, negative sign indicating southward flow. The magnitude of the interhemispherical heat flow is comparable with the transport across 40° parallel which averages over the year to $2{\cdot}1 \times 10^7$ cal cm^{-1} min^{-1} and reaches a maximum of about $3{\cdot}8 \times 10^7$ in January. It may also be mentioned that even while considering the heat balance of any single hemisphere the transport of heat at midsummer should be equatorward at latitudes less than 50°, the magnitude being about $0{\cdot}6 \times 10^7$ cal cm^{-1} min^{-1} at 30°.

The scope of this study is to investigate the processes by which the required heat transport between the two hemispheres is achieved. Attempt is also made to obtain a picture of the seasonal and daily mass transport across the Equator.

TABLE 2. POSITIONS OF STATIONS AND NUMBER OF AVAILABLE WIND OBSERVATIONS

Station	Latitude	Longitude	Month	Hour of observation GMT	Number of observations at Surface	10 km 300 mb	15 km 200 mb	17 km 100 mb
Christmas Island	02° 00′N	157° 23′W	Jan.	0200	31	31	31	31
				1400	31	31	31	31
			Apr.	0000	31	31	31	31
				1200	29	29	29	29
			July	0000	31	31	31	31
			Oct.	0000	31	30	30	30
				1200	31	31	31	26
Canton Island	02° 46′S	171° 43′W	Jan.	0300	31	31	29	24
				1500	31	30	28	22
			Apr.	0300	30	30	30	30
				1500	30	30	30	30
			July	0000	31	30	29	30
				1200	31	30	29	26
			Oct.	0000	31	29	27	27
				1200	31	29	25	24
Singapore	01° 21′N	103° 54′E	Jan.	0300	29	29	29	28
				1500	29	29	29	27
			Apr.	0000	29	28	29	24
				1500	29	29	30	28
			July	0000	26	26	25	21
				1200	29	30	29	26
			Oct.	0000	23	26	25	17
				1200	24	25	24	19
Nairobi	01° 17′S	36° 50′E	Jan.	0200	25	25	25	17
			Apr.	0000	21	20	18	15
				1200	21	21	19	11
			July	0000	24	24	22	19
				1200	26	25	22	15
			Oct.	0000	28	28	28	27
				1200	27	26	25	24
Port Gentil	00° 45′S	08° 45′E	Dec.	0500	31	31	29	23
			July	0500	30	30	22	18
			Oct.	0500	31	30	25	20

3. Data used

Radiosonde and rawin stations close to the Equator are very few. There were only five stations with sufficient radio-wind observations within 3° of the Equator. It is not desirable to take stations farther away from the Equator, as transport within the hemisphere may complicate the picture. All available data of 1957 for the months of January, April, July and October of these stations have been used in the study. Table 2 gives the positions of the stations and the number of wind observations available at selected levels. Equally numerous temperature and humidity observations were available, at least for one synoptic hour, except from Port Gentil which was only a rawin station and in January from Nairobi when ascents were made on very few days. Rawin ascents at Port Gentil apparently commenced with the I.G.Y. period and the data of December have been taken as representative for January.

4. Mass transport

If v is the meridional component of the wind velocity, ρv gives the mass transport across a unit area fixed in the west-east vertical plane. The total meridional mass transport across the entire vertical plane of unit zonal length is given by $\int_0^\infty \rho v \, dz = \frac{1}{g}\int_0^{p_0} v \, dp$. The meridional components at the pressure levels at stations near the Equator would indicate the mass exchange between the hemispheres.

Figs. 1 to 4 show the meridional components of the mean wind velocity in the months of January, April, July and October 1957. Positive sign indicates northward flow. The means of only one year's data seem sufficiently representative to give an idea of the order of magnitude and the pattern of meridional flow. This will be evident from a comparison in Table 3 of the means of July 1957 at Singapore and Nairobi with those of July 1955 worked out in connexion with study of the southwest monsoon circulation.

TABLE 3. Comparison of mean meridional components (kt) of wind velocity in July

Station	Year	Pressure level (mb) 850	800	700	600	500	400	300	200	150	100
Singapore	1955	3·8		2.8	0·3	0·8	0·0	0·7	− 2·4	− 9·7	− 4·8
	1957	3·3		1·1	1·5	2·4	3·9	− 1·9	− 12·5	− 16·2	− 5·1
Nairobi	1955		3·4	1·5	− 7·7	− 4·7	− 1·0	1·5	5·3	0·6	0·3
	1957		3·8	− 1·3	− 5·1	− 1·9	2·0	0·5	3·8	1·0	2·2

At Singapore both years' data show northward flow in lower and mid-troposphere with southward flow in upper-troposphere. At Nairobi, also, there is agreement in having three layers, the highest and lowest with northward transport and in between southward flow. The mean meridional components of Singapore for 1957 presented here are in general agreement with the data discussed by Clarkson (1956). There is also consistency between data of different hours.

From the cross-sections in Figs. 1 to 4 it will be seen that marked meridional flow prevails between 300 and 100 mb, often opposite to that in the lower troposphere. In January from Port Gentil to Canton Island the flow is southwards in the lower troposphere, associated with the northeast monsoon of Asia, weak at Canton Island. This flow rapidly decreases with height and above 500 mb northward flow builds up reaching a maximum near the 200 mb level. The flow pattern at Christmas Island shows a marked southward flow between 300 and 100 mb and a weak flow below. In April the meridional flow weakens except in the upper troposphere and at the surface at Canton Island. In July the northward flow occurs at the surface from Port Gentil to Singapore, associated with the southwest monsoon of Asia and Africa and extends weakly up to Christmas Island.

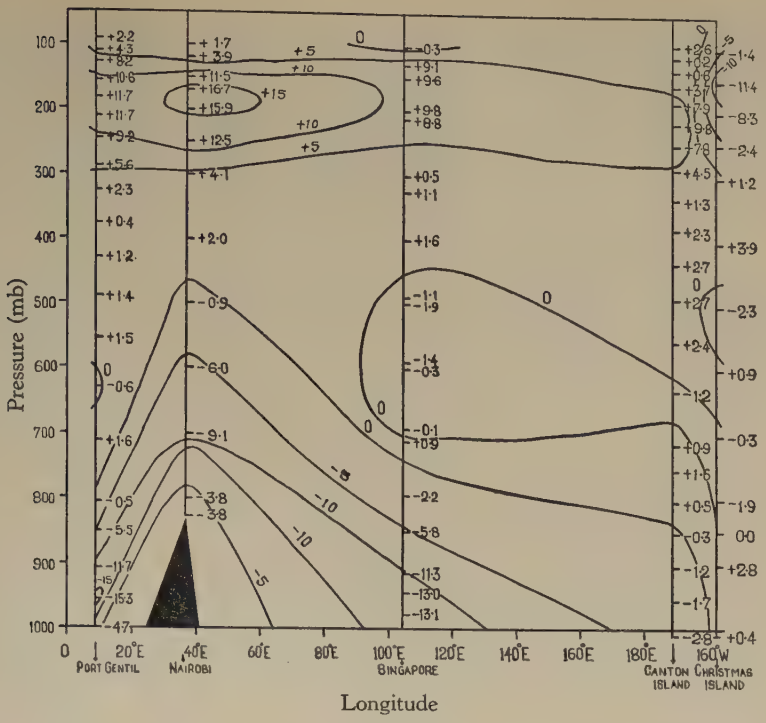

Figure 1. Mean meridional wind components (kt) in January 1957.

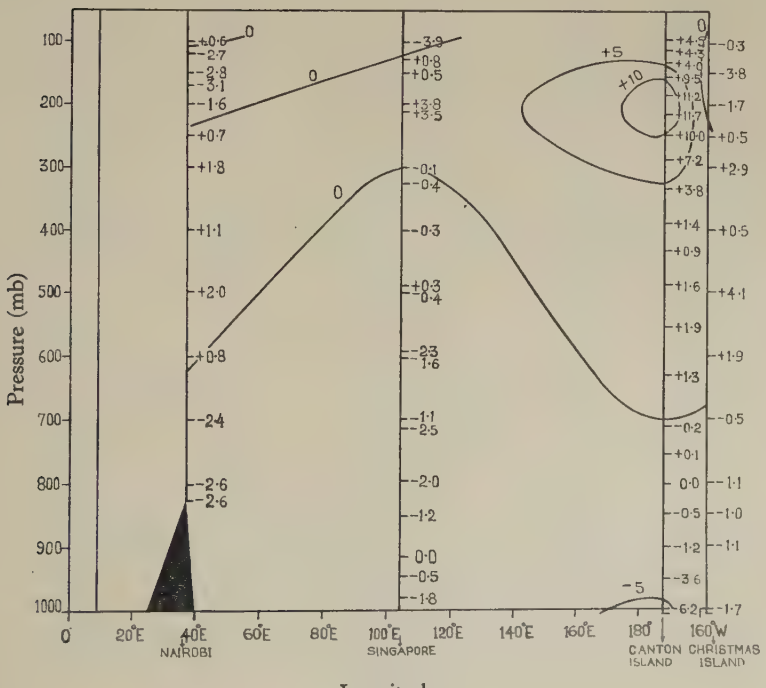

Figure 2. Mean meridional wind components (kt) in April 1957.

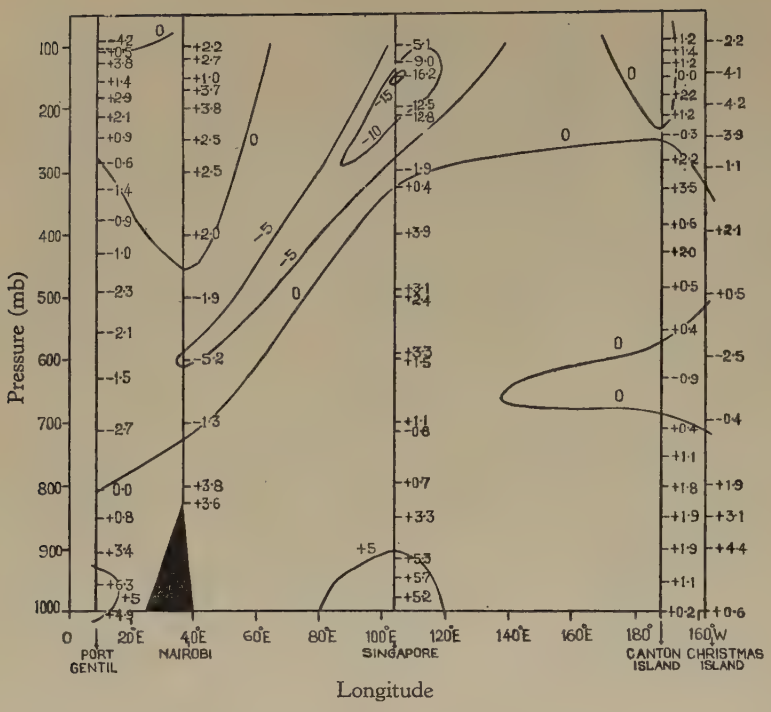

Figure 3. Mean meridional wind components (kt) in July 1957.

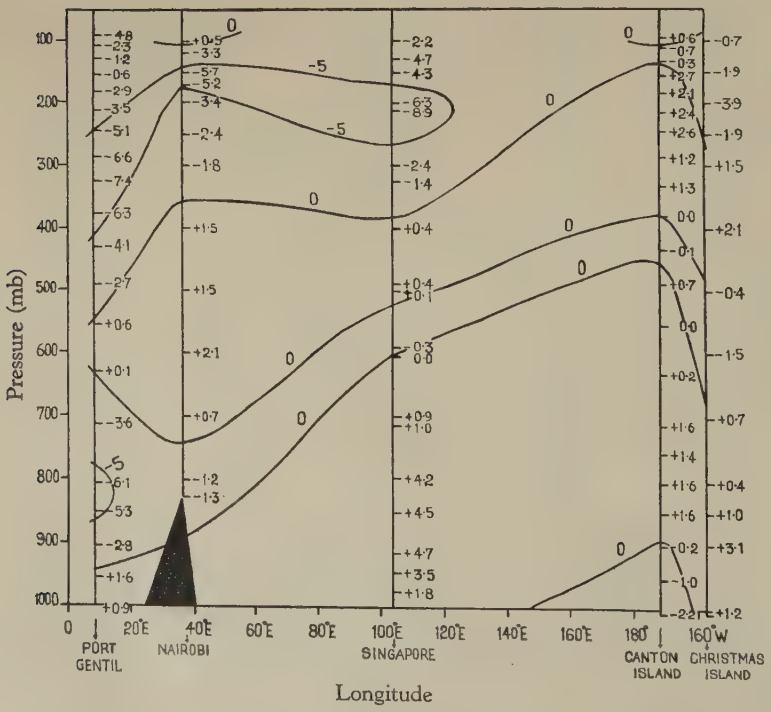

Figure 4. Mean meridional wind components (kt) in October 1957.

At Singapore the northward flow extends up to 300 mb but is replaced by a strong southward flow up to 100 mb. At the other stations the upper tropospheric flow is much weaker than in January and northwards at Port Gentil, Nairobi and Canton Island and southwards at Christmas Island. The upper tropospheric flow is in the same direction as in January at these places. In October the upper tropospheric flow is northwards at Port Gentil, Nairobi, Singapore and Christmas Island.

Table 4 gives the rate of mass transport per month across the Equator between the surface and 100 mb computed from the mean meridional components.

TABLE 4. MASS TRANSPORT ACROSS THE EQUATOR (10^2 METRIC TONS CM^{-1} $MONTH^{-1}$)

Station	Jan.	Apr.	July	Oct.
Christmas Island	− 1000	+ 362	− 140	+ 248
Canton Island	+ 2,260	+ 2,760	+ 1,340	+ 874
Singapore	− 1,130	− 610	+ 564	− 106
Nairobi	+ 1,030	− 300	+ 290	− 375
Port Gentil	+ 652		+ 133	− 4,180

The rate of mass transport in the troposphere at these stations is far in excess, sometimes by a thousand times, of the monthly flow across the Equator given in Table 1. In all the months there are areas of northward and southward flow. In July across half the length of the Equator, from Port Gentil to Canton Island, mass is being transported northward, opposite flow being seen at Christmas Island. In January there are two areas of flow in each direction. The western portions under study show southward flow in both April and October and the eastern portions northward flow. It is interesting to note that at Canton Island the transport is northwards throughout the year. The monthly interhemispherical mass exchange is apparently a small residual of very much larger transports in the troposphere in opposite directions in different longitudes. It is also quite possible that part of the compensation for the tropospheric flow takes place in the stratosphere. Clarkson (1951) concluded that there is subsidence in the lower equatorial stratosphere

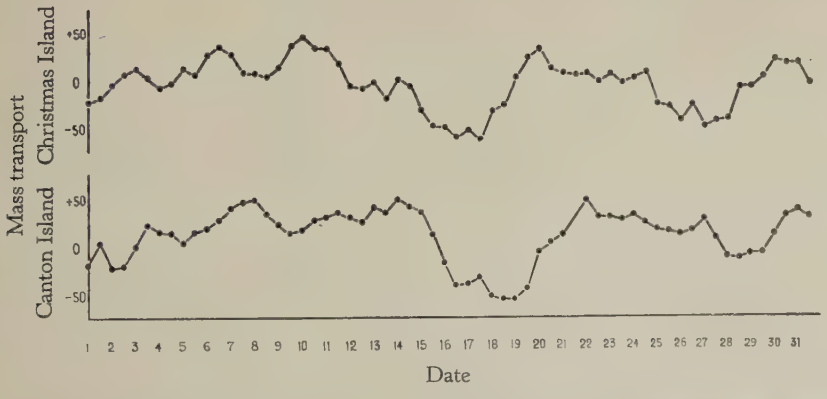

Figure 5. Daily variations in mass transport (10^3 kg cm^{-1} min^{-1}) at Christmas and Canton Islands in January 1957.

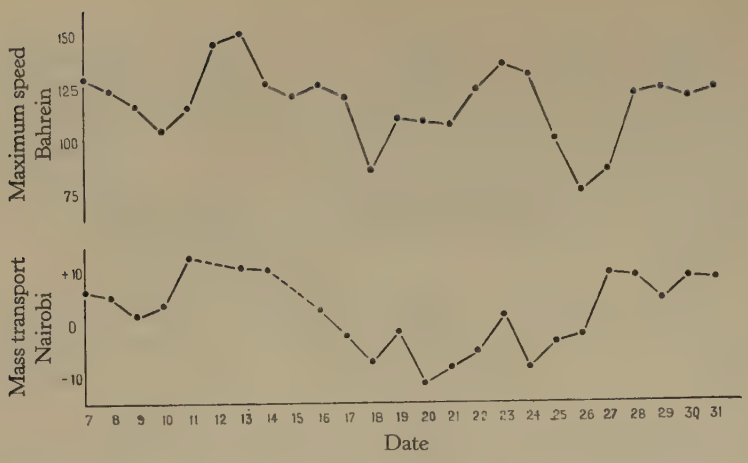

Figure 6. Daily variations in mass transport (10^3 kg cm^{-1} min^{-1}) at Nairobi and maximum wind speed (kt) at Bahrein in January 1957.

of the winter hemisphere. A net transport in the troposphere from the winter to the summer hemisphere compensated by stratospheric flow in the opposite direction may fit with subsidence in the lower equatorial stratosphere of the winter hemisphere.

As large northward and southward mass transport is seen to occur simultaneously in all months it may be right to assume that substantial balance takes place between them and the compensation in the stratosphere is relatively smaller. If this were the case, the longitudes of inflow and outflow of mass along the Equator must be linked in some manner as undue accumulation of mass in any hemisphere is not possible. The sub-tropical jet and the easterly jet may have an important role in this respect.

The net mass transport up to 100 mb was computed for each wind sounding, for January and July at all stations. Fig. 5 shows the daily variations in January at Christmas and Canton Islands. It appears that the pattern of mass transport moves from Christmas to Canton Island in one to two days, suggesting association with travelling disturbances. Similar relationship is not found in July. Other stations are too far apart to show any migratory features. Irrespective of the direction of the mean monthly flow at any station, the daily flow oscillates between positive and negative values. On some occasions the flow is in the same direction from Nairobi to Christmas Island which may continue even for two days. On 7 and 8 January the mass transfer was northwards and on 17 and 18 January, southwards. If the sub-tropical jet is a channel linking zones of inflow and outflow of mass into a hemisphere, fluctuations in the strength of the jet may be correlated with variations in the trans-equatorial flow in the same longitude. Fig. 6 shows the daily variations in January of the maximum wind speed at Bahrein (26°N, 50°E) and the net tropospheric flow at Nairobi, and Fig. 7 shows the same quantities respectively at Hong Kong (22°N, 114°E) and Singapore. Bahrein and Hong Kong are sufficiently near the mean position of the sub-tropical jet to regard the maximum wind speed at these stations as a measure of the jet. The 24-hr changes in the maximum wind speed at Bahrein and meridional tropospheric flow at Nairobi show a correlation coefficient of + 0·29 with 20 cases, which is significant at less than 10 per cent level. Between the same quantities at Hong Kong and Singapore, the correlation coefficient is — 0·42 with 22 cases, which is significant at 5 per cent level. Considering that the mean tropospheric meridional flow in January at Singapore and Nairobi are opposite in direction, the reversal in the sign of the correlation coefficients is interesting. No similar relationship could be found between the maximum wind speeds at Midway Island (28°N, 177°W) and Lihue (22°N, 159°W) and the meridional flow at Canton and Christmas Islands respectively. Nevertheless the relationships observed at Singapore and Nairobi merit further examination with more instances.

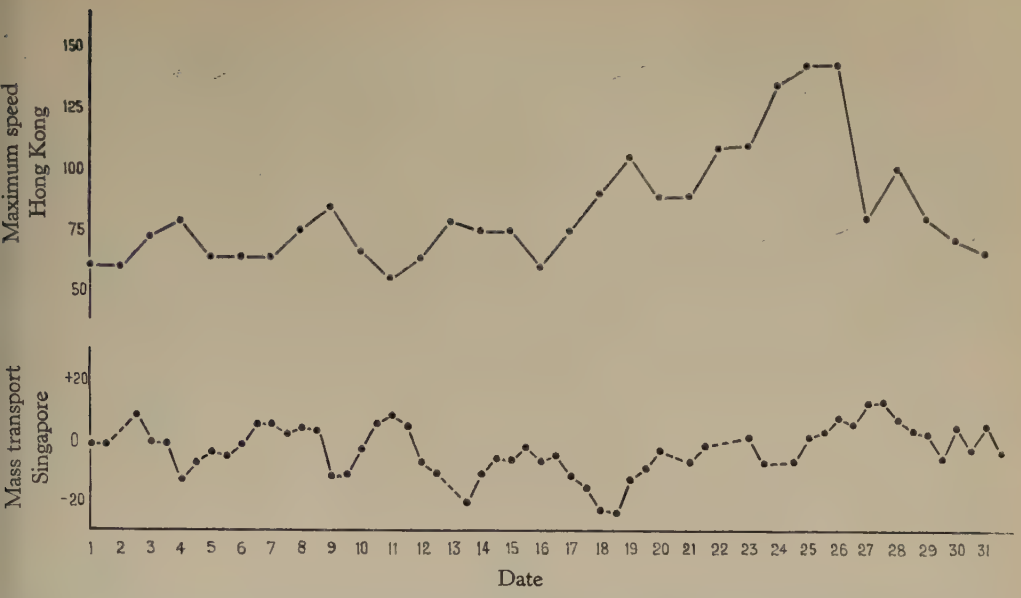

Figure 7. Daily variations in mass transport (10^3 kg cm^{-1} min^{-1}) at Singapore and maximum wind speed (kt) at Hong Kong in January 1957.

5. Eddy transport of heat

The rate of eddy transport of heat across a vertical plane of unit length along the Equator is given by $\frac{c_p}{g}\int \overline{v'\,T' + v'\,x'\,\frac{L}{c_p}}\,dp$ where v', T' and x' are departures from the mean, x being the humidity mixing ratio. Tables 5 and 6 give the values of $\overline{v'\,T'}$ and $\overline{v'\,x'\,\frac{L}{c_p}}$ at different levels, which will respectively be proportional to the eddy transport of enthalpy and latent heat and are referred to accordingly.

TABLE 5. $\overline{v'\,T'}$ (10^2 °A cm sec^{-1}) proportional to eddy transport of enthalpy

Station	Month	Pressure level (mb) 1,000	900	850	800	700	600	500	400	300	250	200	150	100
Christmas Island	Jan.	+ 0·1	− 0·4	− 1·3	− 0·4	+ 0·9	+ 0·6	+ 0·3	− 2·6	− 4·3	− 0·9	− 0·7	− 5·3	+ 0·9
	July	− 0·8	− 0·5	− 0·2	− 0·1	− 0·4	− 0·6	+ 1·5	+ 0·7	0	− 0·4	− 1·6	− 4·0	+ 2·4
Canton Island	Jan.	+ 0·7		+ 0·3		+ 0·9	+ 0·9	+ 0·1	+ 1·0	+ 2·8	+ 3·7	− 2·8	− 6·5	− 0·8
	July	− 0·2		+ 0·4		− 0·1	− 0·5	+ 1·5	+ 1·0	0	+ 1·2	+ 0·9	− 0·2	− 0·2
Singapore	Jan.			+ 1·5		+ 0·3	− 0·4	+ 0·5	− 0·3	+ 1·3		+ 1·2	+ 1·6	+ 0·1
	July			+ 0·3		− 0·3	+ 0·1	+ 0·2	− 0·7	+ 0·6		+ 1·5	− 2·5	− 0·3
Nairobi	July				0	− 0·9	+ 1·7	+ 1·0	− 1·3	− 1·0	+ 5·2	+ 3·8	+ 2·2	− 1·9

TABLE 6. $\overline{v' x' \frac{L}{c_p}}$ (10^2 °A cm sec^{-1}) PROPORTIONAL TO EDDY TRANSPORT OF LATENT HEAT

Station	Month	Pressure level (mb) 1,000	900	850	800	700	600	500
			−		−	−	−	−
Christmas Island	Jan.	0	1·5	0	8·7	2·1	1·8	1·9
		−	−	−	−	−	−	−
	July	1·5	0·6	3·6	0·7	4·6	1·5	7·0
		+		+		−	+	+
Canton Island	Jan.	1·6		2·4		3·6	0·9	1·6
		−		−		−	−	−
	July	0·8		2·8		0·1	0·7	2·9
				−		−	−	+
Singapore	Jan.			1·9		3·9	1·6	2·1
				−		+	+	+
	July			1·5		4·5	3·7	1·2
						−	−	−
Nairobi	July				0	4·0	7·7	3·4

The eddy transport of enthalpy is generally small and there is no consistency in the direction of transport between different levels at any station. Though the eddy transport of latent heat is more uniform in sign between different levels and relatively larger, the magnitude is still not sufficiently large. In some cases the direction of transport of latent heat is opposite to the required flow. In January, Christmas Island shows southward flow which should have been into the winter hemisphere. Table 7 gives the integrated eddy transport of enthalpy between surface and 100 mb and latent heat up to 500 mb. The total eddy transport is not more than ten per cent of the magnitude of the inter-hemispherical heat flow. Also, it is sometimes transporting in the wrong direction, such as at Christmas Island in January which is towards the summer hemisphere.

TABLE 7. TOTAL EDDY TRANSPORT OF HEAT (10^7 cal cm^{-1} min^{-1})

Station	January Enthalpy	January Latent heat	January Total	July Enthalpy	July Latent heat	July Total
	−	−	−	−	−	−
Christmas Island	0·12	0·18	0·30	0·03	0·19	0·22
	+	+	+	+	−	−
Canton Island	0·05	0·03	0·08	0·05	0·10	0·05
	+	−	−	+	+	+
Singapore	0·10	0·14	0·04	0·01	0·10	0·11
				+	−	−
Nairobi				0·07	0·20	0·13

The eddy transports of enthalpy and latent heat are in some cases opposite in sign. The conclusion is that eddy transport cannot account for the seasonal interhemispherical exchange of heat. Owing to lack of radiosonde and rawin stations near Equator in the Indian Ocean it has not been possible to sample the eddy transport in association with the south-west monsoon. But Rao (1958) found that at Indian stations during that period, eddy transport of enthalpy and latent heat was negligible, which may be assumed to be true at the Equator also.

6. Advective transport of heat

The advective transport of heat is given by $\frac{1}{g}\int \bar{v}\,(\overline{C_pT + Lx + Agz})\,dp$. A is the reciprocal of the mechanical equivalent of heat. The decrease of T and x with increase of z is such that $\overline{C_pT + Lx + Agz}$ decreases slightly up to the mid-troposphere and then increases. The range of variation is 8 per cent. Table 8 gives the values of $\overline{C_pT + Lx + Agz}$ at selected levels. It is only in the stratosphere that the quantity increases rather rapidly, with the temperature remaining constant or increasing with height. For computations of the advective transport of heat in the troposphere mean value of 82 cal can be substituted for $\overline{C_pT + Lx + Agz}$ and the expression simplified as $\frac{82}{g}\int \bar{v}\,dp$. The heat transport is thus approximately proportional to the mass transport in the troposphere.

TABLE 8. Values of $\overline{C_pT + Lx + Agz}$ (cal)

Station	Month	Pressure level (mb) 1,000	850	800	500	300	100
Christmas Island	Jan.	81·0	78·3		78·8	80·6	85·1
	July	83·4	79·6		79·5	81·1	85·6
Canton Island	Jan.	81·7	79·1		78·1	80·7	84·5
	July	83·7	80·2		79·1	80·9	85·4
Singapore	Jan.	81·5	79·4		79·1	80·7	84·5
	July	82 1	80 1		79·3	80·7	85·3
Nairobi	July			78·7	78·1	80·2	85·3

The seasonal transport of mass across the Equator has little significance for the seasonal exchange of heat. The maximum transport of $1{\cdot}86 \times 10^{12}$ metric tons from May to June is equivalent to heat flow of $8{\cdot}8 \times 10^5$ cal cm^{-1} min^{-1} which is $\frac{1}{33}$ of the maximum rate of heat transport. Even if this mass is transported in the stratosphere with a higher heat content it makes little difference. Besides, in the months of January and July when the maximum heat transport takes place, mass transport is little, the hemispherical mass being then either maximum or minimum.

It was seen that in all months there are areas along the Equator where northward and southward mass flow takes place in the troposphere. If the heat content of the air flowing in opposite directions is appreciably different, net heat transport is possible even though there is balance in mass flow between the zones. The required difference in heat content can be evaluated for July. Table 4 shows that in this month from Canton Island to Port Gentil (about half the length of the Equator) there is northward mass flow, the mean value being 582×10^2 metric tons cm^{-1} month^{-1}. Over the rest of the Equator we shall assume equal rate of mean mass flow in the opposite direction. To result in the required heat flow of $-2{\cdot}2 \times 10^7$ cal cm^{-1} min^{-1}, the difference in $\overline{C_pT + Lx + Agz}$ between the zones of northward and southward flow should be 34 cal, which is impossible. It will be noticed from Table 8 the total heat content is very uniform along the Equator, irrespective of the direction of mass flow.

The possibility of heat transport through the tropospheric flow being balanced by reverse flow in the stratosphere may now be considered. In July the mean of the tropospheric flow at the five stations studied is $+437 \times 10^2$ metric tons cm^{-1} month^{-1}. If this is representative of the whole Equator, with equal and opposite mass flow in the stratosphere the average heat content of the stratospheric air must be 22 calories higher than that of the troposphere to result in the required heat flow to the winter hemisphere.

As the heat content of the tropospheric air has been taken as 82 calories, the mean heat content of the stratospheric air participating in mass compensation should be 104 calories. At the tropopause the heat content is 86 calories and hence the top of the stratospheric air involved in this flow should have a heat content of 122 calories which is at 15 mb level. Thus a very deep layer of the stratosphere should be involved in mass compensation for the tropospheric flow to result in the required heat transport. If the actual mean tropospheric mass flow across the Equator is greater than that given by the present sample of observations it may correspondingly reduce the depth of stratosphere to be involved in the process. In January the position is less satisfactory. The sample of observations under study gives a mean tropospheric mass flow northwards, which, if representative of the rest of the Equator, with southward mass compensation in the stratosphere, cannot lead to northward heat flow. To evaluate the heat flow from meridional circulation, reliable estimates of the meridional wind components along the Equator are necessary. The present study covering five stations and one year's data can be regarded as giving only a very rough estimate of the meridional circulation. However, from this preliminary study the mechanism of meridional circulation across the Equator transporting the required heat involves a very deep layer of the stratosphere in July and does not appear possible in January.

If the meridional circulation is found ineffective after further study, the sea currents may have to be investigated as a mechanism of transporting heat between the hemispheres. Though the sea currents are very much weaker than the atmospheric winds, a layer of sea-water 2·5 m thick has the same heat capacity as the complete vertical extent of the atmosphere.

7. Acknowledgments

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Observations of precipitation elements in cumulus clouds

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Summary

Cloud particles of diameters greater than about 100 μ can be sampled from an aircraft by exposing within the cloud a thin aluminium-foil surface to the airstream and counting and sizing the indentations made on it. A series of flights has been made through cumulus clouds using this technique with the object of obtaining data on the concentrations of these larger particles in clouds of different size and temperature. A series of observations showing the measured concentrations is presented and discussed. Concentrations range from less than $1/m^3$ in the smaller cumulus clouds up to 1/litre or more in clouds about 1 km thick.

1. Introduction

Sampling of cloud droplets from an aircraft is usually carried out by exposing within the cloud some form of slide with a prepared surface, e.g., soot, magnesium oxide or oil and then counting and sizing the holes made by the droplets on impact or the number of droplets caught in the oil. The spectrum so obtained is limited at its lower end by the collection efficiency of the exposed slide. The limitation at the upper end is usually imposed by the small volume of air that can be sampled without swamping the slide by the smaller-sized droplets. The concentration of cloud droplets measured by this method is usually in the order of hundreds per cubic centimetre and the exposure time and size of the slide are such that a few tens of cubic centimetres of air are sampled. Raindrop concentrations at the surface, however, are usually found to be in the order of 0·1 to 1 per litre, and therefore to measure rain or precipitation elements in the air we need to sample tens or hundreds of litres of cloud air and the resulting record must not be spoilt by the effects of the very much larger concentrations of smaller particles. By precipitation elements we mean, in this context, those particles with diameters greater than about 100 μ, a size where droplet growth by coalescence is very much greater than that by condensation, and approximating to the smallest drizzle size. An instrument has been developed which measures only particles greater than this size and in one of its applications a series of flights has been made in cumuliform clouds of different thickness and temperature to obtain data, useful in cloud-physics studies, on the number of precipitation elements they contain and hence their potentiality of producing rain.

2. The sampling instrument

The instrument used was the aluminium-foil impactor (Garrod 1957) which was based on laboratory work carried out by the Mechanical Engineering Department, Royal Aircraft Establishment, Farnborough, and is now regularly used for sampling rain from aircraft. A photograph of this instrument and typical precipitation records have also been given by Durbin (1958). In order to determine the minimum size of particle which would give a record, a calibration against several hundred samples obtained on oiled slides was carried out in an icing wind tunnel operated at aircraft speeds and with droplet sizes and water contents similar to those found in clouds. The method consisted in counting all marks made in the aluminium-foil surface over a given period and comparing the number against the overall spectrum of all the oil samples taken during the same period to find the particle size above which the total numbers were equal. No great precision is claimed for the method, but the conditions of calibration simulated flight conditions

realistically and it seems very likely that, with care, cloud particles down to 100 μ diameter can be detected. Larger sizes, e.g., greater than 250 μ diameter can be measured easily. With experience some differentiation is also possible between the indentations made by water droplets, snow and ice crystals. Water droplets make characteristic circular marks, ice crystals leave sharper dents or sometimes lines, and snow produces a blurred impression with no distinct outline.

3. Flight procedure

Occasions on which sampling was possible were limited to days when individual cumulus clouds were well separated. This ensured that a series of runs could be made in one particular cumulus cloud, and made positioning of the aircraft easier. A suitable cloud having been selected, a number of runs was made through it at different levels with 500 ft or 1,000 ft (150 to 300 m) separation and at an airspeed in the range of 160 to 200 kt (80 to 100 m sec^{-1}). Owing to rapid changes in the cloud structure it was found necessary to limit the number of runs to about six, but on occasions the clouds dispersed after one or two runs.

The impactor was normally exposed before entering cloud, and withdrawn after leaving cloud, the time in cloud being noted. However, if the cloud was particularly large, or the drop concentration very high, the impactor was exposed for a shorter period inside the cloud. During exposure the aluminium tape was kept stationary, the sampling area on it being equal to that of the aperture, i.e., 3 cm^2, so that the volume of air sampled was 300 litres per km flown.

Other observations included the height of the base and top of the cloud, the mean airspeed during the exposure of the impactor, and the temperature structure in clear air from 1,000 ft (300 m) to the cumulus tops.

4. Results

Appendix 1 gives details of the day's weather and synoptic conditions, with particular reference to the Farnborough area in southern England where all the flights were made. Fig. 1 shows individual observations, with the concentrations in numbers/litre of particles greater than 100 μ diameter plotted at the cloud sampling heights. Base and top temperatures and maximum theoretical adiabatic water content for each cloud are also given. The clouds are plotted in order of size in six arbitrarily chosen groups which are discussed below. It will be noticed that there is great variability between concentrations of precipitation particles even in clouds of the same group. The histograms on the right of the diagram show the total number of observations in each group in which concentrations of nil, 0·001 to 0·01, 0·01 to 0·1, etc./litre were found.

(*a*) *Cumulus clouds of small vertical extent having temperatures above 0°C throughout*

These warm clouds were all less than 2,500 ft (750 m) in vertical extent and had adiabatic water contents of less than 2 g m^{-3}. In general, the smallest clouds in this group did not contain any drops exceeding 100 μ diameter, and even in the largest of these clouds, concentrations of ·01/litre were only exceeded infrequently. No noticeable differences were observed between the small clouds in stable synoptic conditions and the small clouds accompanying larger clouds in less stable situations. No changes in characteristics between morning and afternoon clouds could be detected.

(*b*) *Cumulus clouds of moderate vertical extent having temperatures above 0°C throughout*

The clouds in this group had thicknesses from 3,000 ft to 8,000 ft (1,000 m to 2,500 m) and theoretical water contents of up to about 3 g m^{-3}. Droplets greater than 100 μ diameter were usually present and on a substantial number of occasions concentrations of

0·1/litre or greater were found. In their development from the preceding group the number of precipitation elements had therefore risen to concentrations equal to those commonly found in rainfall at the surface, and some possibility exists of rainfall being produced by the condensation-coalescence process in clouds in this size range. Most of the observations were made in unstable situations with showers, and sometimes thunderstorms were observed in larger clouds during these flights.

(*c*) *Mixed cumulus and stratocumulus clouds with temperatures above 0°C throughout*

It was noticed on some occasions that the concentrations of large droplets were very much greater than those given in groups (*a*) and (*b*) above. These cases could be associated with mixed situations, e.g., cumulus forming from a dissipating stratocumulus layer, cumulus penetrating a stratocumulus layer, or the existence for several hours, of a stratocumulus layer with cumulus forming and continuously spreading out. Evidently these types of situations, as distinct from purely cumuliform cloud cover, are likely to produce rainfall more easily and it is often difficult to decide what is the role of the stratocumulus in the rainfall mechanism when stratocumulus and cumulus clouds are both reported. The case of 25 Sept. 1957 is of particular interest when cumulus clouds with a base at 2,200 ft (700 m) and tops at 8,100 ft (2,500 m) were growing through a stratocumulus layer base 5,000 ft (1,500 m), tops 7,000 ft (2,000 m). Precipitation was observed falling both from the cumulus and from the layer cloud.

(*d*) *Small cumulus clouds with temperatures above 0°C at their bases and below 0°C at their tops*

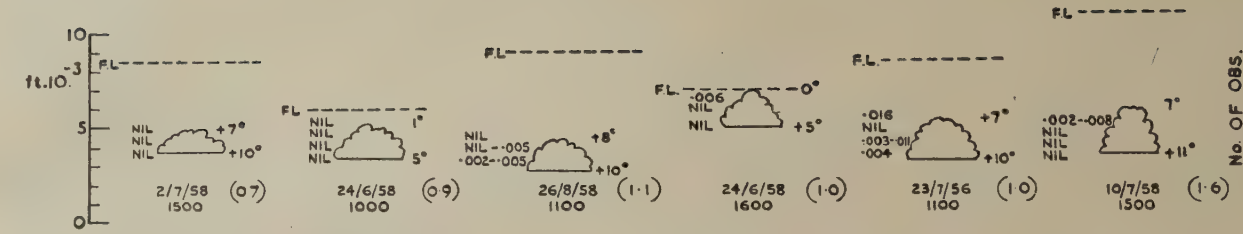
These clouds had thicknesses of 2,500 ft to 5,000 ft (750 m to 1,500 m) with adiabatic water content up to about 2·0 g m^{-3}. Comparing them with group (*b*) it seems that in spite of smaller thicknesses and water content they are at least equally likely to have concentrations of precipitation particles greater than about 0·1/litre. In none of these observations was there any report of ice crystals within the cloud. The lowest cloud top temperature in this group was — 8°C.

(*e*) *Larger cumulus clouds with temperatures above 0°C at their bases and below 0°C at their tops*

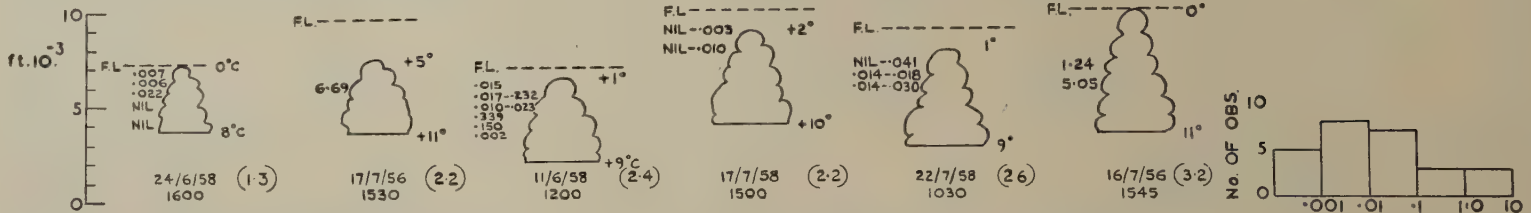
The clouds in this group had thicknesses between 5,000 ft (1,500 m) and 8,000 ft (2,500 m) and theoretical water contents of up to about 4 g m^{-3}. In some cases both supercooled water and ice crystals were observed. The cloud of 13 Feb. 1957 is of special interest because it produced very large concentrations of precipitation particles, mainly ice crystals. The temperature of its top was — 11°C, somewhat lower than the others and it suggests that the Bergeron process was playing an important role in this case. Freezing also appeared to have started in the cloud of 2 July 1958 with a cloud-top temperature of — 6°C, although in this case the concentration of precipitation particles was not high. Discounting the case of 13 Feb. 1957 there appears to be little significant difference between the ability of clouds in groups (*b*), (*d*) and group (*e*) to produce precipitation.

(*f*) *Cumulus clouds with temperatures below 0°C throughout*

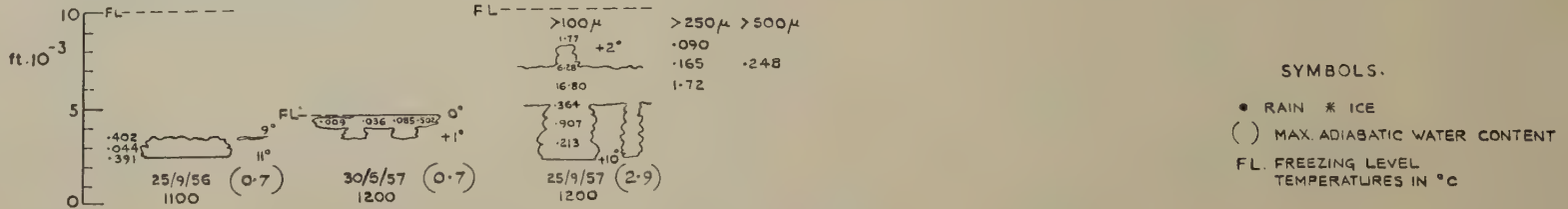
These clouds were from 4,000 ft (1,200 m) to 7,000 ft (2,000 m) thick and had very low adiabatic water content, and 1·8 g m^{-3} being the highest. The low theoretical water content does not seem to be a very important factor in limiting the likelihood of producing precipitation particles as their number is very similar to that in groups (*b*) and (*e*). In two cases, 6 Feb. 1957 when the cloud-top temperature was — 9°C, and 14 Feb. 1957 when it was — 10°C, freezing was apparently taking place and, in the former, there



(a) CUMULUS CLOUDS OF SMALL VERTICAL EXTENT (LESS THAN 2500 FT THICK) AND ABOVE 0° C THROUGHOUT.



(b) CUMULUS CLOUDS OF MODERATE VERTICAL EXTENT (3000FT—8000FT THICK) AND ABOVE 0° C THROUGHOUT



(c) MIXED CUMULUS AND STRATOCUMULUS CLOUDS ABOVE 0° C THROUGHOUT.

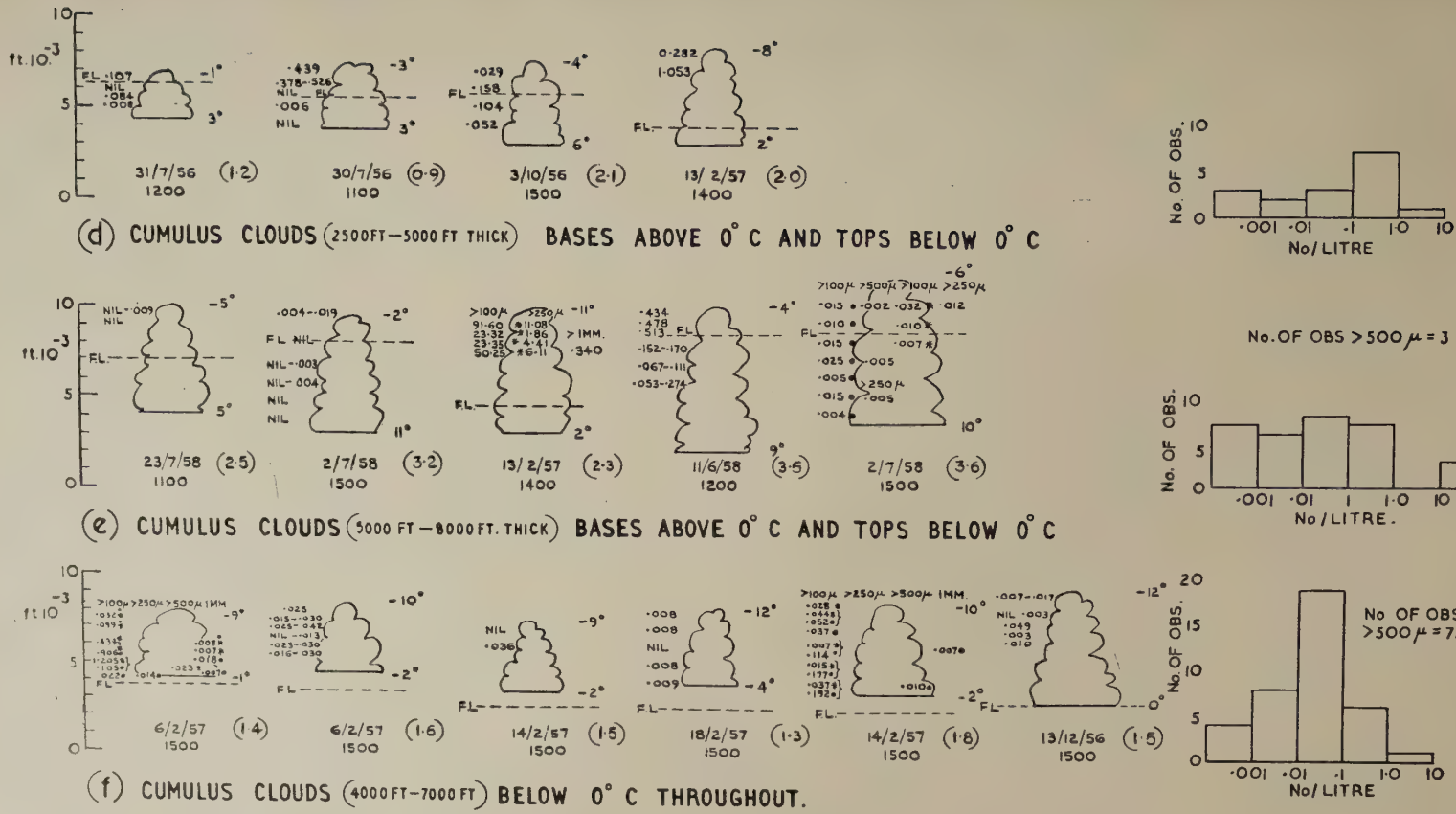


Figure 1 (a), (b), (c), (d), (e), (f). Number per litre of particles $> 100\,\mu$ diameter. Individual observations and frequencies.

was noticeable precipitation. In the latter, in spite of the production of some much larger particles in 500 μ — 1 mm diameter range, the particle concentration was too small to have produced noticeable rainfall at the surface.

In all groups the higher portions of the cloud tended to contain a much larger number of precipitation particles than the lower portions (about 80 per cent compared to 20 per cent below the centre). This would be expected as long as the updraught is capable of supporting the particles. Once drops fall downwards through the cloud, however, they will tend to grow by further collisions and at that stage the larger particles would be expected near the base. In most of the cases examined, the cloud at the beginning of the traverses was hard in appearance and the precipitation process had apparently not developed to this extent.

5. Discussion

The principal factors which determine the growth of droplets in the condensation-coalescence stage are believed to be the population of condensation nuclei, the strength of the updraughts and lifetime of the droplets in the cloud.

Data have been accumulated regarding the larger salt particles (Durbin 1959), from which it appears that near cloud base the average concentration of salt nuclei with mass greater than 10^{-9} g, is about equal to the average concentration of precipitation particles found in the clouds above. This cannot, however, be immediately regarded as establishing a direct relationship between the larger salt nuclei and the precipitation particles and further experiments are necessary to discover whether or not this approximate equality has any physical significance. It has not been possible in this series to demonstrate whether the recent history of the air has any effect on precipitation particle content of the clouds, as in nearly all cases the air had a maritime track. This is, of course, the usual case for the United Kingdom and the type of data given above may be regarded as representative for this country.

The lifetime of a droplet in a cloud is not an independent parameter, being largely, but not entirely, controlled by updraught strength and the size of the cloud. It would be desirable in investigating possible relationships between precipitation particle concentrations and updraughts to be able to measure simultaneously the distribution of updraughts and precipitation particles throughout the clouds. In this investigation it was only possible to make rough estimates of the updraught strength on each occasion in terms of the positive area on the tephigram. No relationship, however, was detected in this way, probably because the sample was too small and the effect overshadowed by other factors.

It would also be useful to be able to relate numbers of precipitation particles as measured in cloud with raindrop concentrations at the surface. Their respective values would be expected to be different as the falling speed of raindrops is so much greater and varies considerably with raindrop size. Raindrop concentration at the surface is usually in the range 0·1 to 1·0/litre, a value exceeded by many of the precipitation particle contents in cloud as measured in this series. In the cases where rain was definitely seen to be falling from the cloud – those of 25 Sept. 1957, 13 Feb. 1957 and 6 Feb. 1957 – the highest measured concentrations of precipitation particles were about 20, 100 and 1/litre respectively. On the other hand, there were several cases where no precipitation was observed below cloud, but the precipitation particle concentration within the cloud was in the range of 1 to 10/litre. In these cases it also seems likely that precipitation must have fallen from the clouds at some stage. It appears that these observations, as well as showing that the condensation-coalescence mechanism is very important in producing precipitation elements, also suggest that it is possible for small amounts of rainfall to fall from warm cumulus clouds of moderate thickness in this country (although the possibility of the rainfall reaching the surface will depend on the amount of evaporation below cloud base).

The few results in group (*c*) indicate that the condensation-coalescence process is more efficient in producing precipitation particles in layer than in cumulus clouds. This might be expected since the updraughts in layer clouds are relatively smaller and therefore a particle takes longer to reach a given height above cloud base. Growth by condensation

nd coalescence continues during this longer period so that more particles reach precipitaion size within a given depth of cloud. Hence, in mixed stratocumulus-cumulus cloud ystems the presence of the stratocumulus may be expected to increase the number of lroplets of precipitation size.

In some of the cases where ice crystals were observed, e.g., 6 Feb. 1957, 14 Feb. 1957 nd 2 July 1958, the concentrations of precipitation particles did not exceed those found n some of the warm clouds. On the other hand, the example of 13 Feb. 1957 which ontained ice crystals had many more precipitation particles than any other case and it nay be that on this occasion the Bergeron process was more fully advanced. Discounting his case the histogram-concentrations for groups (*b*), (*e*) and (*f*) are broadly similar. These results illustrate that in clouds with temperatures entirely above − 10°C there s frequently little sign of any contribution to the precipitation particle concentration rom the Bergeron effect. By the time a cloud has grown to a height where the temperature of its tops is − 10°C it may already have a sizeable concentration of precipitation particles which could possibly lead to some rainfall. The presence of the larger particles may also end to accelerate the freezing process.

The fact that ice crystals were detected on several occasions in these experiments, often in clouds where the temperature of the tops was in the region of − 10°C and nucleaion from other colder sources was unlikely is also noteworthy. This observation is in agreement with a number of previous observations, but it is nevertheless puzzling because neither laboratory observations of the freezing of small drops nor airborne measurements of the numbers of freezing nuclei would lead us to expect any significant number of ice crystals to appear at temperatures above about − 20°C. It has been suggested that the explanation lies in some process of self-multiplication (e.g., splintering) by which a large population of ice crystals could be built up from a very few parent crystals. Possibly this could have happened in the cloud sampled on 13 Feb. 1957.

Acknowledgments

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Appendix

Details of weather and synoptic conditions in southern England

Group (a). Cumulus clouds of small vertical extent and above 0°C throughout

2 July 1958
1500 GMT. There was general stratocumulus cover until 10 hr; thereafter cumulus increased, finally having tops to 15,000 ft or more, with showers. There was variable high cloud and light S'ly winds were reported with a depression off South Devon.

24 June 1958
1000 GMT. Cumulus and stratocumulus were reported until 11 hr. Other clouds with tops to 12,000 ft gave aircraft icing at 10,000 ft and there was some medium and high cloud also. Showers and cumulonimbus cloud were reported after 12 hr. Winds were light W'ly in a col.

26 August 1958
1100 GMT. Cumulus formed from 08 hr onwards in a ridge with light W'ly winds. There were variable amounts of medium and high cloud.

24 June 1958
1600 GMT. As at 10 hr in Group (*a*).

23 July 1956
1100 GMT. Cumulus formed from 08 hr onwards in a ridge. Winds were light W'ly and there were large amounts of upper cloud.

10 July 1958
1500 GMT. Early morning stratocumulus dissipated and cumulus then formed. Winds were light NW'ly in a ridge and there was considerable upper cloud.

Group (b). Cumulus clouds of moderate vertical extent and above 0°C throughout

24 June 1958
1600 GMT. As at 10 hr in Group (*a*). There were many showers and thunderstorms in the afternoon.

17 July 1956
1530 GMT. Morning stratocumulus cloud turned to cumulus around 12 hr. Showers and thunderstorms were observed in East Anglia but not locally. Winds were light SW'ly ahead of a front over Cornwall. Variable medium and upper clouds were reported.

11 June 1958
1200 GMT. There was stratocumulus until 09 hr and then cumulus formed. Some tops reached 10,000 ft and there were a few slight scattered showers. Winds were light and variable in a col or ridge and there were variable amounts of medium and high cloud.

17 July 1958
1500 GMT. Cumulus formed after 09 hr. Winds were light NW'ly and high cloud was reported in a ridge ahead of a warm front. Observations were only possible near the cloud tops as the cloud system was very confused at lower levels.

22 July 1958
1030 GMT. Cumulus formed after 08 hr following morning stratus and stratocumulus clouds. Cumulonimbus developed with showers in the afternoon in a NW'ly unstable air stream behind a cold front. There was an altocumulus cloud around 10,000 ft which gave some aircraft icing.

16 July 1956
1545 GMT. Morning fog lifted to stratus cloud followed by cumulus formation. There were heavy showers in northern England and cumulonimbus clouds were seen over south-eastern England. There was some medium cloud and light variable winds in a col.

Group (c). Mixed cumulus and stratocumulus clouds above 0°C throughout

25 September 1956
1100 GMT. Stratocumulus cloud formed after morning fog which lifted to stratus. During the flight the stratocumulus layer was tending to break into cumulus cloud. Samples were taken in the region of a surface occlusion. There was a trace of rain and winds were light S'ly. Large amounts of medium and high cloud were also reported.

30 May 1957
1200 GMT. Stratocumulus cloud formed during the night and cumulus developed after 08 hr. The cumulus was growing into the stratocumulus base during the sampling period. Winds were light NE'ly in a ridge. There was no upper cloud.

25 September 1957
1200 GMT. There was stratus in the night and stratocumulus from 07 hr. Cumulus was reported from 11 hr and grew through the stratocumulus layer. Precipitation was seen falling from both the cumulus and the stratocumulus clouds. There were light SW'ly winds in a warm sector and variable amounts of medium and high cloud.

Group (d). Cumulus clouds of moderate vertical extent with bases above 0°C and tops below 0°C

31 July 1956
1200 GMT. Cumulus tops spread out to give mixed cumulus-stratocumulus conditions after about 11 hr. There were some showers in East Anglia but not locally. Winds were light W or NW in a ridge. There were no higher clouds.

30 July 1956
1100 GMT. Cumulus formed in the morning and developed throughout the day with showers becoming frequent in the afternoon. There were variable amounts of higher cloud and strong NW'ly winds around a depression off north-eastern Scotland.

3 October 1956
1500 GMT. Cumulus formed about 09 hr and tended to spread out after about 11 hr. There was an inversion at about 9,000 ft and slight showers from clouds about 6,000 ft thick. There were variable amounts of upper cloud and moderate NW winds to the rear of a cold front.

13 February 1957
1400 GMT. Cumulus formed about 09 hr and cumulonimbus was reported after 16 hr with frequent showers in the afternoon. The clouds sampled had ' hard tops ' and gave moderate aircraft icing. There were variable amounts of upper cloud and moderate SW'ly winds with a depression over northern England.

Group (e). Larger cumulus clouds with bases above 0°C and tops below 0°C

23 July 1958
1100 GMT. There were reports of stratocumulus in the early morning and cumulus from 09 hr onwards. The sampling was carried out near the cloud tops which were dense with clear moderate icing. Slight showers were reported locally. There were small amounts of high cloud and moderate W'ly winds with a depression over Scandinavia.

2 July 1958
1500 GMT. As in Group (*a*).

13 February 1957
1400 GMT. As in Group (*d*).

11 June 1958
1200 GMT. As in Group (*b*).

Group (f). Cumulus of moderate vertical extent below 0°C throughout

6 February 1957
1500 GMT. There were reports of cumulus from 10 hr onwards in a ridge with moderate SW'ly winds. There were small amounts of upper cloud. Two clouds which formed part of a cloud street were sampled and precipitation was seen below the base of one. There were surface reports of a few light scattered showers.

14 February 1957
1500 GMT. Cumulus and stratocumulus clouds were reported up to 14 hr and cumulonimbus afterwards with variable medium and high cloud. There were occasional showers giving up to 1 mm precipitation. Winds were moderate W'ly with a depression over Denmark. The clouds sampled had ' hard tops ' and moderate icing and turbulence were reported at all levels above 5,000 ft.

18 February 1957
1500 GMT. High cloud only was reported up to 12 hr and cumulus, stratocumulus and cumulonimbus later with scattered light showers. Winds were light SW'ly in a flat low-pressure area.

13 December 1956
1500 GMT. Cumulonimbus and stratocumulus were reported before 09 hr but cumulus only between 11 and 16 hr. Showers also occurred during the morning. Winds were strong W'ly with a depression near Iceland.

551.574.1 : 551.594.25 : 536.42

The fragmentation and electrification of freezing water drops

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Summary

The freezing of water drops is often accompanied by shattering of the ice shell and the production of small ice splinters. These phenomena have been studied in relation to the drop diameter in the range 30 μ to 1 mm, the nucleation temperature of the drop, the air content and purity of the water. The number of splinters produced is largely governed by the nucleation temperature (the degree of supercooling) which controls the quantity of air released during freezing and is not very dependent on drop size. Slightly supercooled drops of $\frac{1}{10}$-1 mm diameter produced, on average, 20 to 50 splinters. The mechanism of drop fragmentation is discussed and the potential importance of splintering in the ice-nucleus economy of clouds is assessed.

Fragmentation of freezing drops is accompanied by electrical charging, usually with the splinters positively charged and the drop residue carrying a negative charge. The magnitude of the residual charge which, on average, is observed to be about 10^{-3} e.s.u. for a millimetre drop, is related to the same factors which control splinter production. Charging by this mechanism would not appear to be of major importance in the electrification of thunderstorms.

1. Introduction

The production of ice crystals by the bursting of freezing water drops has been observed directly by Mason (1956a), Langham and Mason (1958), and has been inferred from the occasional appearance of small clusters of ice crystals in cloud chambers as noted by Schaefer (1952), Hallett (see Mason 1956a), and Bigg (1957a). Having photographed the ejection of up to 5 ice splinters during the freezing of freely-falling droplets of 50-100 μ diameter, and having observed the production of upwards of 100 crystals during the explosive rupture of suspended millimetre-size drops, Langham and Mason (1958) suggested that this mechanism might be an important factor in the ice-nucleus economy of supercooled clouds. Much more detailed information was, however, required on the rate of splinter production in relation to the size of the drops, their rate of cooling, freezing temperatures, and the purity of the water. The object of the work described in this paper was to provide this information.

It was soon evident that many of the splinters produced by freezing drops were strongly charged and, indeed, the separation of electric charge during the formation of rime, probably caused by the ejection of ice splinters, was noted earlier by Findeisen (1940; 1943) and by Kramer (1948). Because of its potential relevance to the generation of thunderstorm electricity, we have also investigated the electrification of freezing drops in some detail.

PART 1. THE BURSTING AND FRAGMENTATION OF FREEZING DROPS

2. Experimental technique

The drops under test were suspended in the centre of a small refrigerator cell in which the temperature and humidity of the air could be controlled. The cell was a 3·5 cm insulated cube cooled on two sides and the base by chilled petrol circulating between the double metal walls; windows in the other two sides allowed illumination and observation of the drops. The wall temperature and consequently that of the air in the cell was variable over the range 0°C to − 40°C by adjusting the current through a heating coil wound on the copper spiral carrying the cooling fluid. The presence of a temperature inversion in the chamber, causing the air to be stably stratified, damped out convection; the vertical temperature gradient in the middle third of the cell varied with the base temperature but was about 3°C/cm.

After preliminary experiments with a number of supports had shown that, provided the drop was suspended in such a way that it was not greatly distorted from the spherical, the nature of the suspension had no discernible influence on its freezing behaviour, in all subsequent tests large drops, of millimetre size, were suspended from the end of a fine, shellac-coated thermocouple which permitted the degree and duration of the supercooling and the freezing temperature to be measured directly. Smaller droplets, of order 100 μ diameter, were hung from the under-surfaces of fine, horizontally-mounted amyl-acetate fibres, the air temperature at the level of suspension being measured with a thermocouple. The drops were illuminated from the side by a microscope lamp fitted with a heat filter and observed and photographed under a low-power, double-eyepiece microscope.

During the observation of each freezing drop, special note was made of such features as the freezing temperature, the optical appearance of the drop during the various stages of freezing, the appearance of cracks, fissures and spikes, and whether or not shattering of the ice shell occurred. The number of ice splinters produced during freezing was observed by allowing them to fall into a shallow tray of supercooled sugar solution, placed on the floor of the cell, in which the splinters grew into dense white ice crystals of visible size, which could be easily counted. This technique was invented by Bigg (1957b). To prevent the formation of frost splinters from the walls of the chamber, the latter were coated with glycerine so that ice crystals appeared in the sugar solution only during the freezing of the drop.

3. The influence of nucleation temperature, rate of freezing and drop size on the freezing behaviour

Some typical freezing curves for millimetre drops *supercooled to about* $-15°C$ are shown in Fig. 1. They show that freezing occurred in two distinct stages. The first stage involved the formation, within less than one-tenth sec., of a mottled, opaque, outer shell of ice containing numerous tiny air bubbles, and accompanied by a rapid temperature rise towards 0°C. During the second stage, which lasted for 30 to 180 sec depending upon the ambient air temperature, the liquid interior gradually froze as the latent heat of fusion was dissipated mainly by conduction to the air, but also by local convection

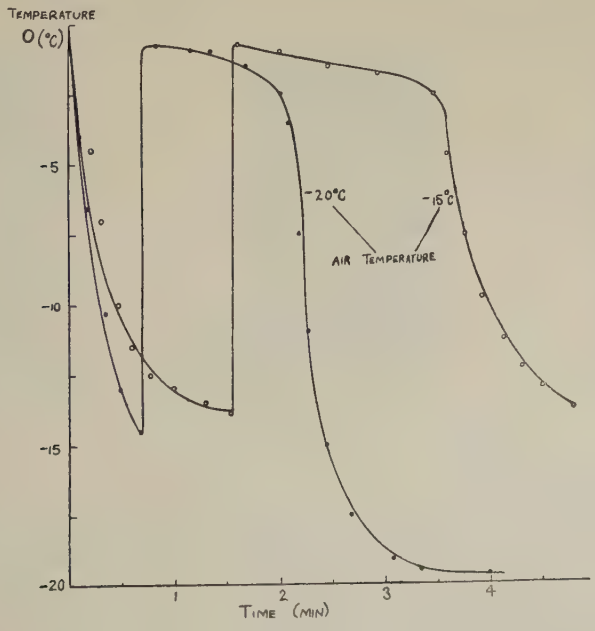

Figure 1. Time-temperature curves for the freezing of 1 mm-diameter water drops.

set up around the drop and by slow evaporation from the warmer ice surface. The observed durations of this second stage, represented by the flat portions of the curves in Fig. 1, are in good agreement with those calculated from the theory of Mason (1956b).

During this phase, liquid from the drop interior seeped through numerous pores in the ice shell to spread and freeze over the surface. As the ice shell thickened, the drop gradually became more transparent. The end of this stage was marked by the appearance of large numbers of interior cracks and by a rapid fall in temperature from near 0°C towards the environmental temperature. A small proportion of drops produced a few spicules, less than one-fifth mm long, towards the end of the freezing period and a few (on average about five) ice splinters – see Table 1.

TABLE 1. STATISTICS ON SHATTERING AND SPLINTERING OF FREEZING DROPS

Nucleation temp. (°C)	Air temp. (°C)	No. of drops	Drops forming spikes No.	Drops forming spikes (per cent)	Drops shattering No.	Drops shattering (per cent)	Splinters produced Total	Splinters produced Maximum per drop	Splinters produced Average per drop
			(a) Drop radius = 1 mm						
0	− 2	14	10	72	4	29	–	–	–
− 1	− 10	90	85	91	42	47	4,350S 485NS	150 16	54
− 1	− 10	41	34	83	18	44	–	–	–
− 2	− 15	42	24	57	9	21	–	–	–
− 2	− 20	18	12	67	7	39	–	–	–
− 4	− 10	45	18	40	10	22	900S 310NS	200 12	27
− 7	− 10	36	3	8	2	6	220S 240NS	160 15	13
− 9	− 17	10	0	0	0	0	44	10	4
− 13	− 15	35	A few tiny spicules only		0	0	141	21	4
− 14	− 20	25			0	0	110	15	4
− 16	− 25	10			0	0	55	18	6
			(b) Drop radius = 300-400 μ						
− 1	− 10	10	9	90	4	40	–	–	–
− 1	− 12	30	30	100	8	27	950S 245NS	200 30	40
− 2	− 20	10	7	70	2	20	–	–	–
− 15	− 20	15	0	0	1	6	50S 28NS	50 8	5
			(c) Drop radius = 100-150 μ						
− 2	− 15	78	30	39	12	15	–	–	–
− 5	− 15	30	8	27	4	12	420	–	14
− 15	− 15	222	15	7	2	1	600	–	3
			(d) Drop radius = 80-120 μ						
0	− 5	216	112	51	55	25	–	–	–
0	− 6	35	21	60	12	35	760	–	22
			(e) Drop radius = 30-80 μ						
0	− 5	92	52	57	24	26	–	–	–
0	− 6	51	22	43	14	27	–	–	–
− 5	− 6	18	—	—	2	11	142	–	8

S = with spikes; NS = no spikes.

When the drops were made to freeze *at much higher temperatures, only a degree or two below 0°C*, either by nucleating them with silver iodide or tiny ice crystals, there were marked changes in behaviour. In the first stage, a thin, transparent film of ice spread rather slowly across the surface of the drop on which ridges and dendritic ice patterns could often be seen. During the second stage, as the ice shell thickened, a bulge often appeared on the surface and developed rapidly into a long spike (see Fig. 2, Plate I),

(i) 10 sec
after nucleation

(ii) 15 sec

(iii) 25 sec

(iv) 35 sec

(v) 50 sec

(vi) 65 sec

(vii) 78 sec

(viii) 78·1 sec

(ix) 78·5 sec

Figure 2. A time sequence of photographs showing spike formation and shatter of 1 mm-diameter water drops. Nucleation temperature — 1°C; air temperature — 10°C.

most frequently in a series of rapid, interrupted steps. In their early stages, the spikes were usually transparent and appeared to consist mainly of liquid within a thin sheath of ice, but, later, freezing of the liquid released air bubbles which migrated sometimes to the tip of the spike and sometimes back into the interior of the drop. The final stages of freezing were usually accompanied by extensive crack formation throughout the drop, followed often by violent shattering into two or more pieces, as shown in Fig. 2 (Plate I). This explosive rupture of the frozen drop almost always produced large numbers of splinters.

Statistics on the formation of spikes, shattering and splinter production are given in Table 1 which clearly show that the frequency of all three processes decreases quite rapidly as the nucleation temperature of the drop falls and is highest in drops which are only slightly supercooled. The rate of freezing was varied by altering the air temperature but this appeared to have little effect.

The effect of the drop size on splinter production was investigated by observing the freezing of several hundreds of drops of radius 80-400 μ, produced by a vibrating capillary, and of 30-100 μ droplets produced by a spinning disc. From the statistics given in Table 1, it will be seen that, in the range 100 μ to 1 mm, the fraction of drops forming spikes and shattering does not greatly decrease with decreasing drop size nor is the number of splinters produced significantly less. For the smaller drops, the nucleation temperature is again the controlling factor.

4. The influence of dissolved air and other impurities on splinter production

Blanchard (1957) pointed out that the formation of clear or opaque ice during the freezing of very large water drops was related, through the degree of supercooling, to the quantity of air dissolved in the water, and Langham and Mason (1958) suggested this might also control the degree of drop shattering and splinter production by influencing the mechanical properties of the ice shell.

In the present experiments, the freezing behaviour of drops of de-aerated water was compared with that of similar drops of distilled water saturated with dissolved air. The de-aerated samples were taken from water which had been boiled and allowed to cool in sealed bottles; because these drops would have again rapidly taken up air while being cooled down to freezing point in the cold cell, the latter was filled with hydrogen whose solubility in water is only half that of air.

The freezing of twenty 1 mm drops of de-aerated water at $-13°C$ was markedly different from that of a similar group of aerated water drops frozen under the same conditions. Eight of the de-aerated drops formed large spikes, four shattered and altogether 400 splinters were produced, an average of 20 per drop. None of the aerated drops shattered and splinter production averaged only 4 per drop. On the other hand, twenty 1 mm drops of de-aerated water, nucleated very near 0°C by seeding them with tiny ice crystals, gave results very similar to those obtained for drops containing air.

Repeated melting and re-freezing of the same drops in air produced no significant changes in their freezing behaviour; presumably the air released during melting was largely re-absorbed during the cooling process. However, this treatment, carried out in an atmosphere of hydrogen, caused a marked increase in the frequency of spike formation and of shattering during freezing.

The freezing of drops containing suspensions of insoluble particles, mostly in the diameter range 0·2-1·0 μ, and in concentrations of up to 10^5 cm^{-3}, indicated that solid impurities had little effect on drop bursting except in so far as they might raise the nucleation temperature. Thus a suspension of silver iodide greatly increased splinter production, but a suspension of inert nuclei had no noticeable effect.

While no discernible changes were produced when distilled water was replaced by tap water or by 10^{-3} N salt solutions, drops of a 0·2 N solution of sodium chloride showed no distortion, produced no spikes and only an occasional splinter when nucleated at

either — 12°C or 0°C. It appeared that freezing, particularly in the later stages, of these concentrated solution drops was accompanied by little change in volume; and indeed, some direct measurements on much larger samples indicated that the overall increase in volume accompanying the freezing of a 0·2 N salt solution is only about one-half of that for pure water.

5. The mechanism of drop fragmentation

The experimental results described above clearly indicate that the extent to which a freezing drop shatters and produces ice crystals is controlled largely by the temperature at which it freezes, the drop radius (over the range 100 μ to 1 mm), and the rate of cooling having little influence. The air content of the water is of great, and probably fundamental, importance; this will be closely controlled by the drop temperature since the solubility of air in water increases rapidly with decreasing temperature, rising by 25 per cent from 37 mg/litre at 0°C to 46 mg/litre at — 10°C.

In general terms, fracture of the ice shell formed during the first stage of the freezing process, is caused by the expansion which accompanies the subsequent freezing of the liquid interior. Nucleation of the drop, at temperature — T_f°C, which may occur either on the surface or in the interior, is followed by rapid solidification of a fraction $T_f/80$ of its mass; the latent heat of fusion released raises the drop temperature everywhere to 0°C. This partial freezing is accomplished by a forest of dendritic crystals spreading very rapidly through the drop and around its surface; these entrap numerous tiny air bubbles which are released because the solubility of the air in ice is only about one per cent of that in the liquid. Further freezing now proceeds radially inwards at a rate controlled by the dissipation of latent heat to the environment. The changes brought about by the consequent increase in expansion depend very much on the nucleation temperature and air content of the water.

When a slightly supercooled drop is nucleated by an ice crystal, freezing occurs initially only in the surface layers, where the very small quantity of air released can escape to the atmosphere, to produce a very thin, transparent shell of ice which is mechanically quite strong. Rupture of the shell, under pressure from the solidifying interior, occurs at a weak spot where there first appears a bulge which may later develop into a spike as liquid from the interior is extruded. If expansion of the interior cannot be accommodated by extrusion through spikes and through the nearly impervious shell, large stresses develop which may eventually cause the drop to explode and produce two or more large fragments and large numbers of small ice splinters.

When a drop freezes at much lower temperatures, larger quantities of air are released and become trapped in the ice to give it a spongy texture. This yields much more readily to the expansion, part of which is taken up by compression of the trapped air. Numerous cracks and fissures appear in the ice shell through which the liquid is exuded but the occurrence of spikes and violent shattering is rare and splintering production is greatly reduced.

6. The meteorological significance of drop splintering

In evaluating the potential importance of drop splintering in the ice-nucleus economy of clouds we recall that the average number of ice splinters produced by a freezing drop is not very sensitive to the drop diameter over the range 100 μ to 2 mm but falls off rather rapidly as the freezing temperature is lowered. For a 1 mm-diameter drop, typical values are :

T (°C)	No. of splinters s
0	50
— 5	18
— 10	6
— 20	1

so $$s = s_0 e^{bT}, \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where $s_0 = 50$ and $b = 0{\cdot}2$.

We consider first, the rate of production of splinters by raindrops freezing through activation of their contained freezing nuclei.

The probability P that a drop of volume V will freeze within t sec at a temperature T°C may be evaluated from the experimental data of Bigg (1953) and of Langham and Mason (1958) on the heterogeneous freezing of water drops, thus

$$P \simeq \alpha V t \, e^{-aT} \quad . \quad . \quad . \quad . \quad (2)$$

where α is a numerical factor depending upon the purity of the water and $a = 1{\cdot}0$. From experiments on the freezing of drops of rainwater, Langham and Mason found $\alpha = 2 \times 10^{-8}$.

If the concentration of raindrops in the cloud is N cm^{-3}, the number of ice crystals produced per unit volume per unit time by freezing raindrops will be

$$\mathcal{N} = \frac{NPs}{t} = N\alpha V s_0 e^{-(a-b)T} \quad . \quad . \quad . \quad . \quad (3)$$

Putting $N = 10^{-4}$ cm^{-3}, drop radius = 1 mm, $T = -10$°C, $\alpha = 2 \times 10^{-8}$

$$\mathcal{N} = 1{\cdot}2 \times 10^{-9} \text{ cm}^{-3} \text{ sec}^{-1}.$$

Thus raindrops freezing on their own account will produce only about one crystal per 10 m^3 at -10°C within 100 sec and increase the normal concentrations of ice nuclei by only about 1 per cent in this time.

Let us now suppose that the supercooled raindrops freeze as a result of capturing ice crystals which have formed on natural nuclei and have grown to be such a size (a few hundred microns) that their collision efficiency for raindrops is nearly unity.

N raindrops per unit volume of radius R, falling at velocity V relative to small ice crystals in concentration n cm^{-3}, will remove crystals at a rate $E\pi R^2 NnV$ cm^{-3} sec^{-1}, and the overall rate of crystal production by splintering will be

$$\left(\frac{dn}{dt}\right)_s = \pi n (s-1) \Sigma ENR^2 V \quad . \quad . \quad . \quad . \quad (4)$$

Putting $E = 1$, neglecting the fall-speed of the crystals compared with that of the raindrops, and using the raindrop-size data of either Best (1950) or Marshall and Palmer (1948) to relate ΣNR^2V to the rate of rainfall, p, in mm hr^{-1}, viz. : $\Sigma NR^2 V = \dfrac{3}{4\pi} \dfrac{P}{3{\cdot}6 \times 10^3 R_n}$ approximately; $R_n = 0{\cdot}46\, p^{1/5}$ where R_n is the median-volume diameter in mm. We have

$$\left(\frac{1}{n}\frac{dn}{dt}\right)_s = 4 \times 10^{-4} (s-1) p^{4/5}, \quad . \quad . \quad . \quad (5)$$

and $$n = n' \exp\left[4 \times 10^{-4} (s-1) p^{4/5}\right] t, \quad . \quad . \quad . \quad (6)$$

where $n = n'$, when $t = 0$. We may take n' to be the initial concentration of natural ice nuclei.

Eq. (6) allows calculation of the factor by which the ice-crystal concentration at a particular cloud level will be multiplied by splintering in the absence of an updraught. At the -5°C level, for which $s = 18$, rain falling at 1 mm hr^{-1} would produce a ten-fold increase of crystals within little more than 5 min. At the -10°C. level the corresponding increase would be only two-fold.

These rates of multiplication are not very impressive, but crystal production could be further increased if a high proportion of the raindrops in one thermal settled into a succeeding thermal, so that the same drops could cross, say, the $-5°C$ level, more than once. For example, we may imagine rain, falling at about 1 mm hr^{-1} in still air, carried up in a thermal to the $-10°C$ level, above which further growth of the cloud is checked by an inversion or a dry, stable layer. The raindrops now fall back at nearly their terminal velocities in still air towards the 0°C level, and continue to cross the $-5°C$ level for 5 min, thereby increasing the crystal concentration there by a factor of ten. We now suppose that much of this rain is caught up in a new thermal (this is likely to occur without serious dilution only in the absence of strong wind shear so that successive thermals will follow each other along nearly vertical paths) and the process is repeated to give another ten-fold increase in crystal concentration at the $-5°C$ level on the upward journey, and a similar increase on the downward journey. Thus, it is not unreasonable to argue that the crystal concentration may be increased by a factor of 10^4, to perhaps one per litre, in about 20 min, and so attain a level sufficient to endow the cloud with signs of glaciation even though its summit temperature is only a few degrees below 0°C.

Such a mechanism might explain the glaciation, noted by Ludlam (1955), Ludlam and Saunders (1956), of weak, shallow cumulus having summit-level temperatures of only about $-5°C$.

Heavy rain, falling through the $-10°C$ to 0°C layer could produce very high rates of crystal production by splintering. For example, rain falling at 10 mm hr^{-1} for 5 min would cause an increase of about 10^5 at the $-5°C$ level and a 50-fold increase at the $-10°C$ level. At this later stage in the cloud development, however, it is unlikely that this process will greatly influence the development of precipitation.

PART 2. THE ELECTRIFICATION OF FREEZING DROPS

In earlier experiments we had observed that some of the larger ice splinters produced by the freezing of water drops in a diffusion cloud chamber were deflected in an electric field and therefore presumably carried an electric charge. As it would have been impracticable to measure the charges on individual splinters, mainly because they either grew or evaporated according to the humidity prevailing in the chamber, we decided to measure the charge on the drop residue to give the total net charge separated during the fragmentation process.

7. Experimental technique

The drops were suspended in the centre of the cold cell (as described in § 2) from the tip of a vertically-hanging, insulated fibre between two parallel brass electrodes, 2 cm square and 0·8 cm apart, across which was applied a steady potential of a few hundred volts. The fibres, drawn from molten polythene tubing, were 3 cm long, 10-50 μ in diameter and thickened at their tips to about 100 μ diameter in order to retain the water drops transferred from a fine-glass pipette. Subsidiary experiments established that the fibres did not become appreciably charged during transfer of the drops, on application of the field, nor by collision with tiny airborne ice crystals.

The procedure involved hanging a drop on the fibre, placing the latter in the cold cell through a small hole in the top, noting the position of the end of the fibre relative to a scale in the eyepiece of the viewing microscope, and continuous observation of the drop while it cooled to the freezing point. There was very little movement of the fibre until freezing followed by shattering of the drop occurred; then a sudden deflexion indicated the presence of an electric charge. The magnitude of this deflexion usually remained sensibly constant for several minutes thus indicating the excellent insulating properties of the fibre. The difference between this final deflexion and that observed at the instant of nucleation was attributed to separation of charge by shattering and splintering of the drop.

The charge q on the residue of the frozen drop is given, in terms of its mass m and its deflexion θ from the vertical, in a horizontal electric field F by the equation

$$Fq = mg \tan\theta \simeq mg\frac{x}{l},$$

where x is the horizontal displacement of the centre of the drop and l the length of the fibre. The mass m of the residue was determined by re-melting it and measuring the diameter of the resulting drop.

8. Results

The majority of the measurements were made on 1 mm-diameter drops of doubly-distilled water filtered through an ion-exchange column to reduce the conductivity to $6 \times 10^{-7}\ \Omega^{-1}\ \text{cm}^{-1}$, but the effect of dissolved electrolytes was investigated with drops containing sodium chloride in concentrations varying from $10^{-6}\ N$ to $1\ N$.

The smallest charge that could be measured with 1 mm drops was 5×10^{-5} e.s.u. No detectable charge was produced unless freezing was followed by shattering of the drops. Table 2 divides the observations into two categories, according to whether more than half, or less than half of the drop-mass was left on the fibre after shattering.

TABLE 2. The electrification of freezing water drops

Purity	D (mm)	T_f (°C)	T_a (°C)	No. of drops	Av. charge per drop (e.s.u. × 10^3)	Shattered drops N_+	N_-	N_T	$\bar{q}_+$	$\bar{q}_-$	$\bar{q}$ (e.s.u. × 10^3)
					(a) Major residues						
Distilled	1	0	− 10	213	− 0·33	22	61	83	+ 1·35	− 1·62	− 0·86
Distilled	1	− 2	− 30	45	− 0·17	2	6	8	+ 0·46	− 1·46	− 0·98
Distilled	1	− 15	− 15	49	− 0·03	0	2	2	–	− 0·72	− 0·72
Distilled	0·35	0	− 6	46	− 0·12	1	14	15	+ 0·08	− 0·41	− 0·37
$10^{-6}\ N$	1	0	− 10	49	− 0·16	2	14	16	+ 0·18	− 0·58	− 0·49
$10^{-4}\ N$	1	0	− 10	45	− 0·04	2	7	9	+ 0·17	− 0·28	− 0·18
$10^{-2}\ N$	1	0	− 10	54	− 0·01	2	6	8	+ 0·05	− 0·11	− 0·07
$1\ N$	1	0	− 10	10	0	0	0	0	–	–	0
					(b) Minor residues						
Distilled	1	0	− 10	27	–	14	13	27	+ 1·22	− 0·67	+ 0·31
Distilled	1	− 2	− 30	5	–	2	3	5	+ 3·68	− 3·45	− 0·60
Distilled	1	− 15	− 15	1	–	1	0	1	+ 0·80	–	+ 0·80
Distilled	0·35	0	− 6	4	–	1	3	4	+ 3·05	− 0·52	+ 0·38
$10^{-6}\ N$	1	0	− 10	6	–	2	4	6	+ 0·32	− 0·47	− 0·21
$10^{-4}\ N$	1	0	− 10	5	–	3	2	5	+ 1·79	− 0·52	+ 0·87
$10^{-2}\ N$	1	0	− 10	1	–	1	0	1	+ 0·03	–	+ 0·03
$1\ N$	1	0	− 10	0	–	–	–	–	–	–	–

Shattering of a drop always produced a charge. Major residues (comprising more than half the original mass) were, on average, negatively charged with negative drops being 3 times more frequent; about half of the minor residues were positively charged. Reasons for this will be discussed in § 9.

Most frequent shattering and greatest electrification was observed with millimetre drops of distilled water nucleated near 0°C. The distribution of charges obtained for 83 major drop residues and 27 minor residues of this kind are shown in Fig. 3. The maximum charge observed was $-7{\cdot}2 \times 10^{-3}$ e.s.u., the smallest $-1{\cdot}1 \times 10^{-4}$ e.s.u. The rather large variation in the charges produced by individual drops is perhaps to be expected in view of the uncontrolled nature of their fragmentation. However, for 50 per cent of the major residues in Fig. 3, the charges lie in the range $-0{\cdot}2$ to $-2{\cdot}0 \times 10^{-3}$ e.s.u., the overall average value being $-0{\cdot}86 \times 10^{-3}$ e.s.u.

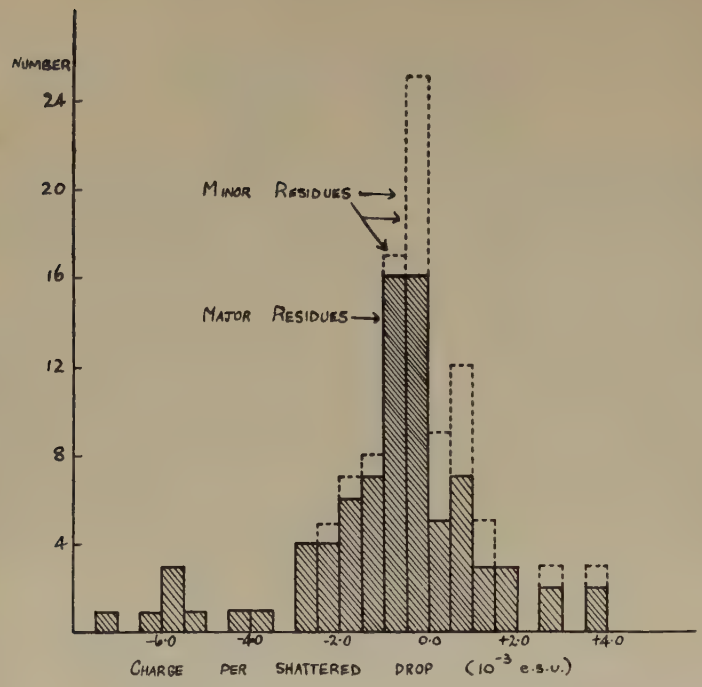

Figure 3. The distribution of electric charges on the residues of shattered water drops.

The freezing, under similar conditions, of rather smaller drops, 0·35 mm instead of 1 mm in diameter, produced charges smaller by a factor of two, but the data are insufficient to establish a size effect. Drops of salt solution, frozen in otherwise similar conditions, not only shattered less frequently as described in § 4, but even for the drops that did rupture, charging fell progressively as the concentration of the solute was increased; shattering and electrification was inhibited altogether when the concentration exceeded about 10^{-2} *N*.

9. The mechanism of electrification

It is suggested that the electrification following the shattering of freezing drops may be explained by the preferential migration of positive ions (protons) down the temperature gradient established in the ice shell during the freezing process. Reynolds's (1954) observation that charge separation occurs when two pieces of ice of slightly different temperatures are brought into momentary contact, the colder ice obtaining a positive charge, is probably another manifestation of the same phenomenon. A detailed theory of the process has been worked out by Mason and will appear elsewhere.

Essentially, the mechanism depends on two facts, (i) the concentration of free H^+ ions and OH^- ions in ice rises quite rapidly with increasing temperature; (ii) the diffusion coefficient of the positive ion is at least 10 times greater than that of the negative ion. Thus the establishment of a temperature gradient in a piece of ice will be accompanied by a concentration gradient of ions of both signs. The more rapid initial diffusion of the positive ions down the concentration (temperature) gradient will lead to a separation of charge with a net excess of positive charge in the colder portions. The electric field associated with this space charge will, of course, tend to accelerate the negative ions and slow down the positive ions and thus oppose further charge separation. If a steady temperature gradient is maintained, an equilibrium state will eventually be reached in which no net flow of current occurs, but with the cold end positively charged and the warm end negatively charged. If, on the other hand, the temperature gradient declines (this will be the case with two pieces of ice of initially different temperatures brought into contact) the separation charge will increase to a maximum and fall to zero when

sothermal conditions are established. Separation of the two pieces prior to this condition eing established will cause a separation of charge.

During the freezing of water drops, a temperature gradient is established across the ce shell as its outer surface cools towards the ambient air temperature and its inner urface is maintained at 0°C by freezing of the liquid interior. Thus, according to Mason's heory, protons will migrate preferentially into the colder, outer layers to build up an xcess of positive charge. If now, shattering and splintering results in pieces being ejected nly from the outer faces of the shell, these will carry away a positive charge and leave corresponding negative charge on the drop residue. If, however, a major fraction of he drop is blown off, the remainder may acquire either a positive or a negative charge lepending upon the fraction of the inner contents retained. The theoretical calculations, nade for very pure water and ice, predict a maximum charge separation of about 10^{-4} e.s.u. n a drop of 1 mm diameter. The measured charges of Table 2 are sometimes an order of nagnitude higher than this, but the extent to which additional charging may result rom the presence of trace contaminants, e.g., dissolved carbon-dioxide and ammonia, rom the selective incorporation of the positive ion into the ice during freezing, as reported by Workman and Reynolds (1950), and from frictional effects accompanying the mechanical upture of the ice has yet to be investigated.

10. Meteorological significance of the electrification of freezing drops

In order to compare the quantity of electric charge which might be generated and eparated by the freezing of raindrops with that involved in thunderstorms, let us suppose hat *all* the raindrops comprising a total rainfall of 2·5 cm were to freeze and splinter before crossing the 0°C level. The total quantity of charge liberated would then be

$$Q = AP\frac{\bar{q}}{\bar{m}},$$

where A is the area of the storm, P the total rainfall, $\bar{m}$ the mean raindrop mass, and $\bar{q}$ the average charge per drop produced by splintering.

Putting $A = 10\ \text{km}^2$, $P = 2{\cdot}5$ cm, $\bar{m} = 10^{-2}$ g, $\bar{q} = 10^{-3}$ e.s.u.,

$$Q = 8{\cdot}3 \text{ coulombs.}$$

This is very small compared with the charges of order 1,000 coulombs which are generated in typical thunderstorms.

It therefore appears that the charging which accompanies the splintering of freezing raindrops does not contribute appreciably to the electrification of the thunderstorms. It is conceivable, however, that the electrification which accompanies the growth of soft-hail pellets by riming may be caused by splintering of *cloud* droplets during their freezing. This, however, requires further investigation.

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ADDENDUM

The fragmentation and electrification of very small droplets

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To examine the freezing of very small droplets, a different technique had to be adopted. A glass-fronted cold chamber was constructed in which a water cloud could be formed by the introduction of moist air. The chamber was first cooled to a temperature below that required for the spontaneous nucleation of the cloud droplets, so that some seconds after the formation of the water cloud, ice crystals began to appear among the droplets. These crystals then fell into a small cell where they could be subjected to a horizontal electric field. Any deflexion of their fall from the vertical then showed the sign and magnitude of any charge carried by the crystals. If charging took place, it could be deduced that splintering accompanies their freezing by spontaneous nucleation.

Two vertical brass electrodes, 2 mm apart, formed the sides of the cell which was 37 mm high and 7 mm deep. The front and rear surfaces were of glass so that a suitable lighting arrangement made it possible to follow individual particles during their fall. Fields of up to 15,000 v cm^{-1} were used during the experiment, and as deviations from the vertical of 3° could be detected with ease, single electronic charges would be apparent on particles of diameter less than 10 μ.

The sizes of the droplets were found by the application of Stokes's law to their fall-speeds as determined by timing against a scale in the cold chamber. For the most part, the diameters lay between 4 and 13 μ, but values between 5 and 6 μ were most common.

The experiments showed no detectable charging of the droplets, indicating complete absence of electrification to at least 10 μ. Very occasionally, singly charged particles were observed, possibly more often positive than negative, but their charges were almost certainly derived from ionization within the cold chamber and may be taken as of no significance.

The complete absence of electrification in these small droplets is consistent with there being no splintering when they freeze by spontaneous nucleation.

551.574.1 : 551.576.11

Size-distribution of ice crystals in cumulonimbus clouds

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(Manuscript received 27 July 1959)

Summary

Ice-crystal size-distributions have been measured in clouds associated with the intertropical convergence zone and more isolated cumulonimbus clouds in temperate zones. Mean size-distributions fit curves $N_D = \frac{1{,}000}{D^3}$ and $N_D = 1{,}000/D^{2\cdot3}$ in tropical and temperate clouds respectively where $N_D\,\delta D$ is the number per m^3 with diameters between D and $D + \delta D$ mm. The tropical curve is a close fit throughout the measured range of D (0·05 to 4 mm) but the temperate curve is applicable in the range up to $D = 2$ mm only, thereafter a curve $N_D = \frac{10{,}000}{D^{5\cdot4}}$ fits the curve for $D = 2$ to 5 mm. In temperate clouds there is little variability with temperature except for a decrease in size of the largest particles observed as temperature decreases.

There is some evidence that the particles caught are graupel of mean density 0·6 g cm^{-3} and, on this assumption, variation with total ice-crystal concentration suggests an empirical distribution law for particles with diameters 0·25 mm or greater of $N_D = 10^{4\cdot1} \exp\left[-\frac{2\cdot67}{W^{\frac{1}{3}}}(D - 0\cdot25)\right]$ where W is the ice-crystal content in g m^{-3}.

1. Introduction

A number of flights by Meteorological Research Flight (M.R.F.) aircraft has been made through cumulonimbus clouds during which ice-crystal imprints have been recorded using the aluminium-foil technique (Bigg, McNaughton and Methven 1956; Garrod 1957). In addition, some records were obtained on Britannia aircraft during proving trials in the tropics and have been made available to the author by the kind cooperation of Bristol Aero-Engines Ltd. and the Mechanical Engineering Department of the Royal Aircraft Establishment (R.A.E.).

Detailed analysis of the records presents a formidable problem on which much time could be spent but for cumulonimbus records it is doubtful whether such expenditure would be justified. A glance at the records reveals, as expected, the extreme variability in number and size of ice crystals not only from flight to flight but also from second to second within a single flight (in one second the aircraft used travelled about 100 m). With extreme variations within this length-scale it is clearly impossible to examine usefully individual clouds with a single aircraft in order to determine variations in time at a particular level within the cloud or in the vertical at a particular time. All one can do is to treat the problem statistically by making sufficient readings at a sufficient number of levels in a sufficient number of clouds.

2. Data available

(*a*) *M.R.F. data*

Flights by M.R.F. aircraft were made on six days; two over the Mediterranean and the remainder over the British Isles. A total of 104 satisfactory records on aluminium foil was obtained with the temperature distribution shown in Table 1. Most of the records were 30-40 cm long corresponding to 5 sec running of the foil past the sampling orifice. At a true airspeed of 200 kt a volume of 0·15 m^3 was sampled in 5 sec.

(b) *Britannia Aircraft data*

Records were obtained in extensive clouds associated with cumulonimbus in the intertropical convergence zone over Africa. The foil (0·0013 cm thick compared with 0·0025 cm on the M.R.F. aircraft) was run almost continuously on a number of flights in cloud. 35 portions of the foil were examined in detail by the Mechanical Engineering Department of R.A.E.

Because of the nature of the proving trials the data obtained were confined to a temperature band in the region of − 20°C (− 12 to − 26°C), and by the use of search radar the most active cumulonimbus cores were avoided.

3. Method of analysis

(a) *M.R.F. data*

5 cm lengths from the records which were judged by eye to be typical of the whole record were examined in detail. It was known from other tests that particles with diameters less than 80 μ would not mark the foil and imprints as large as 1 mm or more could be measured directly. For diameters between 80 μ and 1 mm it was possible to estimate quite closely by eye, imprints of about 0·5 mm and 0·25 mm diameter and counts were made of imprints in the size groups 100 μ, 250 μ, 500 μ, 1 mm, 2 mm, etc. The ' size ' was the estimated diameter of a circle of the same area as the imprint of the crystal on the foil. Since the diameter ranges were uneven the numbers measured were converted to numbers per mm diameter range by assuming that particles described as of diameter 100 μ had, in reality, diameters between 80 μ and 175 μ; particles of 250 μ diameters between 175 μ and 375 μ; particles of 500 μ diameters between 375 μ and 750 μ, particles of 1 mm diameters between 750 μ and 1,500 μ and particles of 2 mm diameters between 1·5 and 2·5 mm etc. Choice of 5 cm lengths limits the volume examined to about 0·02 m^3 so that one particle in the examined length suggests a concentration of about 50 m^{-3}. In searching for particles of low concentration in less variable clouds, it would be preferable to measure the length of foil to catch 10 particles, say, rather than a fixed length.

(b) *Britannia Aircraft data*

35 portions of the foil were photographed and enlarged 40 times. One particle on the length examined usually corresponded to a concentration of about 500 m^{-3}. The size-distribution of each portion was counted in diameter ranges of 0·1 mm, i.e., 0 to 0·1 mm 0·1 to 0·2 mm etc. The minimum marking diameter for the foil used was taken as 50 μ so that the first size group was really in a diameter range of 0·05 to 0·1 mm and this has been taken into account in the analysis.

4. Results

(a) *Mean size-distributions*

One use of the data is to regard each separate distribution obtained as a random sample of the size-distributions present throughout the whole volume of the cloud above the melting level, i.e., regarding the cloud above the melting level as thoroughly stirred by turbulence. This ignores, for example, any systematic variation there may be with temperature or height, total vertical extent of the cloud or temperature at cloud base among parameters which might exercise some influence on the size-distribution.

Because the two types of data available (M.R.F. and Britannia) lend themselves readily to separate analysis this distinction is here retained as giving broadly two types of cloud, that associated with isolated cumulonimbus of maritime extratropical origin and that associated with the intertropical convergence zone.

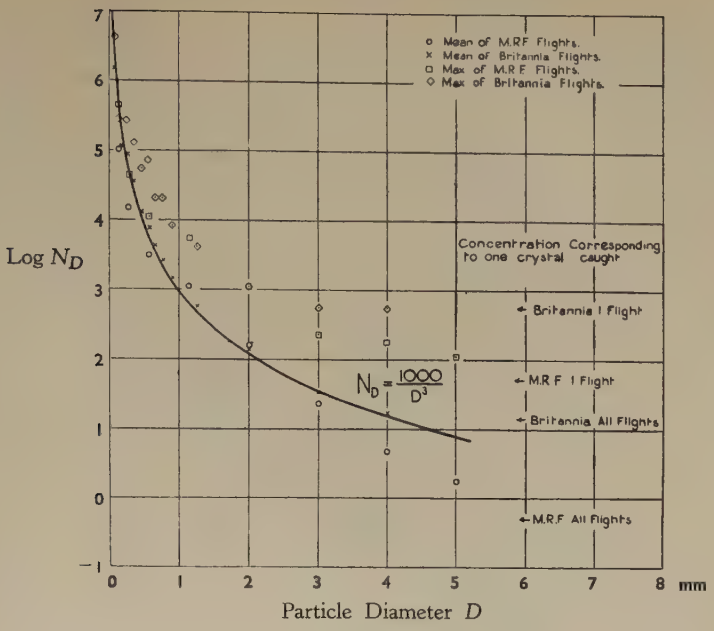

Figure 1. Mean ice-crystal size-distribution.

Mean results of the 104 M.R.F. distributions and the 35 Britannia distributions are given in Fig. 1. The concentrations for both series suggested by only one crystal caught $\left(\frac{50}{104}\Big/ \text{m}^3 \text{ for M.R.F. and } \frac{500}{35}\Big/ \text{m}^3 \text{ for Britannia}\right)$ are also indicated together with the over-all maximum concentrations measured in each size range.

(*b*) *Variation of size-distributions with temperature*

Table 1 shows the distribution of the M.R.F. observations with respect to temperature. Size-distributions obtained for each range of temperature shown in Table 1 are plotted in Fig. 2 in a manner designed to show the temperature variation more clearly. Also shown is the concentration of particles suggested by only one crystal caught in each temperature range $\left(= \frac{50}{N} \text{ m}^{-3} \text{ where } N \text{ is the number of observations}\right)$.

TABLE 1. M.R.F. OBSERVATIONS – DISTRIBUTION WITH TEMPERATURE

Temperature range (°C)	0 to − 4.9	− 5 to − 9·9	− 10 to − 14·9	− 15 to − 19·9	− 20 to − 24·9	− 25 to − 29·9	− 30 to − 34·9	− 35 to − 39·9
No. of observations	7	13	24	25	15	7	11	2

(*c*) *Variation in mean distributions from day to day*

The mean distributions for each of the six days on which M.R.F. sorties were made are shown in Fig. 3. On two of the days (4 Oct. 1956 and 5 Oct. 1956) only the top parts of the clouds were examined but on the remaining days flights were made at many different heights above the melting level.

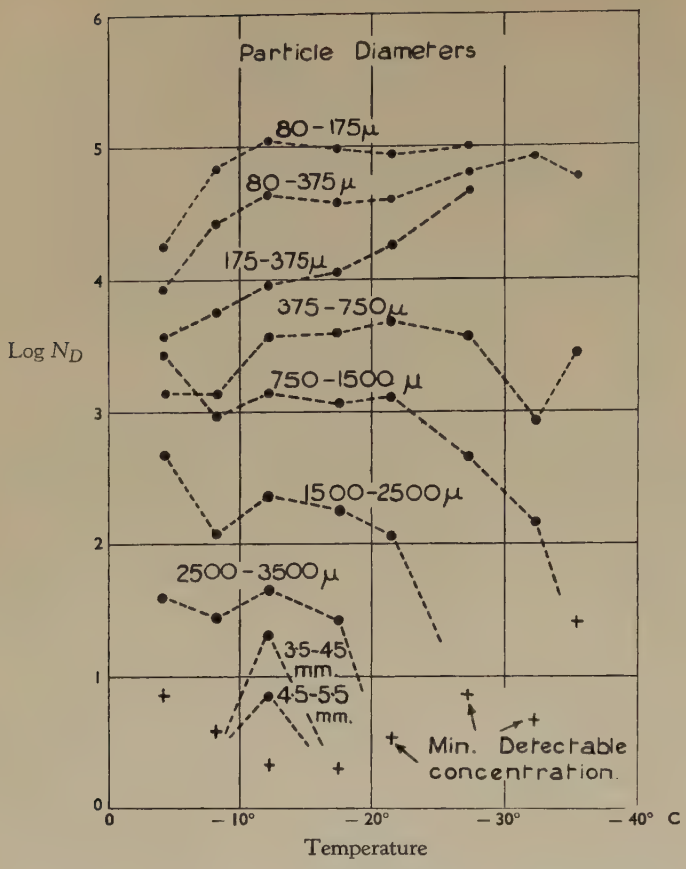

Figure 2. Variation of mean size-distribution with temperature.

(*d*) *Variation with total ice-crystal concentration*

The total ice-crystal concentration was not measured independently on the M.R.F. flights but an estimate of its *variation* might be obtained from the records themselves. It may be reasonable to suppose that in any size-range containing N_D crystals m^{-3} of mean diameter D the total mass of ice crystals will be proportional to $N_D D^3$. (This has some support from Nakaya and Terada (1935) quoted by Mason (1957) who found the mass of graupel particles, probably the most likely form of ice particles in cumulonimbus clouds, proportional to D^3). What the factor of proportionality is, however, is not known with certainty (Nakaya and Terada give 0·065) and may not even be the same for each size-range. To see qualitatively how the size-distributions vary with total concentration the mean distributions have been determined for different ranges of $\Sigma N_D D^3$. The M.R.F. observations divide conveniently into the 10 ranges, with roughly equal numbers (10) in each range, for which the mean size-distributions are plotted in Fig. 4.

Sufficient Britannia observations are not available for a similar division but nine readings fell within the range of $\Sigma N_D D^3 = 1{,}000$-$2{,}000$ mm^3 m^{-3} and showed a remarkably similar distribution to the M.R.F. distribution in this range.

5. Discussion of results

(*a*) *Mean size-distributions*

The M.R.F. and Britannia curves shown in Fig. 1 are remarkably similar for the range of particle diameters from 0·5 to 3·5 mm. There is a suggestion of more large particles (diameter greater than 3·5 mm) in tropical clouds. This is only a ' suggestion ' since the

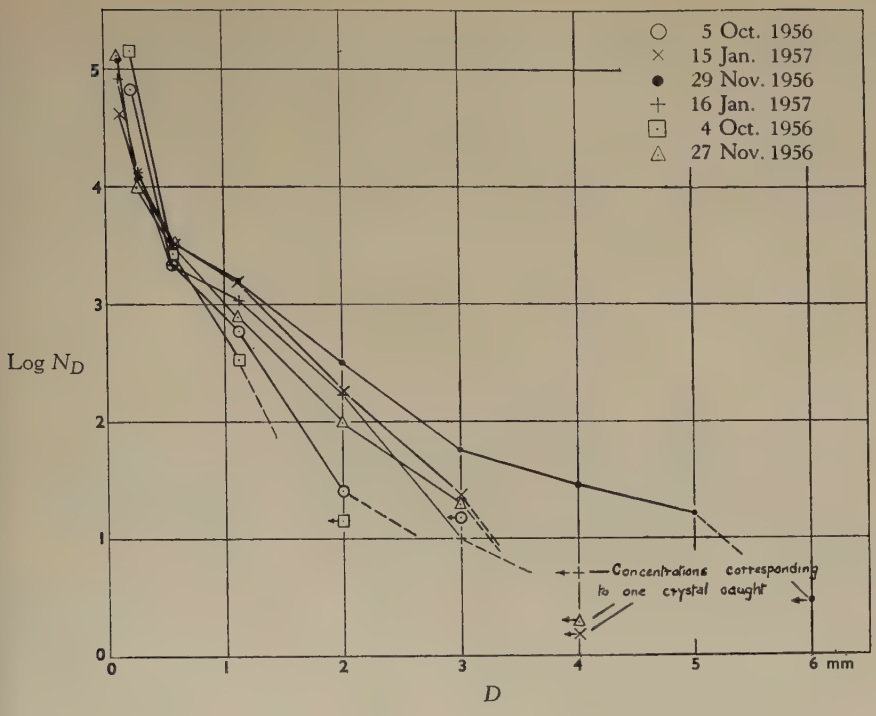

Figure 3. M.R.F. flights, mean size-distributions from day to day.

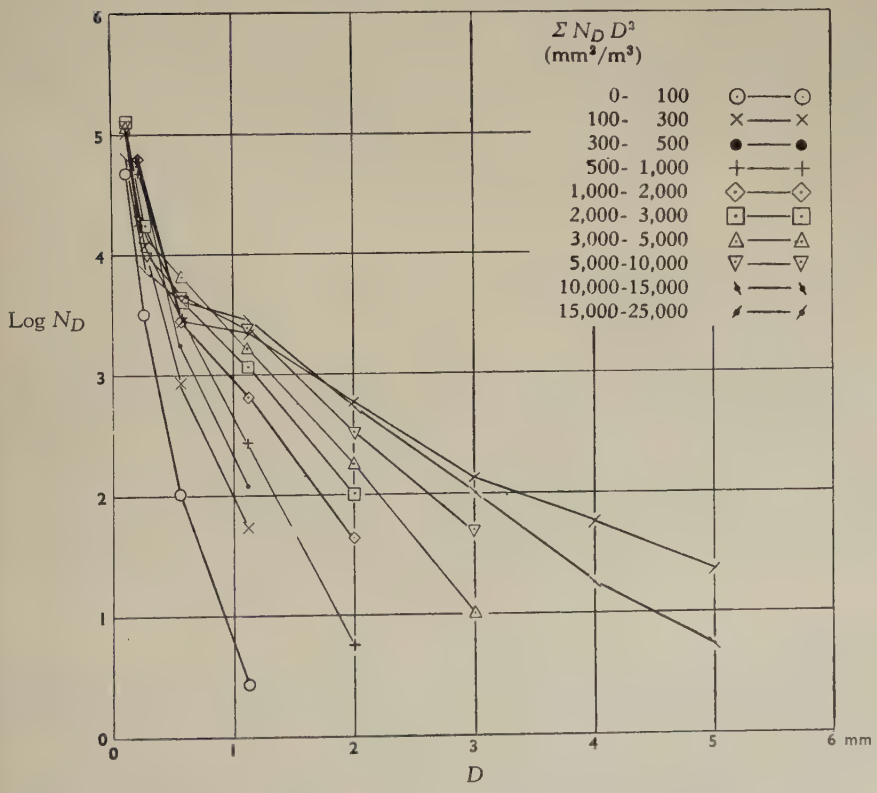

Figure 4. M.R.F. flights, size-distributions for different ranges of $\Sigma N_D D^3$.

curve beyond 4 mm diameter falls below the minimum detectable concentration for all the Britannia samples and in fact one particle of diameter 3·5 to 3·6 mm was the largest measured. (If in three times as many samples as were actually examined there had been no more particles in the 3·5 to 4·5 mm diameter range the Britannia and M.R.F. curves would still be almost coincident in this diameter range).

It can readily be shown (see Fig. 1) that the equation $N_D = 1{,}000/D^3$, where $N_D\,\delta D$ is the number of crystals of diameter between D and $D + \delta D$ mm, is a very close fit to the Britannia curve throughout its range.

For graupel (see Section 4 (d) above) Nakaya and Terada give $m = 0{\cdot}065\,d^3$ where m is the mass of the particle in mg and d in mm is the diameter of the circumscribing sphere, so that the total mass of ice in the cloud would be

$$\Sigma m = 0{\cdot}065\ \Sigma N_D\, d^3\, \delta D \text{ mg m}^{-3}$$

$$= 65\ \Sigma\, \delta D \text{ mg m}^{-3}$$

if we equate diameter of imprint on the foil D to the diameter d as defined by Nakaya and Terada. Thus the total mass of ice in the cloud is evenly divided among equal size ranges and is on the average 0·065 g m^{-3} per mm range of D. This appears to be a very low mean value for such clouds, for which other Britannia observations in similar clouds suggest a mean total concentration of about 1·25 g m^{-3}.

Nakaya and Terada found a mean density for graupel of 0·12 g cm^{-3}, but a density of about 0·6 g cm^{-3} would account for a total concentration of 1·25 g m^{-3} if particles up to 4 mm diameter were present. For a density of 0·9 g cm^{-3}, corresponding to frozen water droplets, the mass of ice in the cloud would be 0·47 g m^{-3} per mm range of D.

For crystals other than graupel Nakaya and Terada give $m \propto d^2$ for rimed plates, powder snow and all forms of dendrites and $m \propto d$ for needles. Needles, which give a characteristic imprint on the foil, certainly made an insignificant contribution to the overall distributions. Putting $m = kd^2$ and $N = 1{,}000/d^3$ we have the total mass of ice given by $\Sigma m = 1{,}000\ k \log d_{max}/d_{min}$ mg m^{-3}, where d_{max} and d_{min} are respectively the diameters in mm of the largest and smallest particles contributing to the total catch. For $d_{max} = 4$ mm, $d_{min} = 100\ \mu$, $\log (d_{max}/d_{min}) = 3{\cdot}7$ giving $k = 0{\cdot}33$ for a total catch of 1·25 g m^{-3}. The mean density ρ of a crystal is given by

$$\frac{6m}{\pi d^3} = \frac{6k}{\pi d} = \frac{2}{\pi d}$$

This implies for particles for which $d < 2/\pi$ mm a mean density greater than 1 g cm^{-3}, which is impossible ($d_{min} = 10\ \mu$ gives $k = 0{\cdot}21$ and $\rho > 1$ g cm^{-3} for $d < 1{\cdot}3/\pi$ mm). It seems most likely, therefore, that the particles having the distribution found were graupel of mean density 0·6 g cm^{-3}.

For particle diameters up to 2 mm the mean M.R.F. curve is closely represented by $N_D = 10^3/D^{2{\cdot}3}$ and for diameters greater than 2 mm by $N_D = 10^4/D^{5{\cdot}4}$. For spherical particles of density 0·6 g cm^{-3} the mean ice-content comprised of particles up to 2 mm diameter would be 0·6 g m^{-3} and also 0·6 g m^{-3} for particles between 2 mm and 5 mm diameter, giving a total content closely similar to Britannia experience. It should be remembered, however, that the M.R.F. aircraft did not have the benefit of search radar and may therefore have investigated more active parts of the clouds than the Britannia.

(*b*) *Variation with temperature*

In considering the curves of Fig. 2 it is necessary to bear in mind that observations in the 0 to − 5°C band were often complicated by the presence of liquid water and that the values in the − 35 to − 40°C band are based on only two flights.

The curves show little variability over a considerable temperature range below 0°C but for particle diameters of about 0·5 mm or greater, there is ultimately a decrease in concentration as temperatures decrease, the temperature at which this takes place being

higher the larger the particle. Thus particles of 0·5 mm diameter have substantially constant concentration with temperature down to about — 30°C and then the concentration falls off rapidly at lower temperatures (ignoring the values in the — 35 to — 40°C band) while for particles of 2 mm diameter the fall-off occurs near — 20°C. Thus relatively few particles of 2 mm diameter, say, were carried up above the — 20°C isotherm by up-currents and relatively few grew to this size at temperatutes below — 20°C. This suggests that up-currents, in general, around the — 20°C level in these clouds, did not exceed the fall-speed of 2 mm diameter particles, about $1\frac{1}{2}$-2 m sec^{-1} (Nakaya and Terada, 1935).

(c) *Variation in mean distribution from day to day*

The flights (4 and 5 Oct. 1956) which were concentrated in the top part of the cloud are clearly distinguished in Fig. 3 by the low frequency of occurrence of particles of diameter 2 mm and greater and by the substantially greater frequency of particles in the 80-375 μ range. The flights on 4 Oct. were all within 3,000 ft of the cloud top but on 5 Oct. two flights were made lower in the cloud than this. The exclusion of these would eliminate all particles of diameter 1 mm and greater, from the results of 5 Oct.

Of the remaining four days three gave closely similar results but the fourth, on 29 Nov. 1956, is outstanding for the large number of particles of diameter 2 mm or greater encountered. This was the only day on which particles of diameter greater than 3 mm were encountered but they were then met on six flights out of seven in the temperature band between — 10°C and — 15°C and not in any other temperature band. Why this should be so is not clear since the distribution of particles of diameter less than 2 mm does not differ from that in the other clouds and the water-content based on adiabatic lifting (wet-bulb potential temperature 2°C) would have been expected to be somewhat smaller on this day than on the other days examined. The ice content, based on $\Sigma N_D D^3$, is even higher than the mean results for the Britannia flying, for which a wet-bulb potential temperature approaching 20°C would not be unreasonable. One possible explanation may be that the low-cloud temperature implied marked instability in the lower levels of the air mass as it crossed the British Isles suggesting that the total ice-content at a particular level is related more to the strength of the probable up-currents than to the content deduced by assumption of adiabatic up-lift. A NNW'ly airstream covered the British Isles and showers of rain, sleet and hail were reported in many areas, particularly western coastal areas and East Anglia.

(d) *Variation with total ice-crystal concentration*

The curves of Fig. 4 show clearly a tendency to radiate from a common point in the region of log $N_D = 4{\cdot}1$, $D = 0{\cdot}25$ mm with much the same distribution of crystals of diameter smaller than 0·25 mm and at least a tendency to a distribution of the form $N_D = 10^{4{\cdot}1} \times 10^{-a(D-0{\cdot}25)}$ for diameters greater than 0·25 mm in such a way that a decreases as the total ice-crystal concentration increases. Putting $\Sigma N_D D^3 = V$ it can be shown that the variation of a is closely represented by $a = 17/V^{\frac{1}{3}}$ when V is in $mm^3\ m^{-3}$.

If, following Section 5 (a), we assume spherical particles with a mean density of 0·6 g cm^{-3} we have $W = 0{\cdot}6\ \pi V/6{,}000$ g m^{-3}, where W is the total ice-crystal concentration of the cloud, and hence

$$a = 17/V^{\frac{1}{3}} = 1{\cdot}16/W^{\frac{1}{3}}$$

A convenient empirical distribution law for particles greater than 0·25 mm diameter would therefore be

$$N_D = 10^{4{\cdot}1} \exp\left[-2{\cdot}67\ W^{-\frac{1}{3}}(D - 0{\cdot}25)\right]$$

where D is in mm, N_D is no./m^3 per mm range of D and W is in g m^{-3}.

6. Acknowledgments

The tedious work of measuring particle imprints was done by Mr. M. P. Garrod for Meteorological Research Flight observations and by Mr. T. J. Methven of Mechanical Engineering Department, Royal Aircraft Establishment, for Britannia observations and their assistance is gratefully recorded. Acknowledgment is also due to Bristol Aero-Engines Ltd. for use of readings obtained on their proving flights.

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Aircraft observations of rain and drizzle from layer clouds

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(Manuscript received 27 July 1959)

Summary

Aircraft observations of droplets greater than 100 μ diameter in 20 low layer clouds indicate that these drops can occur in concentrations of approximately 0·1 per litre to 10 per litre in clouds of 1,000 ft (300 m) to 7,000 ft (2,100 m) thickness. These observations are presented together with data relating the thickness and height of a further 120 layers of low cloud to their ability to produce precipitation at the ground. A diagram is shown which, it is suggested, might be of use as an objective aid for forecasting the occurrence of precipitation from low-layer cloud.

The results are discussed qualitatively in terms of the condensation–coalescence mechanism of droplet growth and demonstrate the efficiency of layer clouds as producers of large drops (and consequently precipitation). There is evidence that, for the range of cloud depths considered, coalescence is the predominant mechanism for drop growth for temperatures down to − 11°C to − 12°C.

1. Introduction

An instrument suitable for the airborne sampling of the larger particles present in clouds has been described by Garrod (1957). The technique is to use an aluminium-foil surface on which big drops or ice crystals impacting at aircraft speeds leave characteristic impressions. For water drops the lower limit of detection has been found to be in the range 80 to 100 μ diameter (Murgatroyd and Garrod 1960). Drops of this size are usually assumed to be equivalent to very small drizzle drops and knowledge of their concentrations and distributions is of importance to cloud physicists as well as to practical synoptic meteorologists.

This paper presents measurements of the concentrations of drops larger than 80 to 100 μ diameter in 20 low layer clouds. With one exception, which had a temperature of − 2°C at the top, they were all entirely below the freezing level. These observations and their relation to the rainfall mechanism are discussed together with 120 sets of observations of layers of low cloud obtained during meteorological reconnaissance flights.

2. Flight procedure

Occasions were selected when there existed in the vicinity of Farnborough a single 8/8 layer of non-freezing cloud with, in general, little or no cloud above or below. On the occasions when there were appreciable amounts of higher cloud care was taken to ascertain that no precipitation was reaching the lower layer from above. On the early flights, (1956-57), several ascents and descents were made through the cloud layers, samples being taken at height intervals of 1,000, 500 or 250 ft (300, 150 or 75 m) depending on the cloud thickness. On the more recent (1958-59) flights, ascents and descents were made with level runs, on reciprocal headings, at the same or similar heights, 3 or 4 samples being taken at each level. The sampling area of the instrument is about 3 cm^2 and the sampling time was generally 3, 5 or 10 sec with aircraft speeds of 130 to 200 kt (65 to 100 m sec^{-1}). One sampling run therefore, samples a volume of air of 60 to 300 litres. The number of samples obtained varied greatly from flight to flight and hence, although consecutive samples at the same height sometimes gave concentrations of drops varying by a factor of 10 to 100, it was considered that the best approach for each cloud layer was to mean all the available samples at each level. Thus the drop concentrations per litre so obtained were in fact derived from volumes of air of several hundred to a thousand litres.

On the majority of occasions drop samples were also obtained below cloud base.

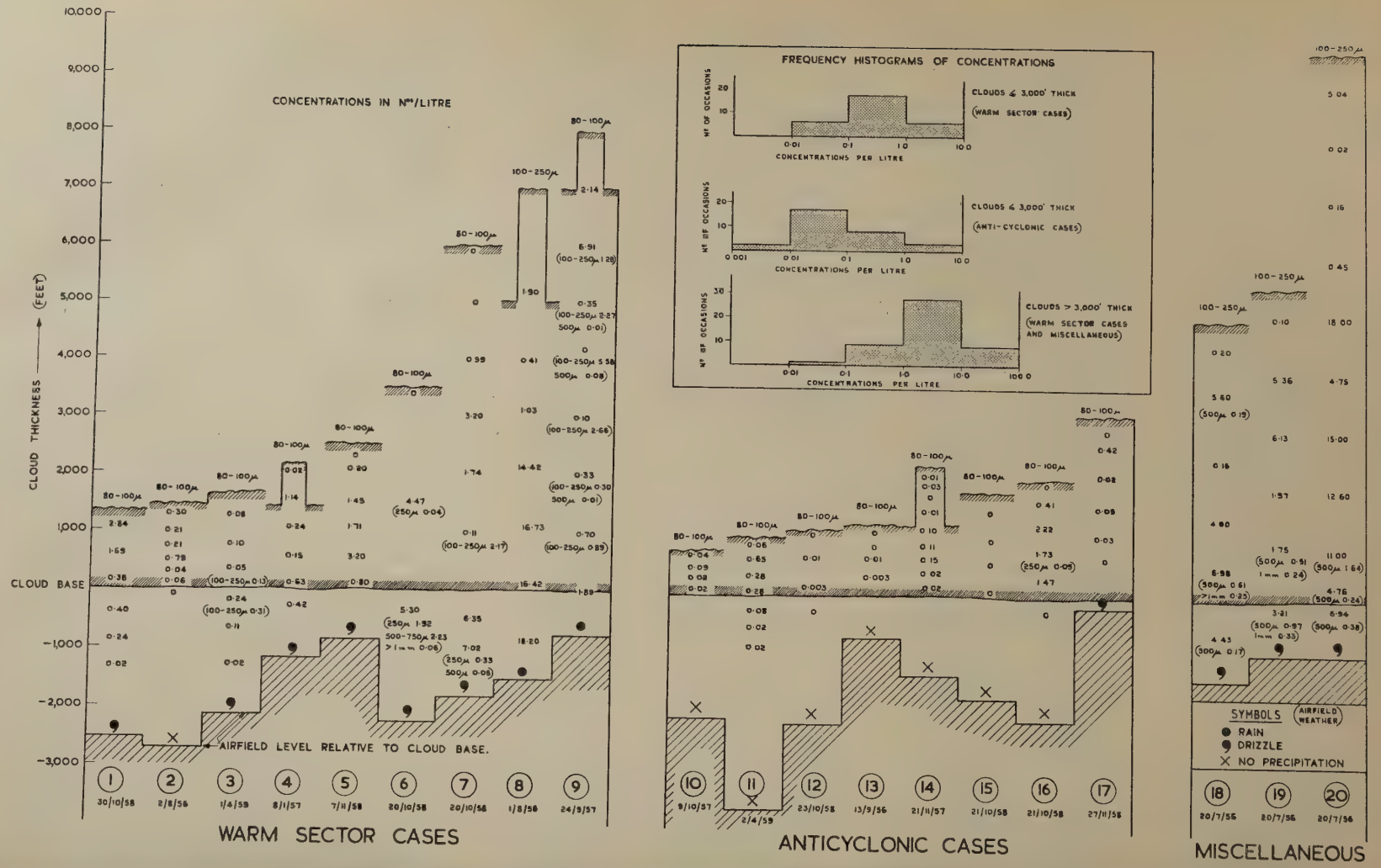

Figure 1. Summary of large drop concentrations in warm layer cloud.

3. Observational data relating to drop samples

The Appendix gives the relevant data regarding the 20 layers of cloud through which sampling flights were made including remarks concerning surface weather and the synoptic situation. All heights are above mean sea level (the station level of Farnborough is about 250 ft (75 m)). Also shown in the Appendix are theoretical values of free water content at cloud top, i.e., the amount of water available assuming adiabatic ascent to cloud top starting with the observed conditions of temperature and pressure at cloud base and assuming no sedimentation.

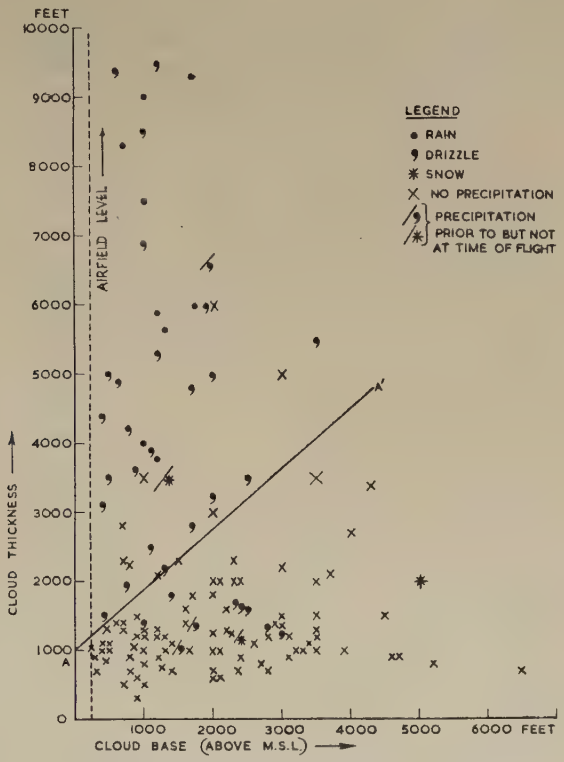

Figure 2. Scatter diagram (all observations) showing Farnborough weather against cloud thickness and height.

Of the different flights nine were either to the immediate rear of warm fronts or in active warm sectors, eight were in anticyclonic conditions or in wide-open warm sectors and three (all on the same day) were in a region of an indeterminate low-pressure area.

The concentrations in number per litre of droplets greater than about 100 μ diameter are shown in Fig. 1 in the above synoptic categories. Frequency distributions of these concentrations for clouds less than 3,000 ft (1,000 m) thick, (warm-sector and anticyclonic cases separately) and all clouds more than 3,000 ft (1,000 m) thick are also shown; all observations irrespective of sampling height were considered in the construction of these histograms. On one flight in the anticyclonic category no large drops were measured, this is not shown in the histogram because of the logarithmic abscissa.

4. Information on cloud layers derived from meteorological reconnaissance flights

In conjunction with the flights described above, an examination has been made of the cloud reports made during week-day morning reconnaissance flights from Farnborough using an uninstrumented aircraft. All the occasions were noted, for the period March

1955 to March 1959, when there existed locally a single 8/8 layer of low cloud with little or no higher cloud within 10,000 ft (3,000 m) of the top of the low-cloud sheet. It was considered in such circumstances, that any precipitation reaching the ground could only have formed in the layer of cloud thus chosen, and occasions were rejected when there existed any reasonable doubt. Except where indicated by the context the word precipitation in this paper shall mean precipitation reaching the ground. Together with the 20 layers of cloud chosen for sampling, data on the thickness of 140 layers of cloud were thus available for study. These observations are shown in Fig. 2 in the form of a scatter diagram with any precipitation reported at Farnborough at the time of the flight being plotted at a point determined by the cloud base and thickness. The line AA′ is seen to give a rough separation of the layers which give precipitation from those which do not do so; 8 layers from 44 on or above AA′ were not observed to give precipitation and only 11 out of 96 below AA′ were observed to give precipitation reaching the ground.

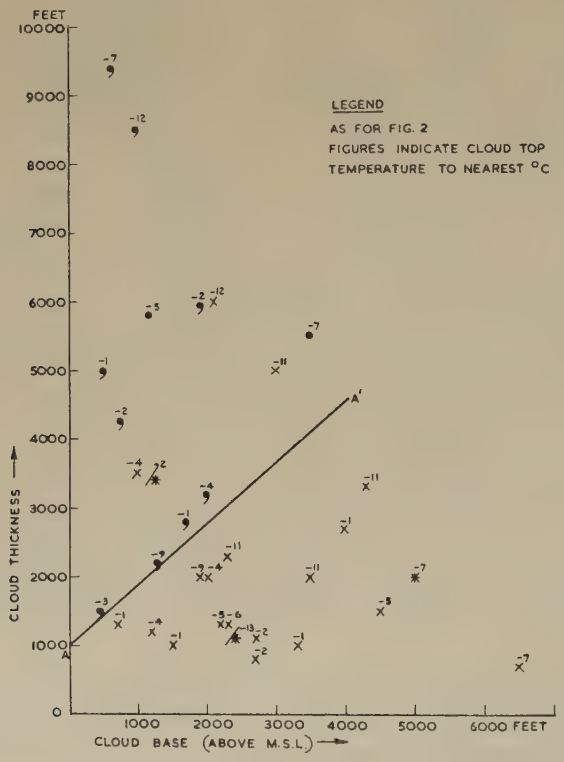

Figure 3. Observations relating to layers of clouds with tops below 0°C.

A similar diagram, for a far greater range of cloud base and thickness, and consequently in less detail over the range presented here, has been derived by Mason and Howorth (1952) from an analysis of routine aircraft ascents at Aldergrove during 1942-44. On this diagram two parallel lines were drawn, above which almost all cloud layers gave precipitation and below which scarcely any did do so, the region between the lines containing nearly equal numbers of observations relating to precipitating and non-precipitating clouds. The line AA′ on Fig. 2 lies in the region bounded by the two lines on the Mason and Howorth diagram. If more observations were available, especially just above and below the line AA′, it would be possible to draw more lines, either to delineate indeterminate regions, as done by Mason and Howorth, or to give areas of percentage probability (within various limits) for precipitating or non-precipitating clouds; these would enhance

the value of Fig. 2 as a tool in objective forecasting. The line AA′ could best be regarded as giving a minimum thickness of cloud necessary for a given cloud-base height to produce precipitation at the ground and which, if used on a day-to-day basis, would be correct on at least 80 per cent of all occasions.

Direct aircraft measurements of temperatures are only available for the 20 layers in which samples were taken. For all but 3 of the remaining layers radiosonde temperatures were obtained, and it was deduced that 32 of the remaining 117 layers had temperatures at their tops below 0°C. These, together with the one occasion of the other 20 layers with a temperature below 0°C have been plotted on Fig. 3 together with the line AA′ derived from Fig. 2. The temperature at cloud top is given for each plotted point. It will be noticed that only 3 out of 15 observations above AA′ did not give precipitation and 2 out of 18 below AA′ did do so.

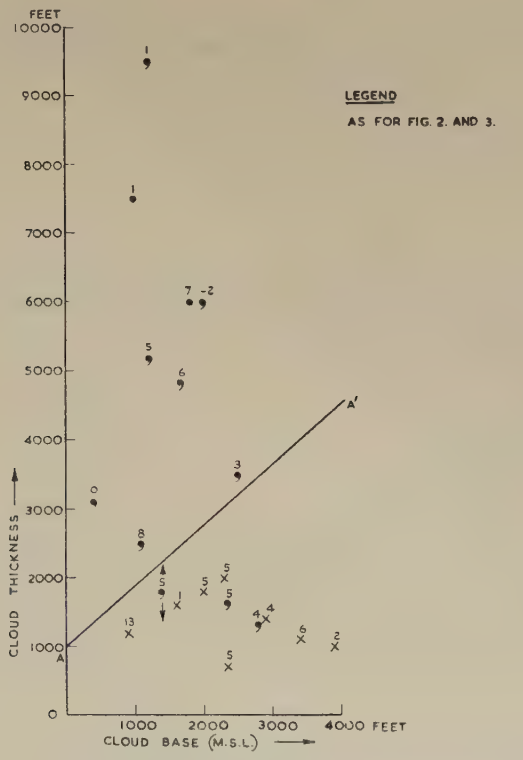

Figure 4. Observations relating to layers of cloud chosen for sampling.

Similar observations relating to the cloud layers chosen for sampling are shown in Fig. 4. Of these, 9 lie above AA′ and all give precipitation, whilst 11 lie below AA′ and of which only 3 were observed to give any precipitation reaching the ground (one of these 3 had a variable top as indicated in Fig. 4). As regards their likelihood of producing precipitation it seems reasonable to assume that these 20 flights are a fairly random sample of the whole population.

All of the 140 layers of cloud had been in existence for at least 3 hr.

5. Discussion

It is evident from these observations that drizzle and occasionally rain from warm layer-type clouds are common phenomena in this country. The Bergeron-Findeisen process being inapplicable, such clouds must produce their precipitation by the condensation-coalescence mechanism. It is suggested that the data given in Figs. 2 and 3 can be

used by forecasters to predict drizzle or light rain, formed by coalescence, from layers of low cloud. Another possible use of these figures is, given the height of the cloud base, to assess the greatest or least possible thickness of a layer by knowing whether it is producing precipitation or not. Exceptions must be expected due to local variations within the cloud layer and also to the rate of evaporation of the drops falling in unsaturated air below cloud. Mason (1952) has shown how the rate of evaporation of falling drops varies with relative humidity. From Fig. 3 it appears that for clouds with temperatures at their tops above about $-11°C$ it will be sufficient to assess their capabilities of producing rainfall simply in terms of cloud base and thickness without any regard to the possibility of extra likelihood of rain due to the Bergeron-Findeisen process. Considering in more detail the positioning of line AA′ with respect to precipitating and non-precipitating clouds and the cases which do not fit the pattern :

(a) On or above AA′ precipitation was observed in 35 cases of which 23 were entirely non-freezing, 7 had temperatures at their tops between $-0{\cdot}5°C$ and $-4{\cdot}5°C$ inclusive, and 5 had temperatures between $-5°C$ and $-12°C$. The cases which did not give precipitation were as follows :

(i) 3 supercooled clouds with temperatures $-4°$, $-11°$ and $-12°C$ respectively at their tops.

(ii) 6 entirely non-freezing clouds.

(b) Below AA′ no precipitation was observed in 85 cases of which 69 were entirely non-freezing, 8 had temperatures at their tops between $0{\cdot}5°C$ and $-4{\cdot}5°C$ and 8 had temperatures between $-5{\cdot}0°C$ and $-11°C$. The cases which did give precipitation were :

(i) 2 supercooled clouds with tops temperatures of $-7°C$ and $-13°C$ respectively.

(ii) 9 non-freezing clouds.

Of the cloud layers which might have contained ice crystals, considered to be those with temperatures below $-5°C$ (following Mason and Howorth) there would appear to be little evidence for regarding their behaviour as any different from clouds at higher temperatures. Probably a more significant fact is that of the 11 precipitating clouds below AA′ 9 were warm. It would, therefore, appear that for low-layer clouds with temperatures between $0°C$ and $-11°C$ any contribution from the Bergeron-Findeisen process is insufficient to compensate for the decrease in the effectiveness of the coalescence process due to the lower-cloud temperatures and consequent smaller water content.

From Fig. 1 it will be seen that comparatively thin clouds can contain large drops in concentrations similar to typical minimum precipitation concentrations observed at the ground, and that wholly non-freezing layer clouds of thickness 6,000 ft to 8,000 ft (1,800 to 2,500 m) may produce appreciable quantities of rainfall. Comparison with the observations of Murgatroyd and Garrod indicates that large drops detectable by the aluminium-foil technique are in general 10 to 100 times more common in layer cloud than cumulus of similar depth. Most of the clouds sampled had drops of size about 100 μ diameter at all levels, there being no obvious relation between numbers of drops and height above cloud base. In the thicker clouds capable of producing $\frac{1}{2}$ mm drops, these larger drops were normally present only near cloud base. The only cloud (that on 24 Sept. 1957) which had $\frac{1}{2}$ mm drops at appreciable heights above the base was reported to have convective cells embedded in it. In order for drops of 100 μ diameter to be in existence near cloud top it is reasonable to suppose that, at this level there must have been updraughts comparable in magnitude to the terminal velocity of these drops, i.e., $\frac{1}{4}$ m sec^{-1}. Similarly on 24 Sept. 1957 typical upcurrents in the convective cells were probably about 1 m sec^{-1}.

The histograms in Fig. 1 indicate that, in the main, anticyclonic layers of cloud have, apparently, fewer large drops than warm-sector layers of the same thickness. This difference is interesting, and, if real, could possibly be explained in terms of the salt-nucleus content of the air; in the warm-sector cases the air would probably have had a longer sea track and been associated with stronger winds (and therefore would have a higher salt content) than in the majority of the anticyclonic cases. An alternative, although more unlikely, explanation could be that upcurrents may be less in warm-sector layers than anticyclonic layers of cloud resulting in an increase of the time available for the growth of the larger drops. The histograms also show that the group of clouds greater than 3,000 ft (1,000 m) thick have roughly 10 times as many drops of diameter 100 μ as

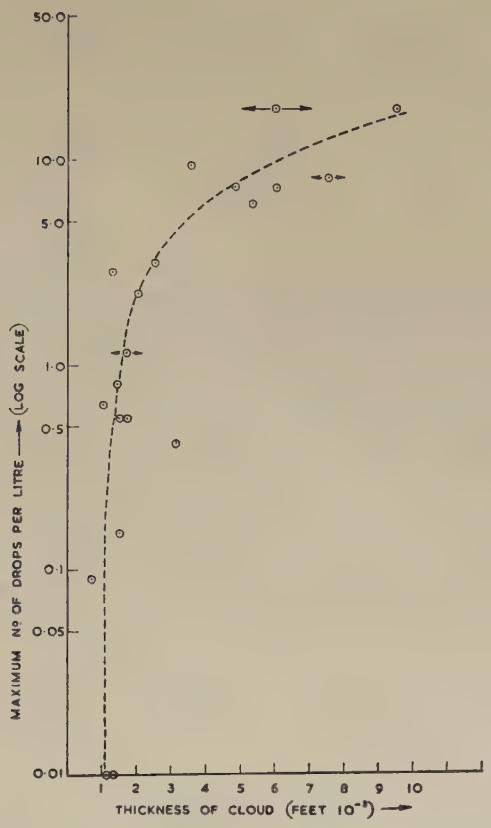

Figure 5. Variation of maximum numbers of large drops with thickness of cloud layer.

the warm sector clouds less than 3,000 ft (1,000 m) thick and 100 times as many drops as anticyclonic clouds less than 3,000 ft (1,000 m) thick. Two apparent anomalies in the observations are No. 11 and No. 17. The former had markedly more large drops than layers of similar thickness in the same category and the latter markedly less. No. 11, although under the influence of an anticyclonic cell was also just to the rear of a warm front and should possibly be in the warm-sector category. No. 17 was a situation when lifted fog merged with a stratocumulus layer, the cloud thickness being given from fog base to cloud top; Fig. 5, which shows maximum drop concentrations plotted against cloud thickness suggests, however, that the cloud was, in fact, only 2,000 ft (600 m) thick.

When considering the theoretical relationships that might exist between condensation nuclei below cloud base and drop spectra within clouds, rather limiting assumptions have to be made and the results always contain a degree of uncertainty. It would be desirable to investigate possible quantitative relationships between the observations presented here and concentrations of different salt-nucleus sizes such as have been obtained at sea level by Moore and Mason (1954) and from aircraft by Durbin (1959). Table 1 shows typical nucleus sizes which occur in similar concentrations to maximum-drop concentrations (derived from Fig. 5) for various cloud thicknesses.

TABLE 1. TYPICAL NUCLEUS SIZES CORRESPONDING TO OBSERVED DROP CONCENTRATIONS

Thickness of cloud	8,000 ft (2,500 m)	6,000 ft (1,800 m)	4,000 ft (1,200 m)	2,000 ft (600 m)	1,000 ft (300 m)
No. of drops per litre	15	10	6	2·5	0·1
Typical nucleus mass in g	10^{-10}	$2{\cdot}10^{-10}$	$3{\cdot}10^{-10}$	$7{\cdot}10^{-10}$	10^{-8}

The effect of differential settling between drops of different sizes may, however, invalidate attempts to equate on this basis the number of the large drops sampled with concentrations of the large nuclei. If this effect is not predominant Table 1 would suggest that salt particles in the range 10^{-8} to 10^{-10} g are likely to have produced all the large drops sampled. It is presupposed that the largest nuclei are responsible for the production of precipitation elements in warm layer-type clouds; this is consistent with Turner's (1955) conclusions based on observations of salinity as a function of drop size. From the above references salt nuclei of 10^{-10} g are usually present over this country in the lower layers of the atmosphere whilst those of 10^{-9} and 10^{-8} g are sometimes definitely not present. The fact that large drops are usually found in clouds 6,000-8,000 ft (1,800-2,500 m) thick, but less frequently, particularly in anticyclonic conditions, in clouds of 1,000 ft (300 m) thick, lends some support to the comparisons given in Table 1. However, Table 1 can only, at this stage, be regarded as suggestive and more work is obviously needed to justify such comparisons. A possible interpretation of the fact that the curve drawn through the observations in Fig. 5 shows a tendency to flatten might be that only the nuclei of mass 10^{-10} g and larger are of importance in determining the precipitation spectrum of these layers.

ACKNOWLEDGMENTS

To the Director-General of the Meteorological Office for permission to publish this paper; to the Officer-in-Charge and staff of Crawley radiosonde station for supplying a large number of temperature observations; to staff and aircrew of Meteorological Research Flight for assistance in compiling the observations, in particular Mr. D. J. Smith who also prepared the diagrams.

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Appendix

Warm sector situations

No.	Date Time GMT	Height (ft), pressure (mb), temperature (°C) Cloud base	Cloud top	Thickness (ft)	W g m^{-3}	Remarks
1	30 October 1958 1100	2,800 917 4·0	4,100 870 4·0	1,300	0·7	7/8 Ci. st with virga, but no precipitation reaching lower layer. Slow-moving cold front over Wales, associated warm front over Scandinavia. Winds light and variable. Intermittent slight drizzle at ground.
2	2 August 1956 1000	2,900 906 8·2	4,300 861 4·3	1,400	0·7	Nil or little upper cloud. Complex warm-sector situation. Winds NW. No precipitation reaching ground.
3	1 April 1959 1100	2,350 942 7·0	3,900 888 4·6	1,550	1·1	Nil or little upper cloud. Winds W. light. Intermittent slight drizzle at ground.
4	8 January 1957 1500	1,400 979 7·0	2,900 to 3700 931 to 903 5-6	1,500 to 2,300	0·8 to 1·2	Nil or little upper cloud. Very wide warm sector (almost anti-cyclonic case). Wind SW. light. Slight drizzle at ground at 1330 hr. Temperatures derived from radio-sonde.
5	7 November 1958 1500	1,100 972 11·0	3,550 890 7·6	2,450	1·4	7/8 Acu, zero count at cloud top indicates no precipitation from above. Complex warm sector, winds SW.-W. Intermittent slight drizzle at ground.
6	20 October 1958 1000	2,500 941 6·4	6,000 827 2·5	3,500	1·7	4/8 Acu above. Just behind slow-moving warm front. Wind NW. light. Intermittent slight drizzle.
7	20 October 1958 1100	2,000 958 7·2	8,000 766 — 1·7	6,000	2·9	About an hour later than No. 6. Intermittent slight drizzle at ground.
8	1 August 1956 1500	1,800 944 8·0	7,000 gen. occ. 9,000 782 7·2	5,000 occ. 7,000	2·5 occ. 3·3	3/8 Ci. Double frontal system, samples taken just to rear of second warm front. Cloud top generally 7,000 ft with occasional local increases to 9,000 ft, apparently not of a cumuliform nature. Winds SW.-W. Continuous rain at the ground.
9	24 September 1957 1400	1,000 977 9·7	8,000 to 9,000 753 to 722 0·8	7,000 to 8,000	3·3 to 3·8	Nil or little upper cloud. Just to rear of very slow-moving warm front. Wind E. becoming SW., light. Continuous slight rain at ground.

ANTICYCLONIC SITUATIONS

No.	Date Time GMT	Height (ft), pressure (mb), temperature (°C) Cloud base	Cloud top	Thickness (ft)	W g m^{-3}	Remarks
10	9 October 1957 1100	2,350 939 6·0	3,050 912 4·5	700	0·6	No cloud above. High, centred over NW. Europe. Winds SE. light. No precipitation at ground.
11	2 April 1959 1100	3,900 895 3·6	4,900 863 2·2	1,000	0·6	1/8 Ci above. Anticyclonic centre over SE. England inside wide warm sector. Winds light and variable. No precipitation at the ground.
12	23 October 1958 1500	3,400 910 6·7	4,500 873 5·7	1,100	0·7	No cloud above. Anticyclone over SW. England. Winds E.-NE. light. No precipitation at ground.
13	13 September 1956 1000	900 990 13·6	2,100 948 12·8	1,200	0·9	Nil or little upper cloud. High to SW. of United Kingdom ridging towards Scandinavia. Winds W.-NW. (A wide-open warm sector). No precipitation at the ground.
14	21 November 1957 1200	1,600 974 1·0	2,800 to 3,800 932 to 898 1·0	1,200 to 2,200	0·5 to 0·9	Nil or little upper cloud. Warm front over Irish Sea preceded by weak ridge. Winds SW.-W. No precipitation at the ground.
15	21 October 1958 1000	2,000 960 8·1	3,800 898 4·0	1,800	1·1	3/8 upper cloud. Anticyclone centred to SW. of United Kingdom quasi-stationary front over NW. Europe beginning to show signs of moving W. Winds N., light. No precipitation at the ground.
16	21 October 1958 1600	2,300 948 8·5	4,300 880 5·4	2,000	1·2	As above. No precipitation at ground.
17	27 November 1958 1100	400 1,012 6·5	3,500 903 0·4	3,100	1·7	Anticyclonic centre to N. Low stratus or fog merging with stratocumulus layer. Winds NE., light. Slight intermittent drizzle at the ground.

MISCELLANEOUS CASES

18	20 July 1956 1400	1,650 955 10·0	6,400 820 6	4,800	2·3	7/8 Acu above 10,000 ft. No precipitation reaching lower layer. Remarks as for No. 19. (3 hr. later).
19	20 July 1956 1030	1,200 976 9·6	6,500 796 4·8	5,300	2·9	Broken Acu and Ci. Weak low-pressure system over Low Countries. Winds NE., light. Continuous drizzle at the ground.
20	20 July 1956 1130	1,200 976 9·6	10,700 680 0·4	9,500	3·7	20 mi W. of above sampling run on same day. Nil or little cloud above. Remarks as for No. 19.

551.510.41 : 551.584.42

The carbon-dioxide flux over a field of sugar beet

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(Manuscript received 5 August 1959)

Summary

The difference of CO_2 concentration at two heights above a sugar beet crop was measured with a sensitive infra-red gas analyser. The ratio of evaporation to humidity gradient and of total heat flux to wet-bulb temperature gradient gave a transport number for calculating CO_2 flux, assumed proportional to CO_2 gradient and wind speed. In August and September 1958 there was often a constant upward flux at night (mean 0·05 mg cm^{-2} hr^{-1}) attributed to respiration by plants and soil micro-organisms; but during daylight, photosynthesis gave a downward flux varying with solar radiation and reaching a maximum of 0·4 mg cm^{-2} hr^{-1}. From 31 July to 11 September the net downward flux of atmospheric CO_2 was 158 $\pm$ 29 mg cm^{-2} compared with an uptake of 217 $\pm$ 38 mg cm^{-2} estimated from dry matter increase of sampled plants. During October there was a net upward flux of 4·9 mg cm^{-2} day^{-1} attributed to a striking increase in amount of CO_2 produced in the soil.

1. Introduction

Throughout the life of a field crop, the atmosphere is a sink for transpired water vapour and a source of carbon dioxide for photosynthesis. Turbulent transport theory has been successfully used by Pasquill (1950) to estimate transpiration from short grass and extension of the theory to taller crops has been described by Rider (1957) and others. Attempts to measure the agriculturally more important carbon-dioxide flux have been fewer, simply because concentration gradients are extremely small and adequate precision is difficult to attain.

Huber (1952) measured the concentration difference of CO_2 between 1 and 22·5 m over a wheat field with an infra-red gas analyser. Multiplying this gradient by wind speed at an intermediate height, he obtained a quantity approximately proportional to flux which showed a marked diurnal variation, changing direction at sunrise and sunset. The height interval of 21·5 m was essential to obtain measurable concentration differences but introduced uncertainty because the relation between flux and gradient depends on stability, as well as on wind speed, unless observations are made close to the surface. Working over a more acceptable height range, Inoue (1957, 1958) measured the carbon-dioxide flux over rice and wheat. Air from five levels below 10 m was drawn into polythene bags which were then removed to the laboratory for standard CO_2 analysis, and fluxes were calculated from CO_2 and wind profiles inserted in an equation similar to that used by Rider for water vapour. The absence of steep gradients between the soil and the top of the crop showed that the CO_2 flux from the soil was negligible, but the evidence of other workers suggests that it may sometimes be important.

For the present study, the Infra-red Development Company produced an analyser which can detect concentration differences of about 0·05 p.p.m. (parts CO_2 per million parts air by volume), in air with a normal CO_2 concentration of about 300 p.p.m. and, with unavoidable breaks, this instrument recorded gradients over a field of sugar beet from August to November 1958.

2. Equipment

The plot of sugar beet used for the experiment adjoins the meteorological enclosure at Rothamsted and the site has been described by Penman and Long (1960). The variety Klein E was sown in drills 60 cm apart on 11 April and seedlings were singled to give a spacing of 23 cm in the rows. The area of the crop was approximately 50 $\times$ 50 m.

The CO_2 analyser, installed in a hut at the side of the field, sampled air from two heights above the crop drawn through 50 m lengths of 12 mm diameter polythene tubing at a rate of 1 litre/min in each line, so that the effective response time was about ten minutes. Gradients of water vapour, to which the instrument is slightly sensitive, were often very much larger that those of CO_2, and the two incoming air streams were thoroughly dried by passage through tubes of silica gel (changed daily) and of magnesium perchlorate (changed weekly). The desiccated air then flowed through the two analysis tubes within the instrument and differences in the absorption of two chopped beams of infra-red radiation were detected by a pressure-sensitive transducer, in the input circuit of a three-stage amplifier. The nominal output was 1 mV for 1 p.p.m. concentration difference, but it was found convenient to work with half this sensitivity and to use a thread recorder giving a final sensitivity of about 1 p.p.m. CO_2 per cm chart. Error caused by zero drift was eliminated by interchanging the two gas streams every two minutes with a solenoid-operated valve controlled by a time-switch. The recorder then displayed a double trace of width proportional to the output signal and independent of zero drift either in the analyser or in the galvanometer of the recorder. During most daylight periods, ' noise ' in the trace, partly of electrical, but mainly of real, atmospheric origin, was about $\pm$ 0·1 p.p.m.

The analyser was calibrated weekly by restricting flow through the two analysis tubes in such a way that air entered each tube at the same rate but with a different pressure. Drawing air from a cylinder of known CO_2 concentration C, the concentration difference between the tubes was $C\,\Delta p/p$ where p was total pressure, effectively atmospheric, and Δp was total pressure difference measured with an alcohol manometer. There was a steady fall in sensitivity over two months but the instrument response remained linear over the range $\pm$ 10 p.p.m.

Other equipment was required to provide estimates of transport constant. In the centre of the plot, the polythene tubes ended in ducts containing wet- and dry-bulb thermocouple units of the type described by Pasquill (1949), and these were used to measure gradients of wet- and dry-bulb temperature with a mirror galvanometer shunted to give a sensitivity of 10 cm/°C. Absolute wet-bulb temperature at one height was obtained on a thread recorder. This system gave mean temperatures for selected periods during which an additional fan-motor provided sufficient aspiration for the couples without disturbing flow through the analyser.

Evaporation was measured directly with a field balance, described by Morris (1959), carrying 18 plants in a soil pan of area 2 m^2 and depth 50 cm, and giving a continuous pen record from which total water-vapour flux could be read for periods as short as a $\frac{1}{4}$ hr. The wet season ensured that the soil within the balance, like that of the surrounding field, was always close to field capacity.

With leaves of the crop reaching 60 cm above the ground, wind speeds at 60, 70, 120 and 170 cm were recorded with the units described by Long (1957); soil-heat flux was obtained with glass flux plates (Deacon 1950) recording on a Kent multi-point potentiometer; and net radiation with a Gier and Dunkle-type radiometer (MacDowall 1955) normally recording on the same potentiometer, but occasionally switched into the galvanometer circuit used for the thermocouples. A Kipp solarimeter on the laboratory roof, $\frac{1}{2}$ mi from the field, gave a continuous record of solar radiation on a second potentiometer.

3. Determination of transport number

The following symbols are used :

F, ϕ = vertical flux and volume concentration of water vapour, CO_2 etc.

K = turbulent transfer coefficient ($cm^2\ sec^{-1}$).

u_i = wind speed at height z_i.

k = von Kármán's constant $\simeq$ 0·41.

d = zero plane displacement.

x = absolute humidity (g cm^{-3}).
λ = latent heat of evaporation.
E = water-vapour flux (g cm^{-2} sec^{-1}).
C = sensible-heat flux (cal cm^{-2} sec^{-1}).
Q = total-heat transfer = $\lambda E + C$.
R = net radiation (cal cm^{-2} sec^{-1}).
G = soil-heat flux (cal cm^{-2} sec^{-1}).
c_p = specific heat of air at constant pressure (cal g^{-1}).
ρ = density of air (g cm^{-3}).
γ = ' constant ' of the psychrometer equation (mb/°C).
T, T' = dry- and wet-bulb temperatures (°C).
$e, e_s(T')$ = actual vapour pressure and saturated vapour pressure at wet-bulb temperature
$\Delta = \delta e_s/\delta T$.

In horizontally homogeneous flow, the exchange of momentum, sensible heat or of a specific gas between the earth's surface and the atmosphere can be related to the vertical gradient of volume concentration by equations of the form

$$F = -K(z)\,\delta\phi/\delta z \,. \qquad (1)$$

Within a few metres of the ground where flux may be considered invariant with height, integration of this equation gives :

$$F/(\phi_1 - \phi_2) = 1 \Big/ \int_{z_1}^{z_2} K^{-1}\,dz \qquad (2)$$

Each side of this equation has dimensions of velocity, and assuming equality of transfer coefficients whatever property is transferred and whatever the form of the profile, a general dimensionless transport number m can be defined by

$$F/(\phi_1 - \phi_2) = mu_3 \,. \qquad (3)$$

For logarithmic profiles

$$m = \frac{k^2\,(u_4/u_3 - 1)}{\ln\dfrac{z_2 - d}{z_1 - d} \cdot \ln\dfrac{z_4 - d}{z_3 - d}} \qquad (4)$$

where it is usual to take $z_1 = z_3$, $z_2 = z_4$. When zero plane displacement and surface roughness are independent of wind speed and when the atmosphere is in neutral equilibrium, u_4/u_3 is independent of wind speed. Then m is also independent of wind speed but is determined by d, and the chosen heights z_1 to z_4.

In the present work, mean hourly profiles of wind speed at four heights above the crop were not sufficiently logarithmic for confident determination of d. The lowest anemometer was at leaf height where it was greatly influenced by local leaf distribution. The wind profile representative of the crop may not have reached the highest anemometer at 110 cm above the leaves with a fetch of only 25-45 m. Two other methods of finding m were therefore explored : first, from measurements of evaporation and humidity gradient using

$$m = E/u_3\,(x_1 - x_2) \,. \qquad (5)$$

and second from the flux of total heat.

Ignoring small changes of heat storage within the crop its heat balance may be written

$$R + G + Q = 0$$

For consistency with Eq. (1), fluxes from surface to air or to underlying soil have positive sign. Omitting a negligible term for the adiabatic lapse rate, Eq. (1) gives

$$\left.\begin{aligned} Q &= -K(\lambda\,\delta x/\delta z + \rho c_p\,\delta T/\delta z) \\ &= -(\delta e/\delta z + \gamma\,\delta T/\delta z)\,K\rho c_p/\gamma \end{aligned}\right\} \quad . \quad . \quad (6)$$

since $x = \rho c_p\, e/\gamma\lambda$

Differentiating the psychrometer equation

$$e = e_s(T') - \gamma(T - T') \quad . \quad . \quad . \quad . \quad (7)$$

and substituting $\delta e/\delta z$ in Eq. (6) gives

$$Q = -K\rho c_p(1 + \Delta/\gamma)\,\delta T'/\delta z$$

showing that the wet-bulb temperature is the appropriate potential for the flux of total heat (McIlroy 1955). Hence

$$m = \frac{(R + G)}{u_3\,\rho c_p(1 + \Delta/\gamma)(T_2' - T_1')} \quad . \quad . \quad . \quad (8)$$

In applying Eqs. (5) and (8) to the estimate of CO_2 flux it was assumed that similar fully developed profiles of temperature, humidity and CO_2 concentration existed within 50 cm above the top of the crop which was approximately 60 cm above the ground. From 19 August to 9 September (Period A), CO_2 and humidity gradients were measured between $z_1 = 68$ and $z_2 = 93$ cm. From 26 determinations over 10 min intervals of the humidity difference $(x_1 - x_2)$, the wind velocity at 70 cm, u_3, and the balance record of evaporation, E, Eq. (5) gave

$$m_A = 0{\cdot}150 \pm 0{\cdot}042$$

On 9 September the sampling heights z_1 and z_2 were changed to 80 and 105 cm, z_3 remaining unchanged at 70 cm. Re-determination of m between 9 and 24 September (Period B) from 21 half-hourly mean values of $(x_1 - x_2)$ and u_3 gave

$$m = 0{\cdot}155 \pm 0{\cdot}032$$

Using 19 half-hourly means of total-heat flux and wet-bulb temperature difference, Eq. (8) gave

$$m = 0{\cdot}165 \pm 0{\cdot}037$$

Giving the two values equal weight the mean is

$$m_B = 0{\cdot}160 \pm 0{\cdot}042$$

In the graphical presentation of these data (Fig. 1) no attempt has been made to distinguish differences in atmospheric stability. All measurements were made in near-neutral conditions with Richardson's number between $-0{\cdot}04$ and $+0{\cdot}01$.

Because there was no measureable change in crop height and little in leaf area after the end of August, it is assumed that the zero-plane displacement, d, remained unchanged from Period A until the beginning of November and the working values of m_A and m_B have been adjusted to satisfy this assumption as follows. Using the appropriate values of z_1, z_2 and z_3 and a measured value of $u_4/u_3 = 1{\cdot}55$ based on 48 hourly means with $z_4 = 120$ cm, it is possible to plot d as a function of m from Eq. (4). This was done for each period and the values of d read off from the curves were

$$d_A = 40 \pm 6 \text{ cm}$$

$$d_B = 44{\cdot}5 \pm 5 \text{ cm}$$

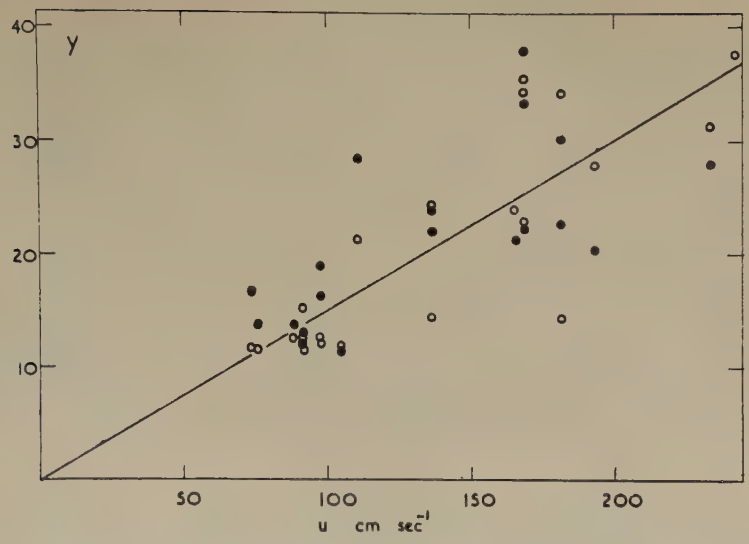

Figure 1. Determination of transport number from Eqs. (5) and (8) and observations between 9 and 24 September.

o $y = E/(x_1 - x_2)$

• $y = (R + G)/\rho c_p (1 + \Delta/\gamma)(T_2' - T_1')$

The straight line gives the mean gradient for all points, $m_1 = 0{\cdot}150$.

By taking the mean as the ' true ' value of d (= 42·3 cm) for both periods, the ' true ' values of m read from the curves are

$$m_A = 0{\cdot}13 \pm 0{\cdot}04$$

$$m_B = 0{\cdot}18 \pm 0{\cdot}04$$

where the changes lie within the standard errors of the original estimates.

In working units, Eq. (3) becomes

$$F = 3{\cdot}6\, m\rho'\, u_3\, \Delta C \quad . \quad . \quad . \quad . \quad . \quad (9)$$

where F = flux of CO_2 (mg cm^{-2} hr^{-1})

ρ' = density of CO_2, (g cm^{-3})

ΔC = concentration difference of CO_2 between z_1 and z_2 (p.p.m.)

The mean temperature assumed for the observations was 15°C, giving $\rho' = 1{\cdot}9 \times 10^{-3}$.

4. Diurnal variation of CO_2 gradient and flux

A gradient record for three August days (Fig. 2) shows typical diurnal variation with change in sign near sunrise and sunset. During the day, the maximum CO_2 inversion produced by assimilation was only 3 p.p.m./25 cm.; but at night, when wind speed was very low, the recorder was often off-scale indicating a lapse of more than 10 p.p.m./25 cm produced by respiration. Using the instrument on a less sensitive range, a lapse of 76 p.p.m./25 cm was recorded at 2100 hr on 24 October.

Substitution of average hourly wind speeds, CO_2 gradients and the appropriate transport number in Eq. (9) then gave the diurnal variation of flux. An attempt was made to use the buoyancy correction derived from the formula of Obukhov and Monin (Sheppard 1958) but this was found to be unnecessary. During the day, the correction was seldom comparable with the error introduced by uncertainty in the transport number. To give an extreme example, between 0900 and 1000 GMT on 17 August, there was a wind

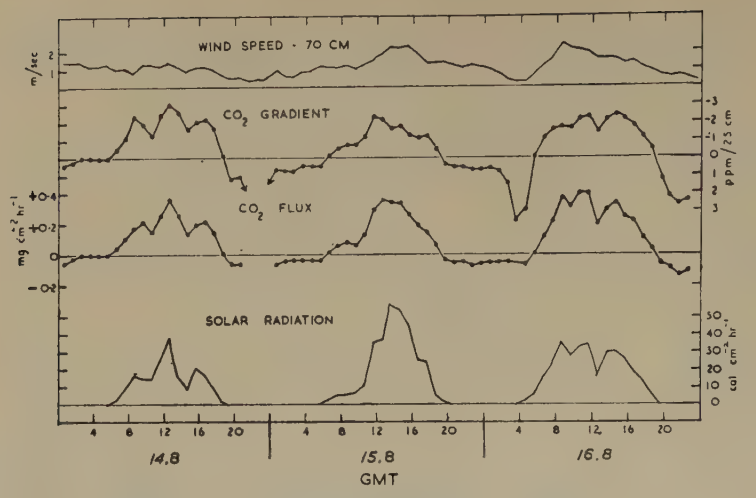

Figure 2. Variation of wind speed at 70 cm CO_2 gradient with $z_1 = 68$, $z_2 = 93$ cm, CO_2 flux and solar radiation for 14-16 August 1958. Crop height 60 cm.

speed difference of 40 cm sec^{-1} and a temperature difference of 1°C between heights of 70 and 120 cm. Correction to the calculated flux was then + 12 per cent. The mean correction for periods of a week was only one or two per cent. At night, important corrections could not be calculated when wind was below the stopping speed of the anemometers. On nights when the presence of cloud and moderate wind prevented the development of a strong temperature inversion (e.g., 15-16 August) the upward flux of CO_2 was uniform, showing that respiration from the crop (and possibly from the soil) was constant. On other nights, e.g., 14-15 August, there were periods when flux could not be calculated and total upward flux (as tabulated in the Appendix) was then obtained from the mean value during periods of constant flux (e.g., 0300-0600 hr 15 August).

Data given in the Appendix show that during August and early September the mean daily downflux of CO_2 on recorded days was 2·5 mg cm^{-2} day^{-1}, the nocturnal upward flux was about 0·6 mg cm^{-2} day^{-1} and the net rate of assimilation of atmospheric CO_2 was therefore 1·9 mg cm^{-2} day^{-1}.

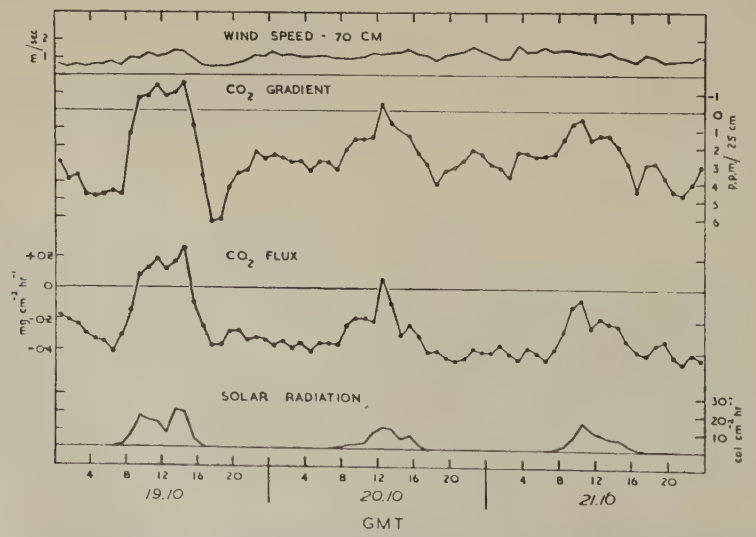

Figure 3. Variation of wind speed at 70 cm CO_2 gradient with $z_1 = 80$, $z_2 = 105$ cm, CO_2 flux and solar radiation for 19-21 October 1958. Crop height 60 cm.

Solar radiation plotted below CO_2 flux, (Fig. 2) shows a striking correlation, particularly on 16 August. While the leaves at the top of the crop were probably light-saturated for photosynthesis even in the presence of cloud, lower leaves were shaded well below the photosynthetic optimum even in bright sunshine. The strong dependence of flux on radiation suggests that assimilation by these shaded leaves dominates the CO_2 exchange between a mature stand of sugar beet and the atmosphere.

Records similar to those of Fig. 2 were obtained during August and much of September. In October, however, the nocturnal loss was sometimes as great as 0·5 mg cm^{-2} hr^{-1} and on dull days, e.g., 21 October (Fig. 3), it was common for an upward flux of CO_2 to prevail both night and day. There was a mean up-flux of 4·9 mg cm^{-2} day^{-1} on the days recorded, far exceeding possible respiration by the crop. It appeared either that the equipment had become defective or that substantial amounts of CO_2 had been released by the breakdown of organic matter in the soil. Evidence for the proper working of the analyser will therefore be presented before considering the alternative source of CO_2.

5. Variation of CO_2 gradient with wind speed at night

If the respiration from a crop remained constant throughout the night, the CO_2 gradient would vary inversely with wind speed according to Eq. (3). Gradients were plotted against $1/u$ (Fig. 4) for the nights 19 to 23 October, chosen because there was little temperature difference from hour to hour or from one night to another. There is much less scatter than for the equivalent water-vapour correlation of Fig. 1. Underestimate of low wind speeds, when the anemometer was stopping intermittently, may be responsible for the scatter of ' low ' points on the right-hand side of the diagram. Ignoring these, a straight line drawn through the remaining points and the origin has a slope equivalent to CO_2 flux of 0·4 mg cm^{-2} hr^{-1}. This result, which would not have been obtained with serious zero error, supports the reality of large nocturnal gradients observed in October.

6. Carbon-dioxide flux and crop growth

The classical method of assessing crop growth is periodically to remove a number of randomly selected plants which can be oven-dried to obtain dry-matter increments. For sugar beet, the main constituent of this dry matter is sucrose ($C_6H_{12}O_6$). Because

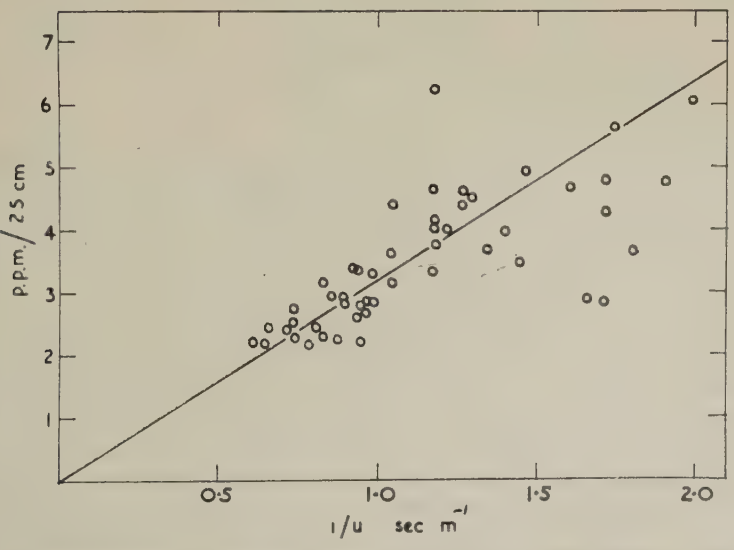

Figure 4. Correlation of CO_2 gradient between 80 and 105 cm with inverse of wind speed ($1/u$).

one sucrose molecule requires the assimilation of six molecules of CO_2 and can produce six molecules of CO_2 in respiration, the *net* uptake of CO_2 by a plant can be obtained from its dry matter.

Between August and November, dry matter and leaf area of the crops were determined by Dr. B. Orchard of the Botany Department for samples of 24 pairs of adjacent plants. Total dry-matter increments in terms of CO_2 equivalent are given in Table 1. Because of successive failures of the equipment it was impossible to obtain unbroken records of CO_2 flux throughout the sampling period but data given in the Appendix were used to compute values given in Table 1 for the uptake of atmospheric CO_2. It was confirmed that mean radiation and temperature for the recorded days were close to the mean for each period. Standard errors quoted for the fluxes are derived from error in transport number and from chart reading error estimated at $\pm$ 0·1 mg cm^{-2} day^{-1}. Errors in dry-matter increment were obtained from the standard deviation of plant weights and are of similar order to the flux errors.

TABLE 1. COMPARISON OF CO_2 FLUX AND TOTAL UPTAKE OF CO_2 BY CROP (mg cm^{-2} day^{-1})

Period 1958	Percent days recorded	Mean daily CO_2 fluxes Downward	Upward	Net CO_2 flux Downward	Net CO_2 uptake determined from dry matter
31 July-21 August	38	$2{\cdot}6_6$	$0{\cdot}6_6$	2·0 $\pm$ 0·6	3·4 $\pm$ 0·6
21 August-11 September	53	$2{\cdot}2_9$	$0{\cdot}7_5$	1·5 $\pm$ 0·4	1·8 $\pm$ 0·7
11 September-7 October	27	$1{\cdot}9_2$	$1{\cdot}2_8$	0·6 $\pm$ 0·3	2·5 $\pm$ 0·6
7 October-14 November	58	$0{\cdot}5_5$	$5{\cdot}4_5$	$-$ 4·9 $\pm$ 1·6	0·4 $\pm$ 0·5

For the first period, the difference between the two determinations is greater than the square root of the sum of the squares of the standard deviations but the eight recorded days may have been unrepresentative of the whole period. Much closer agreement was obtained during the second period when the proportion of recorded days was higher, and taking first and second periods together, the total computed assimilation from 31 July to 11 September was :

158 $\pm$ 29 mg cm^{-2} from atmospheric CO_2 gradients

217 $\pm$ 38 mg cm^{-2} from dry matter increment.

Part of the difference may be attributable to the assimilation of CO_2 released from the soil in the bacterial decomposition of organic matter. Assuming from the work of Lundegårdh (1927) that soil CO_2 production was 0·6 mg cm^{-2} day^{-1}, the net assimilation of CO_2 diffusing downwards from the atmosphere and upwards from the soil becomes 183 mg cm^{-2} in closer agreement with the figure obtained from dry matter. Synthesis of compounds other than sugar would give further undetermined differences between the two estimates.

From 11 September to 14 November there was a large discrepancy between the estimates. Accepting the CO_2 gradients of Fig. 3 as real, there appears to have been a spectacular increase in soil CO_2 production in late autumn. A seasonal increase of CO_2 in the soil atmosphere was noted by Russell and Appleyard (1915) and a possible cause is the very great increase in microbial population that occurs at this time of year (Cutler, Crump and Sandon 1922). But a net up-flux of 4·9 mg cm^{-2} day^{-1} from the crop during a period in which dry matter determinations gave a net assimilation of 0·4 mg cm^{-2} day^{-1} requires the evolution of 5·3 ($\pm$ 1·7) mg cm^{-2} day^{-1}, greatly exceeding the estimates of other workers.

7. Conclusions

With a gas analyser of adequate sensitivity the flux of carbon dioxide above a growing crop can be measured reasonably accurately. In the present work, the greatest source of error was uncertainty in transport number and this error could be decreased by working at a greater height above the crop in a larger field.

Biological interpretation of these and similar measurements must take into account the flux of CO_2 from the soil, but further analysis is showing that the data provide estimates of *gross* assimilation which are highly correlated with short-wave radiation and wind speed.

If the nocturnal flux of CO_2 is frequently as constant as Fig. 4 suggests, it might be exploited to study the effects of thermal stratification on turbulent transport close to the ground in very stable conditions. Nocturnal fluxes of water vapour and of sensible heat are inconveniently small and are too sensitive to changes in surface-heat balance. Above vegetation, the nocturnal flux of CO_2 probably depends less on meteorological conditions than does any other process of atmospheric diffusion.

Acknowledgments

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Appendix

Downward and upward fluxes of CO_2 (mg cm^{-2}) and solar radiation (cal cm^{-2}) for 24 hr periods.

Date (1958)	Solar radiation	Downward flux	Upward flux	Net downward flux
10 August	331	3·5	0·8	2·7
11 August	371	2·4	0·7	1·7
12 August	245	2·5	0·5	2·0
13 August	194	2·8	0·3	2·5
14 August	189	2·3	0·5	1·8
15 August	362	2·3	0·5	1·8
16 August	273	3·3	0·7	2·6
17 August	337	2·2	1·1	1·1
22 August	152	1·5	0·9	0·6
23 August	194	2·6	0·6	2·0

Date (1958)	Solar radiation	Downward flux	Upward flux	Net downward flux
30 August	333	3·4	0·6	2·8
31 August	286	2·6	0·6	2·0
1 September	300	3·1	0·6	2·5
2 September	287	2·8	0·6	2·2
3 September	169	2·0	0·7	1·3
5 September	178	1·7	0·8	0·9
6 September	306	2·1	0·9	1·2
7 September	264	1·7	1·0	0·7
8 September	193	2·2	1·1	1·1
10 September	289	0·8	1·6	− 0·8
11 September	295	0·9	1·9	− 1·0
12 September	250	1·9	2·2	0·3
13 September	287	2·3	1·1	1·2
14 September	263	0·9	1·1	− 0·2
16 September	211	2·8	1·1	1·7
17 September	242	2·0	1·0	1·0
18 September	243	2·4	1·0	1·4
10 October	88	0·5	8·3	− 7·8
11 October	181	1·7	3·9	− 2.2
14 October	214	2·0	5·0	− 3·0
15 October	171	2·3	2·0	0·3
16 October	213	2·1	3·4	− 1.3
18 October	100	0·9	3·4	− 2·5
19 October	106	0·9	5·1	− 4·2
20 October	69	0·1	7·2	− 7·1
21 October	54	0·0	8·1	− 8·1
22 October	32	0·0	8·7	− 8·7
23 October	60	0·4	5·3	− 4·9
26 October	50	0·0	6·2	− 6·2
27 October	116	0·4	6·1	− 5·7
28 October	101	0·0	4·9	− 4·9
29 October	78	0·0	6·2	− 6·2
30 October	30	0·0	5·8	− 5·8
2 November	28	0·0	8·1	− 8·1
3 November	80	0·0	6·8	− 6·8
5 November	56	0·1	5·0	− 4·9
11 November	108	0·5	3·7	− 3·2
12 November	37	0·1	4·2	− 4·3
13 November	88	0·0	2·6	− 2·6

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A surface energy-balance computer

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SUMMARY

A surface energy-balance computer is described. This is fed with such information as dry- and wet-bulb temperatures and temperature differences in the vertical (in the form of fine wire thermocouple outputs) and wind speed differences. The psychrometric equation is solved automatically to yield a vapour-pressure difference and the latent-heat term is then computed on the basis of an aerodynamic equation. The sensible-heat term is similarly made available. The net flux of radiation and the soil-heat flux are also recorded. A determination of each of these four fluxes is made once per minute and the values are integrated and presented on counters which are photographed at desired intervals, the smallest interval so far used being 20 min. Results indicate that for periods of an hour or more the ' positive ' and ' negative ' terms in the surface-heat balance equation so obtained, agree to about 10 per cent except when the heat storage in the first two cm of soil, of which no account is taken, is obviously changing. The only complete 24 hr observational period so far covered indicates a daily balance to a fraction of a mw hr cm^{-2}. Over approximately 80 hr observations, which were not continuous but contained about an equal number of daylight and dark hours, a balance to better than 1 per cent was obtained. A further result, which requires additional confirmation, is that the daily water loss from the centre of an extensive short-grass area which has an ample water supply may be obtained for all practical purposes by measuring the total net radiation and assuming that all is used in the evaporative process. It becomes obvious that if hourly mean values of the profiles of temperature, humidity and wind are used to compute the evaporation then, in general, the results will be in error by up to about 20 per cent, although daily totals may be more accurate than this. The machine as it stands is not reliable for continuous operation owing to the use of a number of electro-mechanical devices which are required to work under conditions for which they were not designed. Some improvements will be required before reliable continuous operation becomes possible.

1. INTRODUCTION

Perhaps the most important single relationship in meteorology is that derived from the application of the principle of the conservation of energy to the earth's surface. Neglecting terms which are of no immediate significance, this is expressed by the equation

$$R + \lambda E + S + Q = 0 \quad . \quad . \quad . \quad . \quad (1)$$

in which R is the net radiative flux, λE is the vertical flux of latent heat, S is the flux of heat in the ground at the surface and Q is the vertical flux of sensible heat, λ being the latent heat of condensation; fluxes directed away from the surface are considered as positive. Techniques to measure these four terms either singly or in combination have been much sought after not only on account of the importance of their absolute and relative magnitudes in theoretical meteorology but also to problems of immediate practical importance. The evaluation of the sum $(Q + \lambda E)$ is a relatively simple matter but the problem of separating the two terms presents some difficulty. The best known method is that due to Bowen (1926). The need to establish an indirect method for the evaluation of natural water loss from land and water surfaces so that irrigation and water conservation practice may be put on a sound footing is perhaps the pre-eminent practical requirement. In the past, evaluation of E has been attempted by either the measurement of R, S, and Q in Eq. 1 (e.g. Lemon, Glazer and Satterwhite 1957), the latent-heat term and hence E being obtained as a difference, or by direct measurement having recourse to turbulent transport equations. These equations have been based on one or the other of the two concepts of the mechanism of atmospheric-turbulent transport but in general they have only proved adequate for the purpose when evaporation over short time intervals has been required. This limitation has arisen partly as a consequence of the very fine instrumentation that is required for successful application (see e.g., Harbeck 1952) and partly owing to the effort needed to handle the lengthy computations that must be completed

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before the desired result is obtained. Recently successful attempts have been made in Australia (Taylor and Dyer 1958) to arrive automatically at values of E from observations of the vertical fluctuations of wind and vapour pressure, and the apparatus to be described here provides the same facilities although the equations which are automatically solved are based on the mixing-length concept of atmospheric turbulence rather than that of continuous mixing. Further the present apparatus evaluates the four terms in Eq. (1) so that a continuous check on the values provided for the individual fluxes is possible on the basis of that equation.

2. General considerations

The rate of evaporation may be obtained from the equation

$$E = \frac{\rho k^2 (u_2 - u_1)(q_1 - q_2)}{\left[\ln \frac{z_2 - d}{z_1 - d}\right]^2} \quad . \quad . \quad . \quad (2)$$

and the sensible-heat flux from the analogous equation

$$Q = \frac{\rho C_p k^2 (u_2 - u_1)(t_1 - t_2)}{\left[\ln \frac{z_2 - d}{z_1 - d}\right]^2} \quad . \quad . \quad . \quad (3)$$

in which ρ is the air density, k is von Kármán's constant, C_p is the specific heat of air at constant pressure, and u_1, u_2, q_1, q_2, and t_1, t_2 are the wind speeds, specific humidities and air temperatures at the heights z_1 and z_2. d is the surface zero displacement. Certain assumptions lie behind these two equations. Some have been fairly conclusively demonstrated; other less so. Eq. (2) has been tested against such direct gravimetric measurements of natural evaporation as have proved feasible and convincing agreement found by Pasquill (1950) and Rider (1954a); Eq. (3) has not been confirmed to the same extent. Sheppard (1958) gives a discussion of the basis and range of validity of these two equations which presents all the essential points. Rider (1954b) gives a short account of the investigation of the basis of Eq. (2). Familiarity with these papers will be assumed and no further comment made here except to note that in order to keep any systematic error due to the disregard of deviations from neutral stability conditions to a minimum, it is necessary to arrange for the heights of measurement of the meteorological elements to be as small as possible. Later it will be shown that both equations have been adequately tested in the results they provide. Thus, accepting the validity of these equations, it is seen that to arrive at values of E and Q it is only necessary to make the indicated measurements of wind speed, temperature and humidity and to carry out the required numerical operations. The numerical work can be done automatically once the measurements have been put into a convenient form, and it only remains, assuming that the measurements can be made, to decide how frequently it is necessary to make independent evaluations of the fluxes. Pasquill (1949) has shown that if, for example, daily mean values of the differences in wind speed, etc., are used, the resulting values of E will be in error compared with values obtained from hourly mean values of the same measurements. Later it will be shown that hourly mean values as used by Pasquill and later by Rider (1957) can themselves lead to errors in E of the order of 20 per cent. Rider (1958) earlier suggested that this might partially explain some anomalous results which he had reported. The frequency with which individual values of E and Q were made was effectively decided by what could be achieved by the anemometers and other apparatus which was available to us or which could be constructed or bought at little cost and also to some extent by what could be achieved is the design and construction of the computer itself in reasonable time with the effort available. The apparatus was made to provide a value for both E and Q once per minute, and the results subsequently obtained are considered to indicate that this frequency was adequate provided mean values for periods of 30 min. or more are required.

Figure 1. General view of the humidity-sensing head.

In our particular case, since we were working over a short-grass surface for which, as will be shown later, the value of d was effectively zero, from Eqs. (2) and (3) we obtain for practical use

$$\lambda E = 1{\cdot}80 \times 10^2 (e_1 - e_2)(u_2 - u_1)/T \quad . \quad . \quad . \quad (4)$$

$$Q = 2{\cdot}23 \times 10^{-1} (t_1 - t_2)(u_2 - u_1) \quad . \quad . \quad . \quad (5)$$

when z_1 and z_2 are 25 and 50 cm. In these two equations λE and Q are in mw cm^{-2}, $(e_1 - e_2)$ which will subsequently be referred to as de is in mb, T is the absolute temperature and $(t_1 - t_2)$ subsequently written as dt is in °F, $(u_2 - u_1)$ is in cm sec^{-1}. λ, the latent heat of condensation, has been taken as 600 g cal g^{-1}.

3. The instruments

The radiation flux R is conveniently measured by a net-flux radiometer. While there is no difficulty in measuring the net-radiative flux at the surface with a flux plate exposed at a convenient height – the change in net flux in the first metre or so above the surface is negligible – certain difficulties arise with the use of a soil-flux plate. The plate must be installed at some depth below the surface. When working over short grass this must be about 2 cm if, after installation, the grass is to continue to grow and have the same appearance as elsewhere. Thus a change in heat storage in the first 2 cm of soil is not accounted for in the flux-plate output, and this will be seen to be reflected in some of the results. Further, the calibration factor is dependent on the thermal conductivity of the material in which the plate is buried, and the factors provided covered a twofold range representative of calibration in different materials. Before observations commenced an estimate of the likely conductivity of the soil on the field site was made, a factor was decided upon and subsequently no change was made.

In Eq. (3) we require a knowledge of the temperature difference dt between the two heights z_1 and z_2; for Eq. (2) the corresponding vapour-pressure difference de is required. The accuracy to which these quantities must be measured, for reasonable accuracy in the computed values of Q and λE, are 0·01°F and 0·01 mb respectively (but see Section 4). It was decided at the outset to measure the vapour pressures psychrometrically by fine-wire thermocouples so that there would be no need to provide forced ventilation, the full wet-bulb depression being obtained in virtually calm air. This has the advantage that disturbance to the natural flow of air is avoided. Further, the use of very small thermocouples will keep radiation errors small without the need for shielding. In this application, in which we are interested mainly in differences in dry- and wet-bulb temperatures, we are not concerned to any extent with the complete elimination of radiation errors. It is only necessary that such errors, if present, be identical on all couples.

Fig. 1, Plate II, is a view of the sensing-head finally adopted after trials of four types. A rectangular frame 35 cm × 20 cm internal dimensions is made from strips of perspex of about 0·5 cm thick and 5 cm wide. Perspex transmits about 90 per cent of incident short-wave radiation, and hence is a very suitable material for this purpose. The thermocouples are constructed from copper-nickel and copper wire. For the dry junctions, wires of 0·0015 in. diameter are used; for the wet junctions, the wire diameter is 0·005 in. It was found impossible to wind efficient wicks on junctions made from the smaller-size wire as will be explained later. The dry and wet junctions are arranged on opposite sides of the frame. Three copper-nickel wires are stretched vertically from top to bottom at each side and the copper wires are placed horizontally to form five dry and five wet T-shaped junctions. The higher and lower junctions are 25 cm apart. Four of each type of junction are connected differentially in series, and the remaining junction of each type is connected differentially to a standard junction maintained in melting ice to yield values of the dry- and wet-bulb temperatures (t and t' respectively) at the lower of the two heights. Fig. 2 shows a simple wiring diagram which will make the arrangement clear.

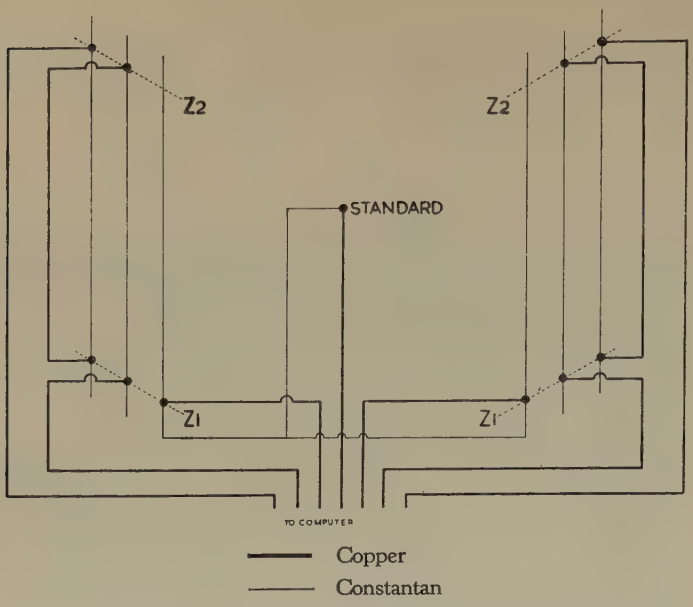

Figure 2. Schematic diagram of the humidity-sensing head.

A 7-core cable leads away from the sensing head to the computer. The wet-junction side of the frame is drilled to house the water reservoirs and the whole is mounted on a balanced perspex wind vane to maintain the plane of the frame at right-angles to the wind. The vertical steel rod seen in Fig. 1 supports the whole on self-aligning ball races so that in very light winds the vane is still capable of efficient operation. The height of exposure of the frame may be adjusted by moving the bearings up and down the steel rod. Throughout the work to be presented this was arranged so that the thermocouples were maintained at 25 cm and 50 cm above the ground. Wet junction feeds were provided by drawing the ends of small test-tubes into capillaries and threading two fine cotton strands through each. On emerging from the glass tubes both threads are taken to the copper-nickel wire at about 2 cm from the junction and wound around and up the wire towards the junction. At the latter they are separated, one thread continuing up the copper-nickel wire while the other is taken along the copper wire for a distance of about 2 cm. They are secured in position by a small drop of perspex cement at the extreme ends. The ' emergent-stem ' error is largely eliminated by covering the wires to some distance from the junction. The preparation of the wick cotton is important, and it was given the following treatment before being wound on the junctions. Washing in trichloroethylene and drying was followed by boiling for 30 min in distilled water to which a little Teepol had been added. The cotton was then finally washed in distilled water and dried. Threading and winding was done as far as possible with forceps to avoid contamination due to handling.

To explain the method of testing adopted for the various types of sensing head that were tried it is convenient here to digress a little and describe the first stage of the computer in outline. The outputs from the head are in the form of small voltages which are proportional to dt and dt'. These outputs are fed to a galvanometer in such a fashion that the deflections L, M, and N which are produced in turn may be written in the form :

$$Z_1 + dt' - dt = L \quad . \quad . \quad . \quad . \quad (6)$$

$$Z_1 - dt' - dt = M \quad . \quad . \quad . \quad . \quad (7)$$

$$Z_1 - dt' + dt = N \quad . \quad . \quad . \quad . \quad (8)$$

Z_1 is the open circuit zero position of the galvanometer, measured from some arbitrary position which is conveniently taken as one end of the galvanometer scale. Switching and other arrangements are made so that values of L, M, and N are obtained once per minute in the form of impulses to electro-mechanical counters. From the equations above it follows that

$$L - M = 2dt' \quad . \quad . \quad . \quad . \quad . \quad (9)$$

$$N - M = 2dt \quad . \quad . \quad . \quad . \quad . \quad (10)$$

$$L - N = 2(dt' - dt) \quad . \quad . \quad . \quad . \quad (11)$$

If the wet junctions are left dry then the mean values of dt and dt' indicated by the counters should agree over a reasonable time. The sensitivity of the circuits was such that a difference in the recorded hourly mean values of these temperature differences of 0·01°F would be indicated by a difference of six counts in the readings of the L and N counters. The procedure followed was to expose the various types of head in this way for periods of an hour and then to compare the two values of dt so indicated. Early designs of head gave differences of several tenths of a degree over hourly periods and were quite useless. A head of the type which has been described above but which was not mounted on a vane gave differences of up to 0·1°F. During several hours of testing the same head mounted on the vane, a maximum difference of five in the L and N counts was recorded, and for many hours there were no differences. Similar tests were then carried out with wet junctions of 0·0015 in wire on both sides of the frame and it was found impossible to get agreement between the two recorded values of dt' presumably because some junctions were not providing a true wet-bulb depression owing to inefficient water feeds. Satisfactory results were only achieved by increasing the wire diameter and size of the wet junctions as has already been indicated. The relevant literature suggests that the provision of wet-bulb feeds to junctions made from wire of about 0·001 in. diameter should present no difficulty, but our experience does not concur. It is only by means of a purely objective test which itself can only be made by some automatic method that disagreements of the order found can be confidently identified. Even when, for example, an automatic chart record is provided there is a tendency to account for such discrepancies as due to personal error or bias in measuring or interpreting the trace. Considering the difficulties associate with the placing of wicks on very fine wire thermocouples, for use in the open, it is perhaps not surprising on reflection that unsatisfactory results were obtained. The need to maintain the frame at right-angles to the wind direction is thought to arise because of the stirring of air around the side members of the frame when once these are inclined to the air flow. In general, more stirring will take place around the couples mounted on one side than those on the other. The results given by the vane-mounted frame are independent of the angle between the vertical planes through the sun and wind direction. If only dry-bulb temperatures were required it would be possible to design a simpler apparatus and dispense with the vane, but supports have to be provided for the water reservoirs and a completely open construction is not possible in the latter instance.

The only anemometers available to us were the familiar Sheppard-type, provided with electrical contacting mechanisms in place of the more usual mechanical counters. The cup-to-contact gearing in eight of these was modified to provide twice the usual number of contacts for a given number of cup revolutions. The calibrations were changed as a result and from the eight so modified it was possible to select two matched pairs after recalibration. The criterion of matching was that at no part of the slightly non-linear individual calibration curves should the wind speed for a given number of counts per minute differ by more than 3 cm sec^{-1}. Over most of the range considered which was from the starting wind speed (about 35 cm sec^{-1}) up to 900 cm sec^{-1} the calibration curves were almost identical for the members of each pair. An increase in the number of counts per minute for a given wind speed was required so that, in a half-minute of running, one count in the difference in counts given by two matched anemometers would represent

a reasonably small wind-speed difference. This difference was about 5 cm sec^{-1}. To avoid damage to the anemometer contacts they were arranged to drive relays fitted with quenching circuits, the relays being chosen to provide the desired number of outputs from each anemometer.

4. The computer

The temperature differences, both dry and wet, which are detected by the humidity sensing head are presented in the form of small voltages and before these can be passed into the circuits which cater for the solution of the psychrometric equation and the arithmetic process required by Eqs. (2) and (3) they must be amplified by some suitable system. Since we wish to detect very small differences in temperature, the voltage available in the thermocouple circuits will, on occasions, amount to fractions of a microvolt. In such cases, particularly when the initial signal is derived from a low-impedance source as in the present case, it is essential to use a primary sensing device which will convert the signal voltage into a form which can be amplified. As a consequence we have developed a system which will digitize and integrate, if required, any signal from the microvolt to the volt range and which should have many applications outside our immediate requirements.* A reflecting galvanometer is used in the following way. The normal galvanometer scale is replaced by a photo-voltaic selenium cell which is covered by a grating consisting of alternate opaque and transparent strips of equal width; in our case 1 mm. The width of these strips is the same as that of the image of the projector lamp slit at the ' scale ' distance. A double slit in which the two bands of light are separated by the width of a grating line is conveniently used. As the image of the slit moves across the face of the grating a series of pulses are produced by the cell, the number of pulses being directly proportional to the distance swept out. Assume that the necessary interconnexions are made to give us the deflection L as defined by Eq. (6). The galvanometer light will take up some position on the ' scale ' according to its free zero position Z_1 and the relative magnitudes and signs of dt and dt'. If now, by some external means, the circuit for L is removed from the galvanometer terminals and at the same time a small current is passed through the galvanometer from a ' bias circuit ' which is sufficient to move the slit of light right off one end of the ' scale ' the number of pulses produced by the cell will be a measure of the deflection L. Similarly by making the interconnexions indicated by Eqs. (7) and (8) and arranging for the ' bias ' circuit to be connected at the appropriate times, the cell will provide a number of pulses proportional to M and N. It now remains to provide a pulse amplifier to raise the output of the cell to a level sufficient to operate electromagnetic counters for recording values of L, M and N and to operate uniselectors (stepping switches) which are employed in the computing circuits.

It is well known that photo-voltaic cells do not lend themselves very readily to incorporation in electronic circuits owing to their low internal impedance. However, a satisfactory amplifier was finally produced. In all, five of these amplifiers together with cells and galvanometers, are used in the complete system and reference to the others will be made later.

At this stage it is convenient to consider some general features of the system employed in the computer. The various components available to us impose certain limitations. For example, we cannot have a galvanometer ' scale ' longer than about 14 cm since cells of greater length than this are not made,† and so by sweeping from one end of the scale to the other we cannot obtain more than 70 pulses. Uniselectors available to us have a limited number of ways, normally 50, and resistance chains (see below) have been made from individual resistors and connexions to them can only be made at the junction of the resistors. For these reasons the accuracy of measurement in a single operation is less than that required in the final result. However, if a sufficient number of determinations

* For example, this apparatus has been satisfactorily employed to provide hourly totals of total and diffuse solar radiation on a horizontal surface.

† Recently larger cells which can be obtained with a stated radius of curvature have become available and should enable larger angular galvanometer deflection to be used without introducing error.

of a quantity that is varying in a more or less random manner are taken, the mean result will be reliable to a significant figure beyond the accuracy of an individual determination. In the atmosphere, vertical gradients of temperature, humidity, etc., are varying from moment to moment and the fine-wire thermocouples and their associated galvanometers follow these fluctuations quite rapidly. The normal diurnal variation is superimposed on this. Thus if a temperature difference between two heights is taken once per minute to 0·1°F we can expect an accuracy of about 0·01°F in an hourly mean. The design of the computer is based on this fact and it is unnecessary to provide for better resolution in the measurements than we have done if minute-to-minute values of the rate of evaporation, etc., are not required. A chain of resistances (the main chain) is provided across which a stabilized voltage is maintained. This is tapped at a number of equal resistance intervals. Any of these positions may be selected electro-mechanically, and any circuits so attached to the main chain always have a high resistance in series, so that the voltage distribution in the chain is disturbed to less than one per cent by any current drains which occur. All elements can be measured in the form of pulses by the means described and can be made to select voltages from the main chain or from some other chain, which may or may not be linear, by the use of uniselectors. The voltages selected are measures of the magnitude of the elements concerned, and all the required numerical operations may be carried out electrically (see, e.g., Soroka 1954).

We can now consider the methods used for the computations of λE and Q in outline. L, M and N, Eqs. (6), (7) and (8), and all other information can be obtained in the form of pulses. As far as λE is concerned the first step is to obtain a value of de. If the vapour pressures at z_1 and z_2 are e_1 and e_2 and the corresponding saturation vapour pressures are e_1' and e_2' the usual psychrometric equations for the two heights are

$$e_1 = e_1' - A\,(t_1 - t_1')$$
$$e_2 = e_2' - A\,(t_2 - t_2')$$

in which the t's have their obvious meanings. Subtraction gives

$$e_1 - e_2 = de = de' + A\,(dt' - dt) \quad . \quad . \quad . \quad (12)$$

de' is determined by the values of t' and dt' and may be obtained from tables of saturation vapour pressure. We can therefore write.

$$de = dt'\,(f(t')) + A\,(dt' - dt)$$

Substituting from Eqs. (6), (7) and (8) we obtain

$$2de = (L - M)\,(f(t')) + A\,(L - N) \quad . \quad . \quad . \quad (13)$$

for technical reasons the machine cannot handle the solution for de as expressed by Eq. (13) and the equation is rearranged as

$$de = L\,(f(t') + A)/2 - M\,(f(t'))/2 - NA/2 \quad . \quad . \quad (14)$$

de is obtained in the form $(Z_1 + de)$ and $(Z_1 - de)$; Z_1 representing the zero position of the galvanometer. Two operations are then carried out, namely $(Z_1 + de)\,(u_2 - u_1)$ and subsequently the multiplication of this product by $1/T$. This yields a value equal to $Z_3 + (Z_1 + de)\,(u_2 - u_1)/T$ and similarly $Z_3 + (Z_1 - de)\,(u_2 - u_1)/T$ is obtained from $(Z_1 - de)$; (here again Z_3 is the zero of the galvanometer used). $c\lambda E$ is obtained from the difference. Since $2dt = N - M$, Q may be obtained by the process $N\,(u_2 - u_1) - M\,(u_2 - u_1)$; $2CQ$ being obtained as a difference in two sets of counts, c and C are calibration factors.

The psychrometric constant, A of Eqs. (12), (13) and (14) varies according to the value of t' and of the barometric pressure. However, the variation is quite small and in any event is slow and it was not considered worth while to provide for the automatic setting of A. Instead, one of seven values may be set on a rotary switch according to conditions prevailing at the start of a run and changed subsequently at any time should conditions demand.

TABLE 1. OBSERVATIONS OF WIND SPEED AND TEMPERATURE, AND COMPUTED HEAT BALANCE TERMS

	Observation date	Period (GMT)	Wind speed cm sec^{-1} u_{25}	u_{50}	t' °F	dt °F	dt' °F	R	λE mw	S cm^{-2}	Q	Balance −	+	Weather*	Cloud*
1	3 September 1958	1300-1400			66·3	− 0·08	+ 0·40	− 20·3	+ 16·1	+ 3·3	− 0·3	20·6	19·4	02	755--
2		1400-1500			66·5	− 0·25	+ 0·19	− 10·7	+ 11·3	+ 2·1	− 4·2	14·9	13·4	03	855--
3		1500-1600			64·7	− 0·22	+ 0·13	− 4·9	+ 2·9	+ 3·4	− 1·9	6·8	6·3	02	855--
4		2030-2130			61·9	− 0·28	− 0·11	+ 1·3	− 1·6	− 0·7	− 1·4	3·7	1·3	02	855--
5		2130-2230			62·5	− 0·08	− 0·12	+ 3·6	− 1·2	− 0·8	− 1·6	3·6	3·6	02	855--
6		2230-2330			60·7	− 0·34	− 0·10	+ 5·6	− 2·0	− 0·8	− 2·3	5·1	5·6	60	855--
7	4 September 1958	1100-1200	219	269	67·4	+ 0·63	+ 0·69	− 28·4	+ 14·8	+ 6·9	+ 6·6	28·4	28·3	02	655--
8		1200-1300	240	296	66·7	+ 0·02	+ 0·41	− 15·9	+ 9·1	+ 4·4	0·0	15·9	13·5	03	655--
9		1300-1400	181	223	66·6	0·00	+ 0·37	− 8·5	+ 5·6	+ 2·1	0·0	8·5	7·7	02	755--
10		1400-1500	194	239	67·8	+ 0·04	+ 0·44	− 14·9	+ 10·1	+ 4·0	− 0·7	15·6	14·1	02	755--
11		1500-1530						− 11·0	+ 10·0	+ 3·0	− 1·1	12·1	13·0	01	755--
12		1530-1630	186	204	67·6	− 0·18	+ 0·28	− 8·6	+ 9·2	+ 2·5	− 1·4	10·0	11·7	01	44600
13		1630-1700	212	261	67·1	− 0·29	+ 0·14	− 4·1	+ 8·8	+ 1·5	− 3·0	7·1	10·3	01	41600
14		1700-1730	163	198	66·8	− 0·29	+ 0·09	− 3·2	+ 2·8	+ 1·1	− 2·1	5·3	3·9	02	41600
15		1730-1830	163	204	65·8	− 0·48	− 0·07	+ 1·3	+ 1·8	− 0·2	− 3·0	3·2	3·1	05	41600
16		1830-1900	163	216	63·5	− 0·53	− 0·20	+ 4·2	+ 0·3	− 1·1	− 4·3	5·4	4·5	02	00900
17		1900-1930	77	106	62·8	− 0·67	− 0·32	+ 4·6	− 2·1	− 1·9	− 3·5	7·5	4·6	02	00900
18		1930-2030	97	128	62·2	− 0·51	− 0·23	+ 4·1	− 0·8	− 2·0	− 2·3	5·1	4·1	02	00900
19		2030-2100	103	132	61·6	− 0·44	− 0·25	+ 3·6	− 1·9	− 2·0	− 2·5	6·4	3·6	02	00900
20		2100-2130	88	117	61·9	− 0·35	− 0·15	+ 3·2	− 1·8	− 1·8	− 3·1	6·7	3·2	02	00900
21		2130-2200	57	81	61·0	− 0·39	− 0·27	+ 3·5	− 1·3	− 2·0	− 2·4	5·7	3·5	02	00900
22		2200-2230	101	133	60·1	− 0·29	− 0·26	+ 4·2	− 0·1	− 2·4	− 2·1	4·6	4·2	02	00900
23	5 September 1958	1200-1300	254	289	71·7	+ 0·57	+ 1·00	− 17·7	+ 15·4	+ 4·3	+ 4·3	17·7	24·0	03	7257-
24		1300-1400	217	252	71·7	+ 0·71	+ 1·08	− 18·0	+ 15·0	+ 4·0	+ 5·6	18·0	24·6	17	6959-
25		1400-1500	150	185	71·8	+ 0·92	+ 1·17	− 10·5	+ 12·5	+ 2·4	+ 3·8	10·5	18·7	17	6959-
26		1500-1600	Stopping		71·6	+ 0·34	+ 0·89	− 9·0	+ 1·5	+ 6·9	0·0	9·0	8·4	01	65631
27		1600-1700	55	74	72·1	− 0·22	+ 0·49	− 1·2	+ 0·8	− 0·2	− 1·7	3·1	0·8	01	55630
28		1700-1800	86	110	71·7	− 0·34	− 0·01	+ 1·4	+ 0·5	− 0·4	− 1·6	2·0	1·9	97	894--
29		2200-2220			61·8	− 0·61	− 0·29	+ 5·6	− 2·3	− 2·1	− 0·9	5·3	5·6	29	39500
30		2220-2240			62·0	− 0·46	− 0·19	+ 4·9	+ 0·4	− 1·5	− 3·2	4·7	5·3	29	29500

* Weather and cloud are given in coded form : ww and $N_h\, C_L\, h\, C_M\, C_H$ following the Washington 1947 Code form FM11A. (See WMO – No. 9 TP4, 1954 edition)

TABLE 1 (continued)

	Observation date	Period (GMT)	Wind speed cm sec^{-1} u_{25}	u_{50}	t' °F	dt °F	dt' °F	R	λE mw	S cm^{-2}	Q	Balance −	+	Weather	Cloud
31	5 September 1958	2240-2300			61·5	− 0·53	− 0·36	+ 4·9	− 1·1	− 1·4	− 2·4	4·9	4·9	13	19500
32		2300-2330	133	158	62·6	− 0·42	− 0·16	+ 4·4	− 0·1	− 1·7	− 2·6	4·4	4·4	13	3653-
33		2330-2400	63	81	59·5	− 0·49	− 0·08	+ 3·4	− 0·7	− 1·8	− 1·2	3·7	3·4	13	765--
34	6 September 1958	0001-0030	119	143	61·7	− 0·33	− 0·19	+ 3·5	0·0	− 1·6	− 1·0	2·6	3·5	13	665--
35		0030-0100	91	121	61·7	− 0·41	− 0·28	+ 4·7	+ 0·1	− 2·1	− 1·1	3·2	4·8	01	36500
36		0100-0130	Stopping		59·0	− 0·98	− 0·87	+ 4·7	− 1·5	− 2·5	− 0·9	4·9	4·7	01	26500
37		0130-0200	Stopping		58·6	− 0·67	− 0·66	+ 3·9	− 0·5	− 2·7	− 0·2	3·4	3·9	01	16500
38		0200-0300	Stopping		58·4	− 0·63	− 0·69	+ 4·4	− 0·3	− 2·1	− 0·8	3·2	4·4	02	16500
39		0300-0330	Stopping		60·7	− 0·33	− 0·28	+ 3·2	− 0·5	− 2·3	− 0·2	3·0	3·2	03	36500
40		0330-0400	91	112	63·9	− 0·38	− 0·45	+ 0·7	+ 0·8	− 0·9	− 1·2	2·1	1·5	41	863--
41		0400-0430	103	131	64·0	− 0·10	− 0·01	0·0	+ 1·3	− 0·4	− 1·0	1·4	1·3	46	862--
42		0430-0700	163	190	65·4	− 0·07	− 0·02	− 1·2	+ 0·2	+ 0·1	− 0·4	1·6	0·3	44	862--
43		0700-0730			67·2	+ 0·05	+ 0·40	− 5·1	+ 1·3	+ 1·1	0·0	5·1	2·4	44	862--
44		0730-1000	270	319	67·9	+ 0·20	+ 0·46	− 10·3	+ 4·7	+ 1·7	+ 2·3	10·3	8·7	28	863--
45		1000-1030						− 16·1	+ 7·8	+ 3·1	+ 4·3	16·1	15·2	02	863--
46		1030-1100	329	368	71·9	+ 0·49	+ 0·81	− 40·0	+ 21·5	+ 4·9	+ 4·9	40·0	31·3	01	26500
47		1100-1130	357	411	70·6	+ 0·13	+ 0·52	− 27·2	+ 19·4	+ 4·6	+ 1·4	27·2	25·4	03	655--
48		1130-1200	428	490	70·2	+ 0·21	+ 0·78	− 41·0	+ 30·5	+ 4·3	+ 5·6	41·0	40·4	01	35601
49		1200-1230	438	500	68·9	+ 0·08	+ 0·68	− 31·4	+ 28·6	+ 4·2	0·0	31·4	32·8	03	55601
50		1230-1430			68·9	+ 0·10	+ 0·66	− 24·2	+ 22·6	+ 3·7	+ 1·9	24·2	28·2	02	55601
51		1430-1500			68·5	− 0·03	+ 0·54	− 31·6	+ 22·1	+ 3·5	+ 0·9	31·6	26·5	01	35601
52		1500-1530	441	507	67·7	− 0·09	+ 0·35	− 23·3	+ 17·8	+ 2·6	0·0	23·3	20·4	02	35601
53		1530-1600	408	472	66·9	− 0·27	+ 0·24	− 13·2	+ 14·1	+ 1·1	− 0·6	13·8	15·2	02	35601
54		1600-1630	417	483	67·4	− 0·19	+ 0·25	− 14·9	+ 13·0	+ 1·4	− 0·7	15·6	14·4	02	35600
55		1630-1700	385	426	65·5	− 0·34	+ 0·07	− 6·1	+ 7·6	0·0	− 0·7	6·8	7·6	02	35600
56		1700-1730	308	355	65·0	− 0·52	− 0·04	+ 0·1	+ 1·6	− 0·7	− 0·5	1·2	1·7	02	35600
57		1730-1800	222	270	64.0	− 0·50	− 0·06	+ 3·2	+ 3·3	− 2·4	− 4·0	6·4	6·5	03	55600
58		1800-1900	279	325	63·2	− 0·50	− 0·12	+ 4·9	+ 4·4	− 1·5	− 1·5	3·0	9·3	02	55600
59		1900-2000	228	266	60·7	− 0·46	− 0·19	+ 6·2	− 0·1	− 1·6	− 1·2	2·9	6·2	01	25630
60		2000-2100	180	212	60·7	− 0·42	− 0·24	+ 5·3	− 2·1	− 2·0	− 2·4	6·5	5·3	02	25600

TABLE 1 (continued)

	Observation date	Period (GMT)	Wind speed cm sec^{-1} u_{25}	u_{50}	t' °F	dt °F	dt' °F	R	λE mw	S cm^{-2}	Q	Balance −	+	Weather	Cloud
61	6 September 1958	2100-2130	221	256	60·2	− 0·73	− 0·20	+ 5·2	+ 0·3	− 2·0	− 4·0	6·0	5·5	02	25600
62		2130-2200	212	250	60·1	− 0·48	− 0·25	+ 6·4	− 0·8	− 1·9	− 3·7	6·4	6·4	01	15600
63		2200-2230	197	230	60·1	− 0·47	− 0·29	+ 6·1	− 1·8	− 2·2	− 4·2	8·2	6·1	02	15630
64		2230-2330	172	201	59·2	− 0·35	− 0·29	+ 5·7	− 2·3	− 1·5	− 2·0	5·8	5·7	03	25600
65		2330-2400	202	235	59·2	− 0·28	− 0·30	+ 6·8	− 2·5	− 2·2	− 2·6	7·3	6·8	01	15600
66	7 September 1958	0001-0030	194	232	59·3	− 0·28	− 0·33	+ 6·9	− 3·4	− 2·3	− 2·3	8·0	6·9	01	00900
67		0030-0100	176	207	59·8	− 0·26	− 0·30	+ 4·4	− 2·7	− 2·0	− 1·4	6·1	4·4	03	15600
68		0100-0130	214	250	59·8	− 0·30	− 0·33	+ 7·1	− 4·4	− 2·0	− 1·6	8·0	7·1	01	00900
69		0130-0200	273	322	59·6	− 0·26	− 0·28	+ 7·0	− 4·6	− 2·0	− 2·6	9·2	7·0	02	00900
70		0200-0230	235	273	59·2	− 0·29	− 0·35	+ 6·8	− 6·1	− 2·0	− 2·3	10·4	6·8	02	00900
71		0230-0300	247	287	59·3	− 0·26	− 0·29	+ 7·0	− 3·3	− 2·0	− 1·9	7·2	7·0	02	00900
72		0300-0330	239	279	59·5	− 0·24	− 0·30	+ 6·3	− 5·0	− 1·5	− 1·6	8·1	6·3	03	15600
73		0330-0400	239	280	59·0	− 0·26	− 0·33	+ 7·1	− 4·5	− 2·0	− 2·1	8·6	7·1	01	00900
74		0400-0430	298	348	58·9	− 0·24	− 0·27	+ 6·9	− 6·8	− 1·9	− 0·3	9·0	6·9	02	00900
75		0430-0500	296	345	57·2	− 0·25	− 0·23	+ 7·7	− 3·0	− 1·9	− 3·0	7·9	7·7	02	00900
76		0500-0600	313	365	58·7	− 0·22	− 0·18	+ 4·8	− 2·7	− 1·8	− 1·1	5·6	4·8	03	15600
77		0600-0630	327	370	59·8	− 0·18	+ 0·01	0·0	+ 1·1	− 1·5	− 2·0	3·5	1·1	02	15600
78		0630-0830	413	478	62·1	− 0·08	+ 0·44	− 10·9	+ 15·7	− 0·1	− 2·5	13·5	15·7	03	32500
79	13 September 1958	0830-0900			62·3	+ 0·05	+ 0·44	− 11·7	+ 10·6	+ 1·2	+ 1·8	11·7	13·6	02	00905
80		0900-1000			62·5	+ 0·21	+ 0·58	− 21·1	+ 14·4	+ 2·3	+ 4·4	21·1	21·1	02	00906
81		1000-1030			63·6	+ 0·34	+ 0·83	− 27·6	+ 17·9	+ 6·7	+ 3·1	27·6	27·7	02	00906
82		1030-1100	236	289	62·9	+ 0·36	+ 0·60	− 20·6	+ 12·1	+ 6·0	+ 2·9	20·6	21·0	03	00907
83		1100-1130	252	309	63·3	+ 0·06	+ 0·59	− 22·0	+ 20·7	+ 5·4	− 1·9	23·9	26·1	02	00907
84		1130-1200	234	298	63·7	+ 0·16	+ 0·63	− 24·7	+ 16·9	+ 6·3	+ 1·4	24·7	24·6	02	00907
85		1200-1300	240	305	64·7	+ 0·24	+ 0·66	− 24·8	+ 19·7	+ 6·4	+ 1·7	24·8	27·8	02	00907
86		1300-1330	260	328	64·7	+ 0·18	+ 0·60	− 25·5	+ 18·0	+ 6·3	+ 0·5	25·5	24·8	02	00907
87		1330-1430	275	334	64·7	+ 0·18	+ 0·56	− 25·8	+ 17·4	+ 5·4	+ 0·9	25·8	23·7	01	00901
88		1430-1500	282	345	64·2	+ 0·10	+ 0·54	− 24·2	+ 19·0	+ 4·4	+ 0·8	24·2	24·2	01	00901
89		1500-1530	267	330	62·9	− 0·21	+ 0·42	− 13·7	+ 14·9	+ 2·5	− 3·7	17·4	17·4	02	00901
90		2100-2130	Stopping		51·5	− 1·22	− 0·80	+ 2·9	+ 0·8	− 3·3	− 0·2	3·5	3·7	02	00900

TABLE 1 (continued)

	Observation date	Period (GMT)	Wind speed cm sec^{-1} u_{25}	u_{50}	t' °F	dt °F	dt' °F	R	λE mw	S cm^{-2}	Q	Balance −	Balance +	Weather	Cloud
91	13 September 1958	2130-2200	Stopping		52·2	− 1·66	− 1·15	+ 3·6	0·0	− 3·5	− 0·5	4·0	3·6	02	00900
92		2200-2230	Stopping		52·4	− 1·20	− 0·84	+ 4·4	+ 0·2	− 3·6	− 0·5	4·1	4·6	02	00900
93		2230-2330	Stopping		53·4	− 0·68	− 0·51	+ 3·2	+ 0·3	− 2·7	− 1·2	3·9	3·5	41	00900
94		2330-2400	Stopping		53·0	− 0·68	− 0·65	+ 2·7	+ 0·5	− 2·5	− 0·5	3·0	3·2	41	00900
95	14 September 1958	0001-0100	Stopping		54·6	− 0·73	− 0·53	+ 2·0	+ 0·3	− 1·9	− 0·6	2·5	2·3	41	00900
96		0100-0200	Stopping		55·2	− 0·47	− 0·36	+ 1·9	+ 0·2	− 1·8	− 0·4	2·2	2·1	41	00900
97		0200-0300	Stopping		53·0	− 0·46	− 0·35	+ 2·7	0·0	− 1·8	− 1·2	3·0	2·7	41	00900
98		0300-0330	Stopping		51·3	− 1·26	− 1·22	+ 2·9	− 1·2	− 2·8	− 1·7	5·7	2·9	41	00900
99		0330-0400	Stopping		51·0	− 1·56	− 1·62	+ 2·7	− 0·5	− 3·1	0·0	3·6	2·7	41	00900
100		0400-0430	Stopping		50·6	− 0·87	− 1·01	+ 2·5	0·0	− 3·1	− 0·2	3·3	2·5	41	00900
101		0430-0500	Stopping		50·3	− 0·92	− 0·78	+ 3·2	+ 0·5	− 2·9	0·0	2·9	3·7	41	25600
102		0500-0530	Stopping		51·9	− 0·64	− 0·56	+ 3·6	− 0·5	− 2·6	− 1·4	4·5	3·6	46	25600
103		0530-0600	Stopping		52·8	− 0·49	− 0·38	+ 2·2	+ 0·5	− 2·1	− 0·7	2·8	2·7	44	00900
104		0600-0700	Stopping		55·4	− 0·07	− 0·02	+ 0·2	0·0	− 0·6	− 0·2	0·8	0·2	45	9-0--
105		0700-0730	Stopping		56·1	− 0·07	− 0·05	+ 0·5	+ 0·5	− 0·3	0·0	0·3	1·0	28	46300
106		2100-2200	134	162	58·0	− 0·34	− 0·23	+ 4·3	− 1·4	− 1·8	− 1·0	4·2	4·3	03	0097-
107		2200-2230	115	144	57·6	− 0·30	− 0·18	+ 2·0	+ 0·3	− 1·6	− 0·7	2·3	2·3	03	0097-
108		2230-2300	107	134	57·2	− 0·38	− 0·27	+ 5·5	− 1·0	− 1·9	− 2·1	5·0	5·5	02	0097-
109		2300-2330	107	134	57·5	− 0·24	− 0·15	+ 1·7	0·0	− 1·6	− 1·7	3·3	1·7	03	5567-
110		2330-2400	119	150	56·9	− 0·26	− 0·17	+ 2·2	− 1·3	− 1·6	− 1·6	4·5	2·2	03	856--
111	15 September 1958	0001-0030	155	191	57·0	− 0·18	− 0·12	+ 1·5	+ 0·3	− 1·2	+ 0·7	1·2	2·5	03	856--
112		0030-0100	152	196	56·9	− 0·13	− 0·09	+ 2·5	− 0·3	− 1·5	− 1·4	3·2	2·5	60	856--

Circuits are provided in the computer to execute the various numerical operations indicated above and the λE and Q terms are presented on counters. The sensitivity of the apparatus was such that a difference of one count on the λE counters was equivalent to 2·52 mw cm^{-2} and the corresponding difference in the Q counters was equivalent to 1·74 mw cm^{-2}. Counters also record the measured values of dry- and wet-bulb temperatures.

The radiative and soil heat fluxes are also presented on counters by using a galvanometer, cell and amplifier for each. To render the results independent of galvanometer zero position, as in the case for λE and Q, pulses which are the measures of $(Z_4 + R)$, $(Z_4 - R)$ and $(Z_5 + S)$ $(Z_5 - S)$ are obtained and recorded on four counters, the fluxes themselves being obtained from the appropriate differences in counts. The galvanometer circuits are arranged with series resistances and shunts so that convenient deflexions are obtained, taking into account the calibrations of the flux plates, the sensitivities of the galvanometers and the ranges of flux to be expected. In the present instance the circuits were such that one count in the appropriate difference represented 1·69 mw cm^{-2} for the net radiative flux and 0·32 mw cm^{-2} for the soil heat flux.

It is obvious that a definite sequence of operations must be followed in order to get the required results and the sequence, which occupies one minute, is controlled by uniselectors of non-standard type and associated relays. An automatic camera controlled by the timing unit is used to photograph the various counters. In addition to the counters previously mentioned a run-counter and counters from five anemometers are provided.

5. Observational programme

Field trials of the whole system were started in August 1958 and by the middle of September results had been obtained covering three periods. The site of the trials was a disused aerodrome some 20 mi west of Cambridge and the sensing instruments were positioned at the centre of a triangular short-grass area which was bounded by concrete runways 30 m wide. The grass was gang-mown at regular intervals and the fetch over a uniform surface was never less than 200 m. During the first period of observation we were unable to obtain an energy balance and spent considerable time in looking for possible faults in the computer. Finally these possibilities were eliminated and suspicion centred on the performance of the net-flux plate. It was found that this was asymmetrical. After correction and recalibration, a satisfactory balance was obtained. Thus, although the values of λE, Q and S were probably reliable in the first period they have not been presented here. Observational work was restarted on 3 September and after the computer had been running for some hours it became apparent that in one particular section the ' engineering ' was not good enough for reliable continuous operation. This arose as a result of the use uniselectors as selection and memory units. They are designed for use in circuits in which the voltage is at least 25 mv; in parts of our circuit the voltage was at times below one millivolt. To work under these conditions the brush contacts, etc., must be set within very fine limits and kept free from all traces of dust and grease. It appears impossible to maintain these requirements for long periods. The uniselectors in question were the five which operate in the wind and $1/T$ circuits. While quite elementary modifications to the apparatus would overcome this fault we did not have the required components available and preferred to use the apparatus as it stood, stopping when necessary to make adjustments, in order to take advantage of such fine weather as remained. (We hoped to get observations when the levels of the heat-balance components were high). Fortunately we were readily able to detect when the machine was not functioning correctly and to be reasonably certain that the results as given in Table 1 were not influenced by this fault. However, there remains the possibility that the occurrence of the fault very infrequently in any period, may have gone unnoticed, and slight error of the order of 1 mw cm^{-2} may be present in the values given for λE and Q. In all, 79½ hr of successful observation were covered in continuous periods of up to 34½ hr. When the wind speed is very low, and the

anemometers at either 25 or 50 cm stop, the construction of the circuits is such that the wind-difference chains automatically lock in their end positions so that overestimation of λE and Q is impossible. Slight underestimation will occur when this happens. During rain the radiometer gives spurious results and continues to do so for a considerable time after it has once been wetted and the plate surfaces appear to have dried. We found it advisable to cover the instrument at the onset of rain so that recording could be started immediately afterwards. No attempt was made to select an occasion of observation according to weather conditions beyond the requirement that rain should not be falling.

6. Results

Values of the heat balance components and the sums of the 'positive' and 'negative' terms together with certain other information are set out in Table 1 for about 80 hr of observation, but before proceeding in a discussion of these results we must first show that we were justified in using a zero value for d in Eqs. (2) and (3). The three observations in the interval 12 to 15 hr on 4 September will suffice. For these three hours $dt < 0{\cdot}04°\text{F}$ and there was considerable wind shear, i.e., $du/dz \gg 0$. The logarithmic wind profile law should therefore hold. Anemometers were exposed at five heights up to 2 m and Table 2 sets out the wind speeds experienced as a ratio of that prevailing at 1 m.

TABLE 2. Wind profiles for three neutral hours

	4 September 1958				
Hour	u_{25}/u_{100}	u_{50}/u_{100}	u_{100}/u_{100}	u_{150}/u_{100}	u_{200}/u_{100}
12-13	0·644	0·796	1·000	1·082	1·140
13-14	0·657	0·805	1·000	1·094	1·192
14-15	0·643	0·807	1·000	1·088	1·137
Means	0·648	0·803	1·000	1·088	1·158

Fig. 3 is a plot of the mean profile against the $\log_{10}(z - d)$ for $d = 0$, and we can see that there is no suggestion of curvature in the curve so that the value of d cannot differ greatly from zero.

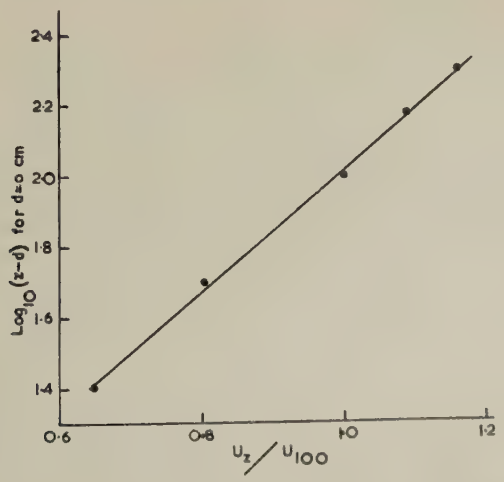

Figure 3. Mean wind profile for three neutral hours (1200-1500) 4 September 1958.

Turning to Table 1 we see that, in general, except when individual values are small, the positive and negative components balance to about 10 per cent in each observation. There are certain exceptions – observations No. (o.n.) 46, 58 and 59 illustrate one cause. We have mentioned that we were not able to account for any change in heat storage in the first 2 cm of soil. Up to the end of o.n. 45 the sky was obscured by low stratus which broke and dispersed by 1035 hr. During the half-hour of o.n. 46 the ground was warming up rapidly and an energy supply at the rate of about 10 mw cm^{-2} is unaccounted for. If we assume that the mean temperature of the first 2 cm of soil rose by the reasonable figure of 6°F and the heat capacity per unit volume of the soil was in the region of 0·35 g cal cm^{-3} $°F^{-1}$, a representative figure for a wet clay soil, then the lack of balance is explained. The reverse process occurred in o.n. 58 and 59 when more heat was used than was apparently available. We also tended to get a poor balance when steady conditions prevailed and when the time over which the observation was taken was less than an hour. This was to be expected. We can anticipate that over a sufficiently long period we shall always obtain a satisfactory balance in spite of difficulties of this type. Unfortunately, we only managed to cover one complete 24 hr period for which we can expect that the change in soil heat in the first 2 cm will be unimportant. As the following figures illustrate, the change in soil heat below the 2 cm level was small, so it appears reasonable to take the change in the thin top layer as being negligible. For 6 September energy used or supplied by the four components (as obtained from Table 1) was as follows :

$$R = -151{\cdot}4, \quad \lambda E = +148{\cdot}8, \quad S = +6{\cdot}6, \quad Q = -4{\cdot}3, \quad \text{mw hr cm}^{-2}$$

and the sums of the positive and negative terms are 155·4 and 155·7 mw hr cm^{-2}. The soil had therefore gained a small amount of heat and the air had supplied a very similar quantity. The evaporation process accounted for virtually all the net radiation, and the agreement with Eq. (1) is very satisfactory. The amount of water lost from the surface was approximately 2·1 mm and Fig. 4 shows a plot of the four terms for the period.

For all the observations listed in Table 1 the total energy used or supplied by the four components was :

$$R = -377{\cdot}6, \quad \lambda E = +384{\cdot}6, \quad S = +28{\cdot}0, \quad Q = -38{\cdot}7 \quad \text{mw hr cm}^{-2}$$

and the sums of the positive and negative terms are 412·6 and 416·3 mw hr cm^{-2}, again a satisfactory result. In this total period there were rather more hours of darkness than of daylight (about $40\frac{1}{2}$ hr as against 39 hr) and there was also a bias towards a greater number of hours of observation after rather than before midday (46 hr as against $33\frac{1}{2}$ hr).

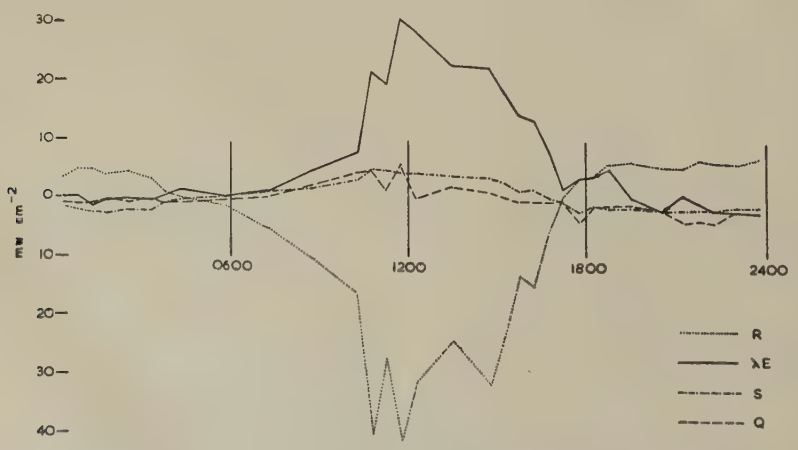

Figure 4. Plot of the energy-balance components 6 September 1958.

Here again we see that for all practical purposes, the energy used in evaporating water is equivalent to the net radiative input. It would therefore appear that satisfactory estimates of the water loss for periods of a day or more from an extensive short-grass area with a plentiful water supply in East Anglia in September can be obtained from an integrated total of net radiation. This result, which will require testing in other seasons and latitudes, could be of considerable practical importance, since it implies that the potential evaporation may be obtained from one instrument exposed over a suitable site and that the complications of the Penman (1948) and similar methods, need not concern us further in day-to-day irrigation practice. Formulae of this type will, however, still be necessary for the historical results. This result on our present site might be somewhat fortuitous since the quantity of heat gained by the soil is approximately balanced by a similar loss from the air. While we had no advection of energy at the level of our instruments (we obtained a satisfactory balance), there is an indication that advection was present at higher levels. One would expect that if the soil were gaining heat (over a period of days) the air would do likewise. In our case the site was surrounded at some distance by fields of stubble or standing ripened wheat and we think it probable that a supply of warm air, i.e., air which was warmer than it would have been had there been no advection, was present aloft. If our observations had covered a period in which the change in soil heat had been zero, or nearly so, then the energy used in evaporation might have exceeded the supply by radiation by about 10 per cent.

One or two interesting features were noticed on a minute-to-minute basis. On a clear day if the sun is suddenly obscured by a cloud, the rate of evaporation continues at, or near, its previous level for some minutes in spite of the immediate reduction in net radiation. The required balance of energy comes from the soil or from a reduced positive value of Q which may even go negative. Certainly on all occasions when this occurs the percentage of the radiative energy used in evaporation increases and we have noticed at least one instance when a reduction in R has resulted in the absolute magnitude of λE increasing. The same tendency can be seen in the results for longer periods. In o.n. 44, 45, and 46 when R was increasing (becoming more negative) about 50 per cent was used in evaporation but later in the same day when R was decreasing, this rose to around 90 per cent and exceeded 100 per cent in one observation (o.n. 53). We have gained the impression that once the wind speed reaches a reasonable level, further increase does not result in a very much enhanced value of evaporation. However, if advection is present this may not remain true, as the increase in wind speed will make more sensible heat available directly above the surface and this in itself will tend to give a greater rate. The grass surface was cut between 7 and 13 September and there is no evidence that this had any material effect on the rate of evaporation.

The total dewfall during the whole period covered in Table 1 amounted to 0·067 cm and by far the greater part of this (0·045 cm) fell on the night of 6/7 September (o.n. 59-76). Monteith (1957) states that the annual dewfall on short grass in the British Isles probably amounts to 0·25 to 0·5 cm and on the basis of the present observations this may well be an underestimation, although it must be recorded that the period September to October is the most favourable for large falls. The highest rate of fall was 9·7 mg cm^{-2} hr^{-1} in o.n. 74 when the wind at 2 m was 4·56 m sec^{-1} and the estimated wind at 10 m was 5·75 m sec^{-1}. Frankenberger (1955) is quoted by Monteith (1957) as stating that with wind speeds above 5 m sec^{-1} at 10 m evaporation is always observed, which is not supported here. The wind speed at 2 m did not drop below 2·6 m sec^{-1} (o.n. 64) and it appears that in order to obtain any quantity of dew, wind speeds of this order are required. (Compare the rates of fall for the night of 5/6 September – Table 1).

If mean hourly values of the meteorological elements are used in Eqs. (4) and (5) errors may result in the calculated values of λE and Q. In Table 3 we have extracted information from Table 1 and expressed the two terms in mw hr cm^{-2} as obtained from the machine and by calculation using the mean values of the elements which were available from counters. As far as λE is concerned the amounts agree in the individual periods to about 20 per cent except when they are small. The periods on 6 September show

considerably better agreement than those for 13 September and we are not able to give an explanation at this stage. The whole-period totals shown at the bottom of Table 3 are reasonably close for 6 September but there is a difference of 20 per cent on 13 September. The sensible-heat amounts are not in agreement and there is a difference of up to 250 per cent in the totals quite apart from the difference in the individual periods. We hope to investigate these features more fully.

TABLE 3. COMPARISON OF CALCULATED AND COMPUTED VALUES OF LATENT AND SENSIBLE HEAT IN MW HR CM^{-2}

Date	Period (GMT)	Calculated λE	Computed λE	Calculated Q	Computed Q
6 September 1958	1030-1100	9·6	10·75	2·1	2·45
	1100-1130	9·9	9·7	0·8	0·7
	1130-1200	16·5	15·25	1·45	2·8
	1200-1230	14·9	14·3	0·55	0·0
	1500-1530	8·85	8·9	− 0·65	0·0
	1530-1600	7·15	7·05	− 1·9	− 0·3
	1600-1630	6·15	6·55	− 1·35	− 0·35
	1630-1700	2·6	3·8	− 1·55	− 0·35
	1700-1730	2·2	0·8	− 2·7	− 0·25
	1730-1800	1·85	1·9	− 2·65	− 2·0
	1800-1900	2·0	4·4	− 5·1	− 1·5
	1900-2000	− 0·5	− 0·1	− 3·9	− 1·2
	2000-2100	− 1·9	− 1·05	− 3·0	− 2·4
	2100-2130	+ 0·7	+ 0·15	− 2·85	− 2·0
	2130-2200	− 0·85	− 0·4	− 2·0	− 1·85
	2200-2230	− 1·2	− 0·9	− 1·7	− 2·1
	2230-2330	− 2·8	− 2·3	− 2·3	− 2·0
	2330-2400	− 1·75	− 1·25	− 1·05	− 1·3
13 September 1958	1030-1100	8·05	6·05	2·15	1·45
	1100-1130	10·55	10·35	0·4	− 0·95
	1130-1200	11·75	8·45	1·15	0·7
	1200-1300	25·0	19·7	3·5	1·7
	1300-1330	12·4	9·0	1·35	0·25
	1330-1430	19·6	17·4	2·4	0·9
	1430-1500	10·75	9·5	0·7	0·4
		Totals			
6 September 1958		73·4	77·6	− 27·8	− 11·8
13 September 1958		98·1	80·5	11·7	4·5

7. CONCLUSIONS

The results so far obtained are considered to compare favourably with those reported in the past (see for example, Lettau and Davidson (1957)). Though relatively small in number compared with the potential output of the machine they are enough to show that the whole system, which comprises the original equations on the one hand and the method of using them on the other, is sound. We have obtained a satisfactory balance in the components for periods of an hour or more and once the engineering of the present apparatus has been improved in the sections indicated we believe that semi-continuous operation will prove possible. For long periods of exposure it would be desirable to use wires of larger diameter for the dry thermocouples (the size adopted for the wet junctions would be adequate) and there is no objection to this from the point of view of radiation error, etc.

A simpler machine could be made to give results of practical significance, that is, values for the latent and sensible heat terms which would not be more than 5 per cent in error. The $1/T$ chain could be replaced by a fixed resistance corresponding to the mean temperature likely to be experienced. The variation in the psychrometric constant is not important and the A and $A/2$ chains could be replaced by single resistors. If a rather larger error could be tolerated, the wind linearization circuits could also go. In addition to this, one term in Eq. (1) could be obtained from a knowledge of the other three and since computation of latent heat requires the most complicated circuitry as well as a sensing-head with wet thermocouples it might well be that in a practical application this could be dispensed with. In certain situations, for example in a study of the influence of films of such substances as cetyl alcohol on the control of free water evaporation for which the present apparatus should be suitable, it would prove difficult to measure the equivalent of the S term and the evaporation would best be dealt with directly. In all cases it is obviously desirable to evaluate all four terms so that a direct check on the results is immediately available. We can also note that in situations in which the zero-plane displacement and/or the heights of instrument exposure are not known, we can evaluate the denominator of Eqs. (2) and (3) if it is assumed that Eq. (1) holds.

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551.511.6 : 551.524.4 : 551.551.8

A determinant hypothesis for the superadiabatic wind and temperature profiles

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SUMMARY

The view is advanced that values of the Monin-Obukhov coefficients, as they are obtained from field data, are products of smoothing rather than true physical constants of a steady regime intermediate between free and forced convection. This hypothesis provides a satisfactory prediction of the first coefficient and, if it is correct, reduces the dual problem of heat and momentum transfer in superadiabatic conditions to the specification of four constants : von Kármán's constant, Priestley's free convection constant, and the ratio K_H/K_M in neutral and in free convection conditions.

In inversion conditions it has yet to be shown whether the constancy of stress and heat flux extend very far, if at all, above the level where significant departures from the neutral profile begin. Thus the basic assumption of some common current approaches still awaits examination.

1. INTRODUCTION

Currently the form most used for the temperature profile near the ground in conditions of quasi-forced convection is the Monin-Obukhov (1954) expansion

$$-\frac{\partial\theta}{\partial z}=\frac{1}{\gamma}\frac{H}{\rho\, c_p\, k u_* z}\phi\left(\frac{z}{L}\right)$$

$$=\frac{1}{\gamma}\frac{H}{\rho\, c_p\, k u_* z}\left(1+\frac{\alpha_1 z}{L}+\frac{\alpha_2 z^2}{L^2}+\ldots.\right) \quad . \quad . \quad (1)$$

where $$L=-\frac{u_*{}^3\, T_0\, \rho\, c_p}{kgH}.$$

The notation, unless otherwise stated, is standard. The profile is a generalization of that applying to the condition of fully forced convection, where

$$-\frac{\partial\theta}{\partial z}=\frac{1}{\gamma}\frac{H}{\rho\, c_p\, k u_* z} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

In both equations γ has been used to denote the unique value of K_H/K_M in the fully forced condition; any variation of K_H/K_M with z/L is regarded as incorporated in ϕ. There are grounds for believing γ to be unity, but this need not be assumed here. The wind profiles may be expressed in corresponding functional forms which will be distinguished by dashed symbols (Eq. 4).

The most recent analysis by Taylor (1960) of the rather complete observational material of Swinbank (1955) in slight to moderate superadiabatic conditions gives $\alpha_1' = 6{\cdot}1 \pm 1{\cdot}5$; while his re-analysis of the original data of Monin and Obukhov gives, less directly but with somewhat better precision, $\alpha_1' = 6{\cdot}0 \pm 0{\cdot}9$. No values have yet been put forward for the coefficients of higher order terms.

2. SUPERADIABATIC CONDITIONS

The temperature profile in conditions of free convection (Priestley 1954) may be written as :

$$-\frac{\partial\theta}{\partial z}=C z^{-\frac{4}{3}}=h^{-\frac{2}{3}}(H/\rho\, c_p)^{\frac{2}{3}}(g/T_0)^{-\frac{1}{3}} z^{-\frac{4}{3}} \quad . \quad . \quad . \quad (3)$$

h is a constant whose value, as judged from all available evidence, is about 0·9 (Priestley 1959). There is evidence from heat-flux measurements (Priestley 1955; Taylor 1956) and temperature profiles (Webb 1958) that Eq. (3) holds to a good approximation above $z/L \approx -0{\cdot}03$ for about a thirty-fold range of heights, above which it becomes invalid.* Taylor's most recent analysis indicates that the same form holds for the wind profile through more or less the same height range.

Eqs. (2) and (3) represent fundamental modes of behaviour, but it is possible to regard Eq. (1) as representing no more than a compromise between them. Both Eqs. (2) and (3) are turbulent regimes and moving elements will traverse the junction between them, so tending to smooth out any discontinuity which might otherwise exist there. The height of the junction, proportional to L, is governed by external conditions and their variability within any finite observing period will also represent a smoothing influence. Further smoothing processes are introduced as data are analysed, both by averaging within groups and by 'contamination' when profiles are assigned to the wrong group through erroneous determination of the grouping parameter such as L or Ri.

Given Eqs. (2) and (3), the simple hypothesis that smoothing exists, whether natural or artificial, is sufficient to determine a complete set of values of α_r.

With no smoothing, equating gradients of profiles (2) and (3) determines the junction height, i.e., $z/L = -\gamma^3 k^4 h^{-2}$. We shall adopt $\gamma = 1$, $k = 0{\cdot}4$, $h = 0{\cdot}9$ in the numerical illustrations which follow, whence $z/L = -0{\cdot}0316$ at the unsmoothed junction.

We now replace Eq. (2) by Eq. (1), retaining terms as far as α_r. In what may be termed the r-th smoothing we may then equate $\partial\theta/\partial z$ and its first r derivatives at the junction, so determining a new junction height and the values of α_1 to α_r inclusive. The numerical results for $r = 1$ to 5 are given in Table 1.

TABLE 1. COEFFICIENTS OBTAINED BY SUCCESSIVE SMOOTHING

	α_1	α_2	α_3	α_4	α_5
First smoothing	3·3				
Second smoothing	4·2	10·1			
Third smoothing	4·6	16·1	23·1		
Fourth smoothing	4·8	20·0	45	42	
Fifth smoothing	5·0	22·6	63	95	62
Final smoothing	5·63	36·3	191	See below	

This calculation involves the straightforward solution of r simultaneous equations in the r unknowns α_r. After a certain stage the pattern of the solution becomes clear, so that the general case may be solved by induction. The forms so obtained can be shown to converge as r tends to infinity, the values of each coefficient then being given by

$$(\gamma^3 k^4 h^{-2})^i \alpha_i = \frac{(0{\cdot}7121)^i}{(3i+1)\,i!}$$

where the number 0·7121 arises as

$$\operatorname*{Lt}_{p\to\infty}\left[\left(\frac{3^p \cdot p!}{4.7.10 \ldots . (3p+1)}\right)^3 p\right].$$

With γ, h, k assigned numerical values as above, this becomes

$$\alpha_i = \frac{(22{\cdot}54)^i}{(3i+1)\,i!},$$

* What happens at higher levels will be excluded from present consideration.

so that $\alpha_1 = 5{\cdot}63$, $\alpha_2 = 36{\cdot}3$, $\alpha_3 = 191$, and so on. Since z/L is negative, the final series for ϕ consists of terms of alternating sign which tend to zero for large i, so that convergence is proved for all finite z. When it is taken into account that k has been conventionally included in the denominator of L, it is seen that all the numerical terms in ϕ are determinate from a single constant $h^2(\gamma k)^{-3}$.

The analysis of the wind profile may be made to follow the same lines. Corresponding to Eq. (1),

$$\frac{\partial u}{\partial z} = \frac{u_*}{kz}\phi'\left(\frac{z}{L}\right) = \frac{u_*}{kz}\left(1 + \alpha_1'\frac{z}{L} + \ldots.\right) . \quad . \quad . \quad (4)$$

Through a range more or less the same as that of Eq. (3), Taylor's work indicates that $\partial u/\partial z \propto z^{-\frac{4}{3}}$. Where this and Eq. (3) and the constancy with height of τ and H all apply, the ratio K_H/K_M will be constant and will be denoted by γ'. The wind profile is then given by

$$\frac{\rho\dfrac{\partial u}{\partial z}}{\tau} = -\gamma'\frac{\rho c_p\dfrac{\partial\theta}{\partial z}}{H} = \gamma' h^{-\frac{4}{3}}\left(\frac{gH}{\rho c_p T_0}\right)^{-\frac{1}{3}} z^{-\frac{4}{3}}$$

or

$$\frac{\partial u}{\partial z} = (\gamma' h^{-\frac{4}{3}} k^{\frac{4}{3}})\frac{u_*}{kz}\left(\frac{z}{L}\right)^{-\frac{1}{3}} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The blending of Eq. (4) into Eq. (5) is then identical with that of Eq. (1) into Eq. (3), with γ' replacing γ, so that ϕ' is uniquely determined from the single constant $h^2(\gamma' k)^{-3}$. Thus each α_i'/α_i is dependent solely on γ'/γ, and ϕ'/ϕ is a continuously-varying function of z/L: it would assume a constant value (unity) only if $\gamma = \gamma'$.

If the hypothesis is correct, therefore, the entire problem of heat and momentum flux in superadiabatic conditions is reduced to the determination of the four constants k, h, γ and γ'.

Let us look more closely at the quantitative implications, in terms first of fluxes and then of gradients. For the flux, we construct the graph of $\overset{*}{H}$ against Ri, or z/L, where

$$\overset{*}{H} = \frac{H}{\rho c_p\left(\dfrac{g}{T_0}\right)^{\frac{1}{3}}\left|\dfrac{\partial\theta}{\partial z}\right|^{\frac{2}{3}} z^2}.$$

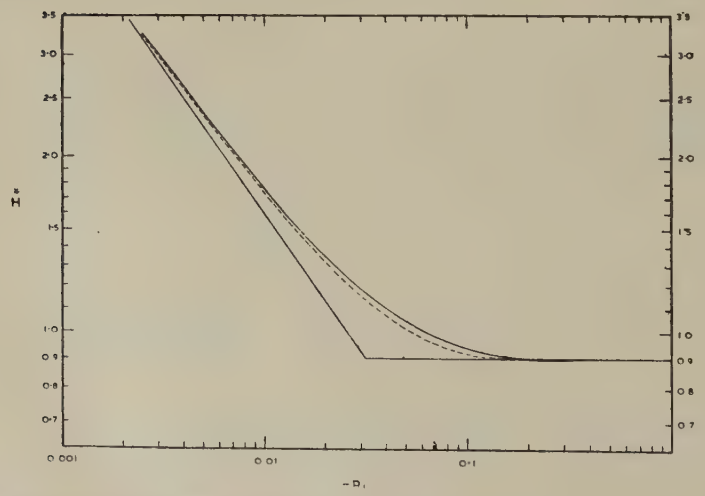

Figure 1. Smoothed and unsmoothed heat-flux values. Broken line indicates the second smoothing and full curved line the fifth smoothing.

$\dot{H}$ is thus a more general dimensionless form of heat flux, equal to the constant value h in free convection conditions only. With $k = 0{\cdot}4$, $h = 0{\cdot}9$, $\gamma = \gamma' = 1$ the 0th, 2nd and 5th smoothings are shown in Fig. 1, illustrating the order of differences between smoothings and the rate of convergence. Concerning the preference for Ri over z/L as abscissa for this diagram, it follows at once from Eqs. (1) and (4) that

$$\dot{H} = \frac{\gamma k^2}{\phi\phi'}\left|Ri\right|^{-\frac{1}{2}} = \frac{\gamma^{\frac{1}{2}} k^2}{\phi^{\frac{1}{2}}}\left|\frac{z}{L}\right|^{-\frac{1}{2}} \quad . \quad . \quad . \quad . \quad (6)$$

so that the choice appears immaterial on fundamental grounds. So long as ϕ and ϕ' are nearly equal, the separation between curves would be less with z/L than with Ri as abscissa. For practical purposes, however, it is preferable to display the heat flux as a function of the gradients rather than in terms of a quantity which itself contains H.

The important practical consideration is the extent to which the gradient, or flux, is modified in the region around the original junction, where the smoothing has the greatest effect. Table 2 gives, as values of ϕ at $z/L = -0{\cdot}0316$, the relative values of the gradient from successive smoothings. We see that complete smoothing nowhere involves a change of gradient of more than 15 per cent. Moreover, more than two-thirds of this change is achieved in the first smoothing, and one may describe the subsequent steps as mere ' trimming.' This is a significant feature, since from the nature of atmospheric phenomena one might regard high-order smoothing as a rather far-fetched procedure. By the same token it is of mainly academic interest that the junction, or point at which the gradients are actually equated, moves steadily upward (to the right in Fig. 1) and passes to infinity at the final smoothing.

TABLE 2. SMOOTHED VALUES OF ϕ AT ORIGINAL TRANSITION POINT

Number of smoothings	0	1	2	3	4	5	∞
Value of ϕ	1·000	0·896	0·877	0·870	0·867	0·863	0·853

The upper limit of 15 per cent for the gradient modification is so low as to invite the following views : that such a degree of smoothing is not out of line with the inevitable smoothing involved in finite-duration measurement under the conditions of space and time variability which nature imposes, and in the subsequent reduction and bulking of data; that α_1, α_1', etc., as obtained from field data are the products of these variabilities of circumstance and the processes of data treatment; and that they do not have the status of basic physical constants such as might be obtained from a sufficiently detailed and precise study of a truly steady intermediate regime. Some of these points may be debatable, and the last leads into deeper philosophical questions about turbulence. What requires to be emphasized is that we are not here attempting to provide *physical* justification for interpolation between two regimes across an intermediate one : rather are we concerned that the tools of interpretation should be fashioned to the nature of the material being interpreted.

It may be pointed out that prior analyses of other data imply α_1 values comparable with those given above (we shall not here distinguish between the profiles of different elements). Using Holzman's (1943) empirical expression

$$\frac{\partial u}{\partial z} = \frac{u_*}{kz}(1 - \sigma Ri)^{-\frac{1}{2}} \quad . \quad . \quad . \quad . \quad (7)$$

and expanding in powers of z/L it is seen that $\alpha_1 = \frac{1}{2}\sigma$. Pasquill (1949) gave $\sigma = 12$ and Deacon (1955), from various other sources, found consistent but somewhat smaller values. Thus if Holzman's formula is preferred to Eq. (1), as a way of smoothing between Eqs. (2) and (3), σ like α_7 is governed by the more fundamental constants k and h.

3. Inversion conditions

The inversion case is quite different. Apart from the passage into laminar flow there has been no suggestion of any mechanism capable of interrupting the steady increase of Ri with height which must take place if the wind and temperature profiles are even approximately similar. The evidence is that Ri does increase steadily with height in this manner. Thus laminar profiles might well replace Eq. (3) of the foregoing treatment and the values of α_τ be deduced in terms of k, γ, κ/ν and Ri_{crit}, the Richardson number at which turbulence is damped out. For reasons which follow, however, this would be likely to prove no more than a formal exercise with little or no physical relevance.

The correct value of Ri_{crit} is uncertain (see, e.g., Sutton 1953). The largest value proposed is unity which, with typical magnitudes of Ri measured on clear nights, indicates that Ri_{crit} should often be reached at heights around 10 or 20 m, except with strong winds. With lower Ri_{crit}, the heights would be correspondingly lower. For τ and H to be maintained through such a transition, or even partial transition, the wind and temperature gradients would be required to increase rapidly with height, and there is no evidence that this happens. Alternatively we must presume that even the quasi-constancy of τ and H with height is lost well before this height is reached. The longer-period intermittency which is characteristic of both wind and temperature fields in inversion conditions illustrates the danger of assuming balanced conditions for the shorter periods.

It seems that the concept of just-no-turbulence or of any rapid reduction of turbulence with height may be irreconcilable with the constancy (and hence uniqueness) of τ and H, even at much lower levels. This constancy is basic to the Monin-Obukhov approach, among others. Accordingly the applicability of these approaches to moderate and strong inversion states, as they occur in nature, must be seriously questioned. Nor is it certain that the Monin-Obukhov approach is valid even to provide the first-order correction, i.e., the a_1 term, at small values of z/L. This will depend on whether or not the constancy of τ and H holds up to the level at which the correction first attains a significant magnitude. More observational studies are required, and of a different character from many in the past which have tended to concentrate too much on the profiles at the expense of the fluxes themselves and the more intimate details of the turbulent structure.

Acknowledgment

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A problem in baroclinic stability

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SUMMARY

Linearized equations are used to formulate the theoretical problem of the stability of large-scale zonal flow with vertical shear. The problem is solved in some generality and illustrates the connexion between the solutions already obtained by Rossby, Charney and Eady.

1. INTRODUCTION

The purpose of this study is to extend the class of stability problems as posed by Charney (1947) and Eady (1949). A broad steady zonal baroclinic current is perturbed by a small-amplitude large-scale wave motion. Consideration of the stability characteristics of this system yields information concerning the initial stages in the growth of baroclinic disturbances.

The equations are simplified so that only those processes directly relevant to the scale of motion are included. The appropriate analysis has already been given in detail (see, e.g., Eady 1949) and will not be repeated here, although the principal steps are indicated.

The main assumptions are that the fluid is frictionless, the motion adiabatic, and that the Richardson number $Ri = gB \Big/ \left(\frac{dU}{dZ}\right)^2$ is large and positive. It can then be shown that the horizontal motion is nearly geostrophic, and that the appropriate vorticity equation (Eady 1949, p. 47, Eqs. 8 and 16) is of the form

$$\frac{D\zeta}{Dt_h} + v\beta = \frac{f}{\rho}\frac{\partial}{\partial Z}(\rho w) \quad . \quad . \quad . \quad . \quad (1)$$

where the suffix h denotes horizontal and

$\frac{D}{Dt_h}$ = differentiation following the horizontal motion

ζ = vertical component of relative vorticity

v = meridional velocity

$\beta = \frac{\partial f}{\partial y}$ = latitude variation of the Coriolis parameter

ρ = density

Z = height

w = vertical velocity

It is convenient to assume a ' standard ' distribution of density with height $\rho_0(Z)$ and the corresponding static pressure $P_0(Z)$, such that they describe the mean state of the fluid. We then consider the pressure and density fields as being given by the standard value plus a deviation, and note that the deviations consistent with the observed large-scale tropospheric motion are much smaller than the mean values.

Thus we write, pressure $= P_0(Z) + \delta P(x, y, Z, t)$

density $= \rho_0(Z) + \delta\rho$

and entropy$/C_p = C_v/C_p \log P - \log \rho \simeq \phi_0(Z) + \delta\phi$

with $\delta\phi/\phi_0 \ll 1$

The conservation of entropy can then be written

$$\frac{D}{Dt_h}\delta\phi + Bw = 0 \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where

$$B = \frac{d\phi_0}{dZ}$$

It is supposed that the deviation pressure and density fields are in static equilibrium

$$\frac{\partial}{\partial Z}\delta P + g\,\delta\rho = 0 \quad . \quad . \quad . \quad . \quad . \quad (3)$$

This set of equations, together with the geostrophic approximation, allows us to eliminate all the dependent variables in terms of the deviation pressure and defines the model fluid. It is, however, more convenient to write the equations in terms of the dimensionsionless function ψ defined by

$$\delta P = \rho_0(Z)\, gH\, \psi(x, y, Z, t)$$

where H is a geometric scale height. The geostrophic approximation becomes

$$\mathbf{V}_h = \frac{gH}{f}\mathbf{k} \wedge \nabla\psi$$

(where $\mathbf{k}$ is unit vertical vector)

and

$$\delta\phi = H\frac{\partial\psi}{\partial Z} - HB\,\psi \simeq H\frac{\partial\psi}{\partial Z}$$

Note that ψ behaves like the classical 'non-divergent' stream function in the horizontal, but with the vertical gradient corresponding (very nearly) to log potential temperature.

Eq. (1) becomes

$$\frac{D}{Dt_h}\left\{\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{f\beta}{gH}y + \frac{f^2}{gB}\left(\frac{\partial^2\psi}{\partial Z^2} + \frac{1}{\rho_0}\frac{\partial\rho_0}{\partial Z}\frac{\partial\psi}{\partial Z}\right)\right\} = 0 \quad . \quad . \quad (4)$$

and

$$w = -\frac{H}{B}\frac{D}{Dt_h}\frac{\partial\psi}{\partial Z} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

We now consider a steady zonal baroclinic motion $U(Z)$ disturbed by a small-amplitude perturbation. It is convenient to specify the perturbation in the form :

$$\text{perturbation } \psi = \text{Real part of } \left\{F(Z)\exp i\lambda(x - Ct)\begin{matrix}\sin\\ \cos\end{matrix}\mu y\right\}$$

Where $F(Z)$ is a complex function describing both the phase and amplitude of ψ, and the complex number $C = C_r + iC_i$ describes the phase velocity C_r and rate of amplification $\exp \lambda C_i t$ of the perturbation. The coordinate system is rectangular with the x-axis horizontal and directed towards the East, the y-axis horizontal and directed towards the North and the z-axis vertical. The variation of the map-scale with latitude has been

neglected as is customary in meteorological dynamics. Thus λ is the east-west wave number, and μ is the South-North wave number. It is convenient to re-write the equation in non-dimensional form and we therefore define

$z = Z/H$ where H is a geometric scale height in the fluid

$u = U(Z)/\Delta U$ where ΔU is a typical velocity difference

$c = C/\Delta U$ wave velocity

$p^2 = \frac{gB}{f^2} H^2 (\lambda^2 + \mu^2)$ horizontal wave-number

$\kappa = -\frac{H}{\rho_0} \frac{d\rho_0}{dZ}$ mass divergence parameter

$\gamma = \frac{gB}{f^2} H^2 \frac{\beta}{\Delta U}$ generalized β parameter

The non-dimensional perturbation form of Eq. (4) then becomes :

$$(u - c)(F'' - \kappa F' - p^2 F) + (\gamma + \kappa u' - u'') F = 0 \quad . \quad . \quad (6)$$

where the primes denote differentiation with respect to z, and the function F defines the perturbation quantities via

$$\text{perturbation pressure} = \rho_0(Z)\, gH \text{ Real part} \left\{ F(z) \exp i\lambda (x - Ct) \begin{matrix} \sin \\ \cos \end{matrix} \mu y \right\}$$

2. The parameters of the motion

For the ranges of pressure and temperature in which we are interested, the fluid must be considered as compressible and this affects three distinct physical processes :

(*i*) The ' dynamic compressibility ' is associated with the elastic forces in the equations of motion and sound waves. The speed of sound is much greater than the wave speeds with which we are concerned and this type of compressibility has been excluded (effectively by making the speed of sound in the model infinite).

(*ii*) The ' static stability ' is a measure of the static forces that oppose vertical movement and is represented by the vertical gradient of entropy (or of potential density), not by the gradient of temperature (or density) only.

(*iii*) The ' mass divergence ' effect is associated with the variation in the volume of a given mass of fluid. The vorticity equation contains a term $f \operatorname{div} \mathbf{V}_h$ that cannot be determined accurately by using the geostrophic approximation. Hence the continuity equation is used, in the form :

$$\operatorname{div} \mathbf{V}_h + \frac{\partial w}{\partial Z} = -\frac{1}{\rho} \frac{D\rho}{Dt} \simeq -\frac{w}{\rho_0} \frac{d\rho_0}{dZ}$$

Thus the difference between the continuity (or divergence) of mass and volume is expressed in terms of $\kappa = -\frac{H}{\rho_0} \frac{d\rho_0}{dZ}$, which is referred to as the mass divergence parameter.

The horizontal scale of the motion is represented by the non-dimensional parameter p, and $\lambda^2 + \mu^2$ appears in that combination throughout, except for the amplification which is represented by a factor $\exp \lambda C_i t$. It follows that if all the parameters of the motion except μ are fixed, then the rate of amplification will be a maximum when $\mu = 0$. This is assumed throughout so that the perturbation motion consists of troughs and ridges of infinite North-South extent. This is not a particularly apt approximation because of the distortion of the coordinate system in extreme latitudes. It might be added that baroclinic zones are observed to be of small meridional extent. However, other studies (e.g., Eady 1949) have indicated the kind of modification to be expected in these more realistic cases.

In defence of the present theory it might be noted that $\mu = 0$ in this context really implies $\mu \ll \lambda$ and that the perturbation for any μ can be recovered from the solutions given here. The most significant advantage is that with $\mu = 0$ the perturbation solution is (technically) a solution of the full non-linear Eq. (4). Thus the transport properties calculated from correlations between the linearized (first order) quantities are not distorted by contributions from second-order effects.

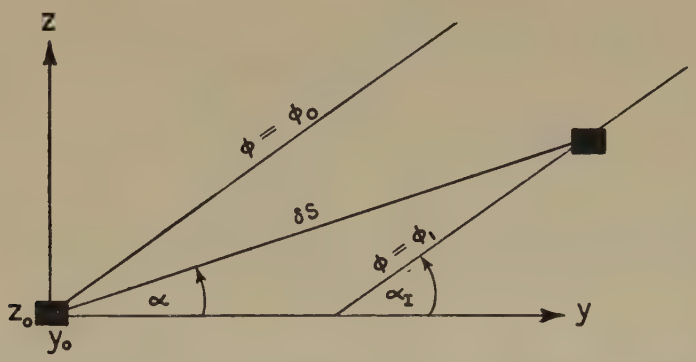

Figure 1. Schematic meridional section of the field of motion, showing the unperturbed adiabatics, and illustrating the parcel interchange.

In the unperturbed system there is potential energy which is available for conversion into perturbation kinetic energy. Let the (unperturbed) entropy gradients at some point (y_0, Z_0) be $\frac{\partial \phi}{\partial y} = -A$, $\frac{\partial \phi}{\partial Z} = B$ with the isentropic slope $\alpha_I = A/B$ (see Fig. 1). Now suppose that we take a mass M_0 of fluid from (y_0, Z_0) to $(y_0 + \delta S \cos \alpha, Z_0 + \delta S \sin \alpha)$ and return the displaced mass M_1 to (y_0, Z_0). Then neglecting the change in the mean pressure field, the change in the potential energy is

$$\Delta = g\, \delta S \sin \alpha\, (M_0 - M_1)$$

If this interchange is carried out adiabatically so that the parcel of mass M_0 is associated with entropy ϕ_0 throughout then

$$M_1/M_0 = \exp(\phi_0 - \phi_1)$$

and

$$\Delta = gM\, (\delta S)^2 \sin \alpha\, (B \sin \alpha - A \cos \alpha) + 0\, (\delta S)^3 \quad . \quad . \quad (7)$$

where

$$2M = M_0 + M_1$$

We see that for a given δS the maximum energy release occurs when $\alpha \simeq \frac{1}{2} \alpha_I$ and is given by

$$\Delta_{\max} = -gM\, (\delta S)^2 \frac{A^2}{4B} = -\frac{M\, (\delta S)^2}{4} \frac{f^2}{gB} \left(\frac{\partial U}{\partial Z}\right)^2 \quad . \quad . \quad (8)$$

assuming $\alpha_I \ll 1$, and $A \delta S \ll 1$.

We suppose this interchange to be consistent with an exponentially amplifying motion whose horizontal speed is given by, say : $V = \delta V \exp \lambda C_i t$ and displacement $\delta S = \int_{-\infty}^{t} V\, dt$ The change in the kinetic energy of this motion is given by :

$$\Delta_{\text{obs}} = 2\, (\tfrac{1}{2} M\, (\lambda C_i\, \delta S)^2) = (2p\, c_i)^2\, \Delta_{\max}$$

We see that pc_i is a measure of the efficiency of conversion of potential into kinetic energy, a more refined analysis confirming the maximum possible value of 0·5.

The variation of the Coriolis parameter with latitude appears in the ' generalized β parameter ' γ, together with the unperturbed entropy gradients. With α_I as the slope of the unperturbed isentropics, we can write

$$\gamma = \frac{H\beta}{f\alpha_I} = v\beta \Big/ \frac{v\alpha_I}{H} f$$

Now the paths of particles in the meridional plane tend to have the ' optimum ' slope of $\frac{1}{2}\alpha_I$ in the middle levels of the fluid. Hence $v\alpha_I/H$ is an estimate of $\frac{\partial w}{\partial Z}$, and $v\alpha_I f/H$ is the corresponding rate-of-change of vorticity due to the vortex stretching. Thus the generalized β parameter gives a measure of the effect of the variation of the Coriolis parameter with latitude, in comparison with that of the baroclinic vortex stretching, in producing vorticity changes.

The solutions to Eq. (6) cannot in general be expressed in analytic form. Even in the special case $u = z$, when such a solution is possible (see Charney 1947; Kuo 1952) the labour involved in calculating the eigen values by hand is prohibitive. Accordingly, it was decided that a high-speed computing device was necessary, and this being so, the solution should remain in numerical form throughout. The actual technique employed is not relevant here although a short account is given in § 7.

3. Linear velocity profile

If the entropy gradients are independent of height (A and B constant), and if we neglect the mass divergence effect (put $\kappa = 0$), then Eq. (8) shows that potential energy would be equally available everywhere were it not for constraints of a kinematic nature. An assessment of the simplest possible constraints that can limit the motion has led to two previous analyses of this problem. We are not concerned at this point with the details of these treatments, so that only their essential features are indicated.

Charney (1947) and Kuo (1952) considered the fluid to be bounded below by a rigid horizontal surface but unlimited above. They found that if the shear is westerly and β positive, then growing waves are possible.

Eady (1949) considered the fluid to be bounded, above and below by rigid horizontal surfaces and found growing waves with $\beta = 0$.

These simplifications illustrate the decisive nature of the constraints, for if we remove the upper lid in Eady's treatment or put $\beta = 0$ in Charney's, the problems become identical, and all waves are neutral. Thus the motion is rendered unstable by the action of these constraints.

4. Linear profile – β effect

The work of this section describes the transition from the Eady to the Charney solution, for motion between two rigid boundaries as the β-effect becomes important. We suppose the motion to be bounded, above and below by smooth, rigid, horizontal surfaces, and neglect the mass divergence effect (put $\kappa = 0$).

If $\gamma = 0$ (Eady), then for each wavelength there are two solutions. For long wavelengths this pair of solutions corresponds to amplifying and diminishing waves travelling with the mean velocity of the undisturbed flow. All the wave properties are either symmetrical or anti-symmetrical about the mid-level of the fluid, and the growth-rate is a maximum at a finite wavelength. Very short waves are neutral, and the pair of solutions then correspond to waves whose steering levels are displaced symmetrically about the mid-level (see Figs. 2 and 5).

With $\gamma \neq 0$, the differential equation is no longer symmetrical, and calculation shows that :

(*i*) There exists a growing wave for nearly all wavelengths – there is no short-wave limit to instability;

(*ii*) An additional long-wave solution is possible;

(*iii*) If $\gamma > 0$ ($\beta > 0$ and westerly shear), then the steering level is below the mid-level for all wavelengths.

In describing the nature of these results it is convenient to *make* a distinction between what may be loosely called thermodynamic, and kinematic processes. If the motion is to make a release of potential energy possible then particle paths must have (on the average) a slope between that of the isentropics and the horizontal. This is a necessary condition because all other orientations of particle paths would require work to be done against the static stability (see, for example, Eq. 7). It is not a sufficient condition because the relative phase of the temperature and velocity fields is not specified.

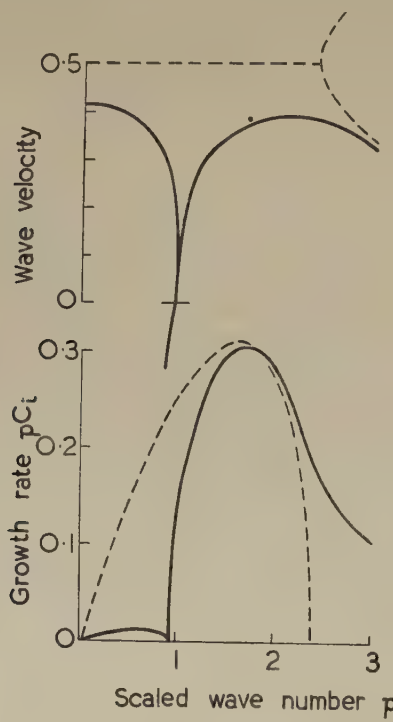

Figure 2. Growth-rate pc_i, and wave velocity c_r as a function of wave number for the linear profile $u = z$.
$\gamma = 0$ (dashed line) $\gamma = 0{\cdot}5$ (full line)
The wave-speed is equal to the height of the steering level/fluid depth.

The appropriate vorticity equation ($\kappa = 0$) expresses a balance between the vorticity changes due to the horizontal advection of absolute vorticity and those derived from the stretching of the vortex :

$$\frac{D\zeta}{Dt_h} + v\beta = f\frac{\partial w}{\partial Z}$$

Now for wavelike motion the first term in this equation is small relative to v near the steering level where (by definition) there is no zonal advection relative to the wave. Here then the

balances must be maintained between the vorticity changes due to meridional advection, and those due to the vortex stretching :

$$\text{i.e., } v\beta \simeq f\frac{\partial w}{\partial Z} \text{ at the steering level} \quad . \quad . \quad . \quad (9)$$

Now suppose that this steering level is near a lower lid. Then $w = 0$ at the lid, and $\frac{\partial w}{\partial Z} > 0$ for $v > 0$ at the steering level. Thus if $\frac{\partial w}{\partial Z}$ changes slowly in between, then $w > 0$ for $v > 0$ at the steering level, and a low steering level therefore implies $\overline{vw} > 0$. This is consistent with the necessary condition if the thermal wind is westerly. Whether this motion will be amplifying or diminishing depends on the phase difference between the velocity and temperature fields.

Conversely, steering level near an upper lid would imply $\overline{vw} < 0$ which does not satisfy the necessary condition with a westerly thermal wind, but does so if the thermal wind is easterly. Alternatively we can suppose that the correlation between v and w, *and* the variation of w with height are implied by the nature of the energy transformations and that the steering level satisfying the condition (9) is then selected.

This bias has an important effect on the short waves, for if $\gamma = 0$ the very short waves form a neutral pair with steering levels close to the boundaries. With $\gamma > 0$, the lower wave becomes an amplifying-diminishing pair, and there is no upper wave (see Fig. 2). If $\gamma < 0$ there is conversely no lower wave – a somewhat disconcerting state of affairs that may be of significance in numerical forecasts since the large-phase velocity of an upper wave could lead to computational instability.

A third major effect arising when $\beta \neq 0$ is the existence of a critical wavelength for which the waves are neutral, and stationary relative to the surface wind. For shorter wavelengths there is a pair of solutions corresponding to growing and diminishing waves, but for longer (than critical) wavelengths there are at least three solutions. Two form a conjugate pair as before, but with rather different structure and a much smaller growth-rate. The third solution is neutral and moves retrograde with respect to the surface wind.

If the fluid is deep or the shear small (i.e., γ large), then other critical points may occur at longer wavelengths, and at each critical point an additional neutral wave is encountered. For example, in the problem discussed by Charney, the fluid depth was infinite, and there are an infinite number of critical wavelengths – corresponding to polynomial solutions of his differential equation.

This behaviour of the longer waves is best appreciated by considering the form of the perturbation equation :

$$(z - c)(F'' - p^2 F) + \gamma F = 0 \quad . \quad . \quad . \quad (10)$$

where the primes denote differentiation with respect to the scaled height z. If the wavelength is large then $p^2 \ll 1$, so that in order to balance the last term, $F'' - p^2 F$ can be small if $(z - c)$ is large, that is, c must be large, and the variation of $(z - c)$ is then irrelevant. Thus we identify an inertial wave with a phase velocity $c \simeq -\gamma/p^2$ i.e. a Rossby wave.

An alternative is that F''/F should be large compared to p^2, and c finite, or that the motion should balance the vorticity changes due to meridional advection and vortex stretching at *all* heights, or

$$v\beta \simeq f\frac{\partial w}{\partial Z} \text{ for all } Z.$$

This is a very stringent constraint for :

$$vw \simeq \frac{f}{\beta}\frac{\partial}{\partial Z}(\tfrac{1}{2}w^2)$$

Thus the total $\int_0^H vw\,dz$ between rigid lids is zero. In fact the correlation between v and

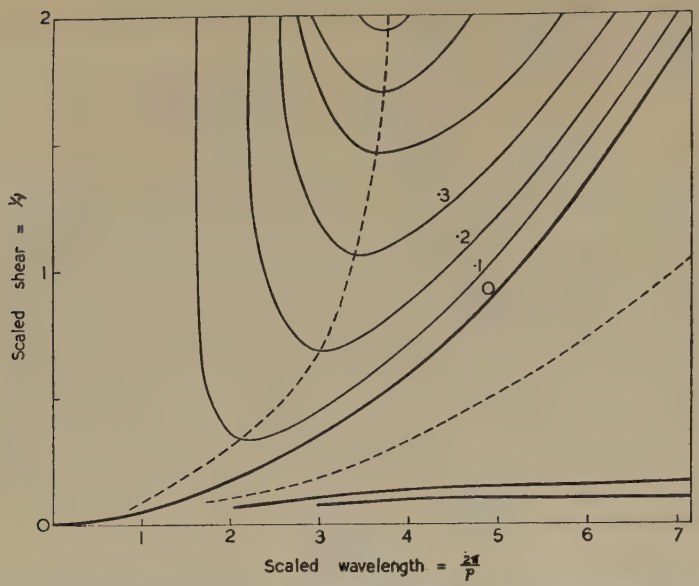

Figure 3. The variation of growth-rate as a function of wavelength and generalized β-parameter. Isopleths of pc_i/γ (growth-rate) against $1/\gamma$ (shear) and $2\pi/p$ (wavelength). All waves are unstable, or neutral (thick lines). The almost neutral waves ($1/\gamma < 0{\cdot}1$) have been omitted.

w is attained by a reversal of the meridional velocity in the upper layers. The necessary condition $\overline{vw} > 0$ is only just attained and the growth-rate is correspondingly small (Fig. 2 and 6).

The broad features of the growth-rate spectrum are illustrated in Fig. 3. The efficiency of conversion of potential energy into kinetic energy that is measured by pc_i maximized with respect to the wave-number p, is very insensitive to variations in the β-effect. For example, maximum pc_i is 0·310 at $\gamma = 0$, 0·286 at $\gamma = 10$. For comparison, maximum pc_i would be reduced to about 0·27 by a ' frictional boundary layer ' at the lower surface (kinematic eddy viscosity $= 10^5$ cm^2 sec^{-1}, $\Delta U = 30$ m sec^{-1}, $H = 10$ km, $(gB)^{\frac{1}{2}} = 10^{-2}$ sec^{-1}, $f = 10^{-4}$ sec^{-1} and $\gamma = 0$). The wavelength at which the maximum rate of growth occurs (the dominant wavelength) varies markedly with the shear, in the sense that longer waves are dominant when the shear is large. This is consistent with the general stabilisation of the longer waves arising when $\beta \neq 0$.

5. Linear profile – stratosphere

The solutions of Eq. (10) with a wavelength less than the first critical wavelength, have an exponential-type variation in the vertical, so that waves of large horizontal extent also extend, in general, to greater heights. Thus the upper boundary of a rigid lid is not ' realistic ' for the long waves and some calculations were therefore made with a superimposed semi-infinite layer of large static stability instead of the upper layer with infinite static stability (lid).

The unperturbed tropopause was supposed horizontal, the static stability of the stratosphere four times that of the troposphere, and the linear velocity profile was extended indefinitely upwards. It can be seen from Eq. (6) that with u increasing indefinitely with height, all solutions are ultimately of exponential form, so that only the decreasing solution is physically relevant. Further, even if u were to attain a finite value the solutions would be oscillatory only with c real, i.e., for critical waves only.

A decrease in the static stability of the upper layer results in an upward displacement of the steering level. With $B_S/B_T = 4$, and $\gamma = 0$ the speed of the dominant wave is 0·72 and maximum $pc_i = 0{\cdot}29$ in the units of Fig. 4. However, with $B_S/B_T = 4$ and

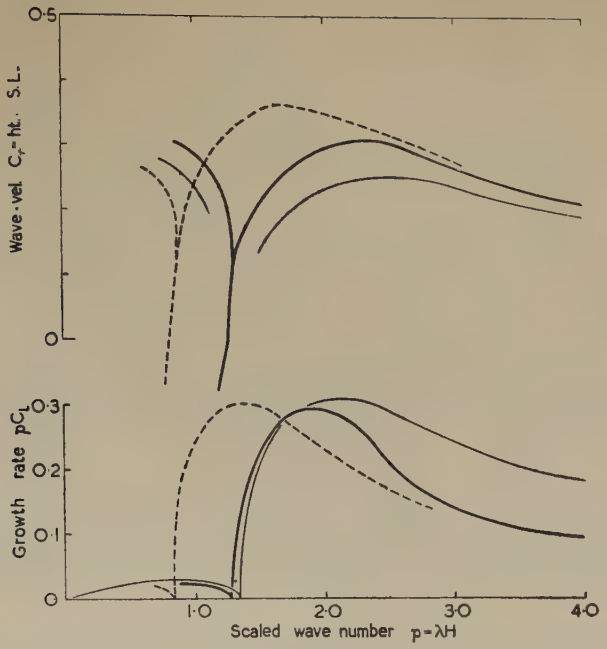

Figure 4. Growth-rate pc_i and wave speed as a function of wave-number for the linear velocity profile with $\gamma = 1$ and

Two rigid boundaries	$\kappa = 0$ (full lines).
Two rigid boundaries	$\kappa = 1$ (thin line).
Stratosphere with ratio of static stabilities 4 : 1	$\kappa = 0$ (dashed line)

$\gamma = 1$, the steering level is kept below the mid-level, and the spectrum is similar to the corresponding rigid lid spectrum, though with a bias towards longer waves (see Fig. 4 and Table 1). It is interesting to note that for $\gamma = 1$, the presence of this stratosphere leads to a slight increase in the maximum growth-rate, which is associated with the more symmetrical position of the steering level.

Unfortunately no calculations were made with $\gamma < 0$ which would correspond to easterly shear. We would expect that the waves would have their steering level and maximum amplitude displaced further towards the tropopause, with small amplitude at the ground.

6. Linear profile – mass divergence

The effect of the variation of density with height on the continuity equation is expressed in terms of the mass divergence parameter, κ. This mass divergence effect is implicitly included when pressure is used as the vertical coordinate, although there is then a corresponding complication in the form of the thermal wind-equation. Expanding the right-hand side of Eq. (1) we have :

$$\frac{D\zeta}{Dt_h} + v\beta = \frac{f}{\rho}\frac{\partial}{\partial Z}(\rho w) \simeq f\frac{\partial w}{\partial Z} + \frac{f}{\rho_0}\frac{d\rho_0}{dZ}w$$

It is easily seen that if v and w are positively correlated then the second term on the right enhances the β-effect. It has already been noted that the short waves are made unstable by the β-effect and we find this effect intensified for $\gamma\kappa > 0$ (see Fig. 4).

The steering level of the long waves is practically unchanged with respect to the pressure coordinate, so that with $\kappa > 0$, the steering level is lower. However, for these long waves it is found that the mass divergence does not contribute to the effect of the

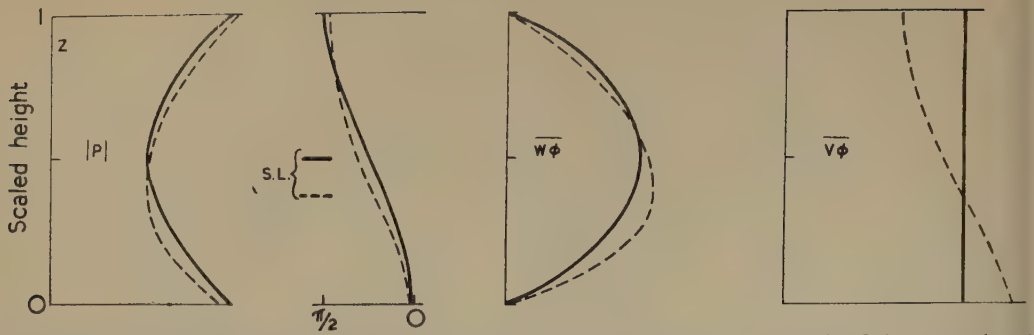

Figure 5. Perturbation pressure : amplitude and phase. Average over a wavelength of the upward and northward entropy transport for the dominant waves between two rigid boundaries when the unperturbed velocity profile is linear. With $\gamma = 0$ (full line) and $\gamma = 0{\cdot}5$ (dashed line). The amplitudes are expressed in arbitrary units and the steering levels, from Fig. 2, are denoted by S.L.

generalized β parameter given by the term $(\gamma + \kappa u')$ of Eq. (6). For example, with $\gamma = 0$ and any κ there is *no* critical wavelength, though if $\kappa \neq 0$ the short waves are unstable (contrast Fig. 2 where $\gamma\,\kappa = 0$ and the short waves are stable).

The amplitude of all waves is distorted (relative to $\kappa = 0$) so that the specific kinetic energy $\frac{1}{2}\rho\,\mathbf{V}^2$ is insensitive to the mass divergence effect.

7. Linear profile – the structure of the dominant wave

(*a*) *Shallow fluid*

With $\gamma \ll 1$ corresponding to a large shear, the proximity and nature of the boundaries is important. For example, with two rigid boundaries and dominant wave number p_x, then

$$p_x \simeq 1{\cdot}60 + 0{\cdot}6\,\gamma^2 \qquad (\gamma \ll 1)$$

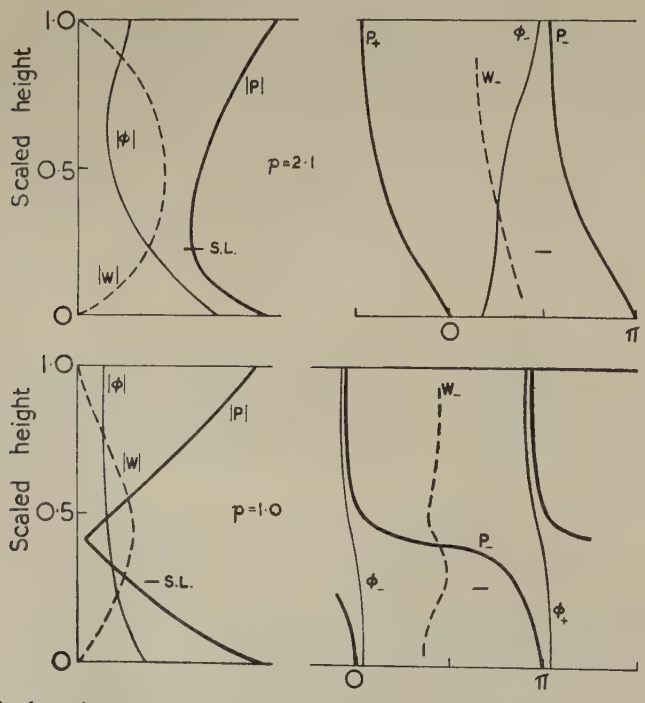

Figure 6. Amplitude and phase of perturbation pressure P, potential temperature ϕ, and vertical velocity w. Motion between two rigid boundaries with $\gamma = 1$, and linear velocity profile. Dominant wave $p = 2{\cdot}1$, 'Super-critical' wave $p = 1{\cdot}0$ (see Fig. 4).

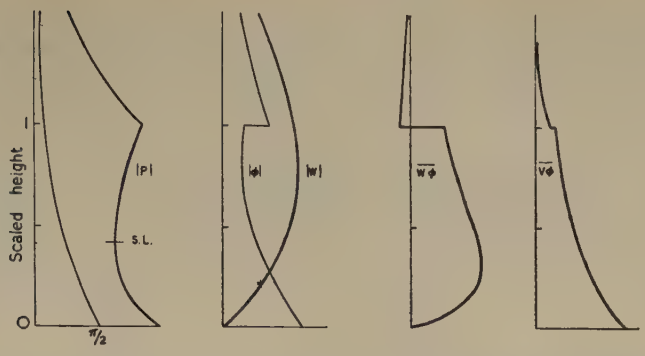

Figure 7. Phase and amplitude of pressure perturbation, amplitude of potential temperature and vertical velocity perturbations. Average upward and northward entropy transport (amplitudes on arbitrary scale). Dominant wave with stratosphere $B_S/B_T = 4$, $\gamma = 1$, and linear velocity profile.

With $\gamma < 1$, the pressure perturbation is nearly symmetrical about the mid-level of the fluid. The bias due to the β-effect occurs in the temperature field (through the stretching terms), and in the velocity field and shows more clearly in the entropy transport (see Fig. 5).

With $\gamma > 0$, and for near-dominant wavelengths, the wave amplitude decreases exponentially throughout the stratosphere, though rather longer waves may have maximum pressure perturbation just above the tropopause. Note (in Figs. 7 and 8) the temperature discontinuity at the tropopause level ($z = 1$). This indicates displacements of the tropopause that are in phase with the vertical displacement of a particle travelling (west-east, left to right) through the wave. Consequently the displaced tropopause is lowest near the upper trough. In the stratosphere the vertical velocity decreases less rapidly than the meridional velocity because of the large zonal particle velocity implied by the assumption of a constant thermal wind. Thus at great heights the particle paths will be almost vertical and the speeds small. In fact the calculations indicate a small downward flux of entropy in the stratosphere (Fig. 7).

(*b*) *Deep fluid*

For $\gamma \geqslant 1$, corresponding to small shear, the scale length is that associated with the β-effect, for the isentropic surfaces and therefore particle paths must be very nearly

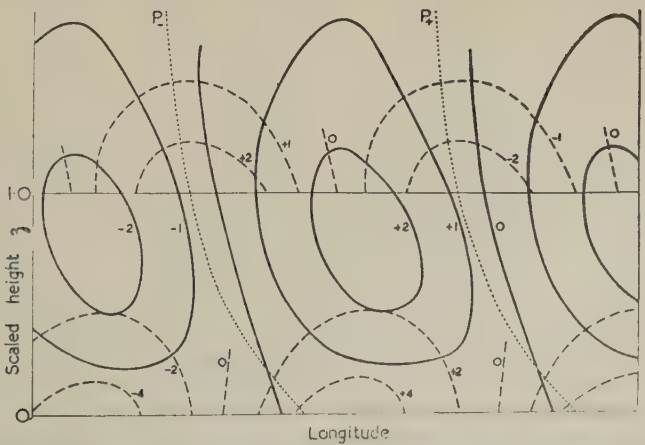

Figure 8. Zonal cross-section of the dominant wave, $\gamma = 1$ with 4 : 1 stratosphere as Fig. 7. Isopleths of potential temperature (dashed lines), and vertical velocity (thin lines), are expressed in arbitrary units.

horizontal. Thus the changes in the vorticity of a parcel of fluid are mainly those associated with meridional displacement and the variation of the Coriolis parameter, and only very close to a boundary does the stretching become significant. Thus for the dominant wave-number p_x,

$$p_x \simeq 0{\cdot}83\,\gamma\,(\gamma \gg 1).$$

The dominant wave is concentrated in the region up to the (low) steering level (Fig. 9) and variations at the upper boundary are unimportant.

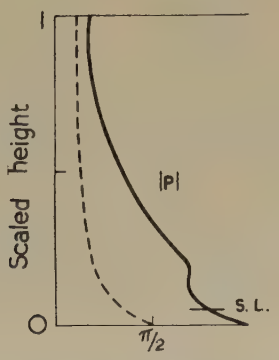

Figure 9. Pressure perturbation for the dominant wave, $\gamma = 5$, linear velocity profile. Phase (dashed) and amplitude (full line).

The maximum pc_i (conversion efficiency) varies very little: for $0 < \kappa < 1$, $0 < \gamma < 10$, and the presence of the 4 : 1 stratosphere, the variation is about 5 per cent.

The trough lines slope backwards (westwards) with height, and the total phase-lag varies from zero for very short waves to π (half-wavelength) for very long waves. However, the dominant waves are characterized by a phase-lag of nearly $\pi/4$ for large variations in the constraints. Table 1 gives a short list of dominant wave numbers as calculated for the linear velocity profile. p_1 is appropriate when the motion is bounded by two rigid boundaries, p_2 when the upper boundary is replaced by a semi-infinite layer with a static stability four times that of the lower fluid.

TABLE 1. WAVELENGTH OF THE MOST RAPIDLY AMPLIFYING WAVE, FOR DIFFERENT MODEL ATMOSPHERES

Dominant wave-number		p_1 with two lids	p_2 with 'stratosphere' 4 : 1
γ	κ	p_1	p_2
0	0	1·61	1·15
1	0	1·93	1·35
0	1	1·70	
1	1	2·18	

8. CURVED VELOCITY PROFILES

We now consider some modifications caused by the variations of thermal wind with height. For simplicity the mass divergence effect is neglected ($\kappa = 0$), and the motion is supposedly bounded by two rigid lids. Strictly the static stability is a function of height and latitude, but a constant value is assumed. This is consistent with the Richardson number being large, and is additionally appropriate because we shall be concerned with rather short waves in this section.

We may readily distinguish between two effects of shear variation.

(*a*) *Isolated wave*

The velocity profile may be such that a wave can exist without reference to material boundaries. The criterion for such isolated waves is similar to that given by Rayleigh for barotropic instability, i.e., growing waves may exist if $(\gamma - u'')$ changes sign, and the wave must have its steering level and maximum amplitude near the level of the zero. Note that only if $\gamma = 0$ does this correspond to a stationary value of the shear, or of available potential energy.

This class of wave is unlikely to be of direct importance for the one-dimensional profiles considered here, because the fluid is so shallow that the influence of at least one of the boundaries predominates.

(*b*) *Boundary waves*

A change in the sign of the shear may make it possible to have independent boundary waves such as the lower boundary wave discussed in connexion with the linear profile. Analysis of the short waves is instructive because they are able to 'pick out' ranges of height favourable for growth, and an analytic solution valid for very short waves has been derived. Criteria applicable to a two-lid model are given below :

$$\left.\begin{aligned} &\text{if } u'(\gamma + \kappa u' - u'') > 0 \text{ at } z = 0 \text{ (bottom)}, \quad \left(c \sim u_0 + \frac{1}{p} u_0' + i c_i\right) \\ &\text{if } u'(\gamma + \kappa u' - u'') < 0 \text{ at } z = 1 \text{ (top)}, \quad \left(c \sim u_1 - \frac{1}{p} u_1' + i c_i\right) \end{aligned}\right\} \quad . \quad (11)$$

then corresponding unstable solutions exist, with $c_i \simeq \frac{\pi}{e^2 p^2} |u'' - \kappa u' - \gamma|$ at the appropriate boundary. These formulae are valid as $p \to \infty$ and the mass divergence effect has been included temporarily.

Some calculations were made using a parabolic velocity profile, so that the isolated wave would not be present. The profile $u = 1{\cdot}5\, z^2 - z$ was chosen as representing roughly the easterly maximum in the Trade winds (Fig. 10). Note that $\int_0^1 (u')^2\, dz = 1$ so that the efficiency factor pc_i is comparable with that for the linear profile. Note however that the efficiency defined in the introduction properly refers to the *local* (maximum) entropy gradient.

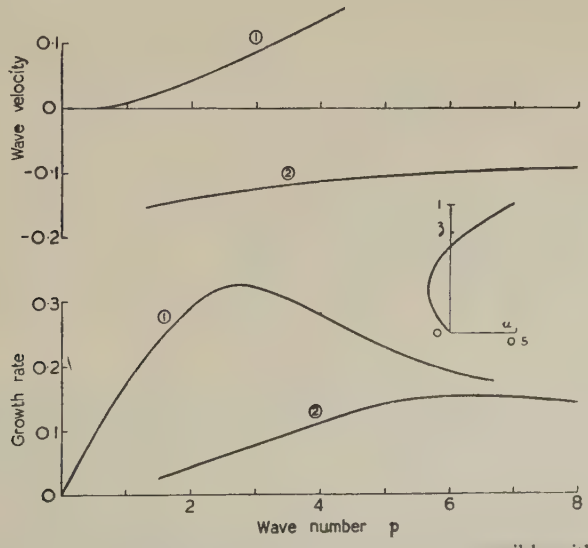

Figure 10. Growth-rate and wave velocity of each of the two waves possible with $u = 1{\cdot}5\, z^2 - z$, $\gamma = 0$, $\kappa = 0$; (1) upper wave; (2) lower wave.

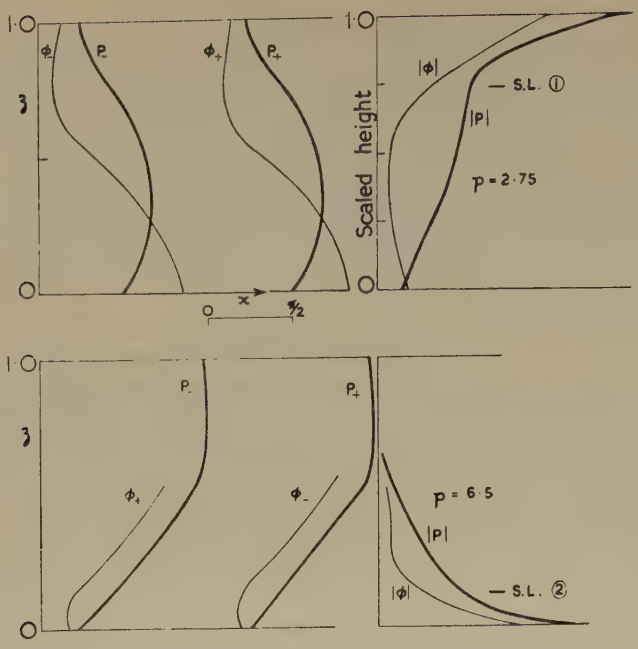

Figure 11. Phase and amplitude of pressure and potential temperature perturbations $u = 1{\cdot}5\,z^2 - z$, $\gamma = \kappa = 0$; (1) upper wave; (2) lower wave.

Each wave of Fig. 11 is similar to the uniform shear wave, but with the amplitude decreasing in the region where the Richardson number is large. The northward transport of entropy is in the opposite sense to the trough-line slope, so that the trough-line has, in general, the same shape as the mirror image of the velocity profile. Thus the lower wave (2) transports heat southwards, especially in the lower layers. Upper wave (1) transports heat northwards, especially in the upper layers. Note the short (horizontal and vertical) wavelength of the lower wave.

9. Note on the calculations

Most of the calculations were made using a layer-type model with simple (3 point) finite-difference approximations to the differentials. The differential equation of the form

$$F'' + \left(\frac{\gamma - u''}{u - c} - p^2\right) F = 0 \qquad . \qquad . \qquad . \qquad (12)$$

is replaced by a set of finite-difference equations :

$$F(z + h) - \left\{2 + h^2 p^2 - \frac{h^2 (\gamma - u'')}{u - c}\right\} F(z) + F(z + h) = 0 \qquad . \qquad (13)$$

$z = nh$, $0 \leqslant n \leqslant N$ where $Nh = 1$ and h is the grid size.

The two boundary conditions in finite-difference form enable us to eliminate $F(-h)$ and $F(1 + h)$ (i.e., for motion between two rigid boundaries), thus leaving a set of $(N + 1)$ simultaneous equations for $(N + 1)$ values of $F(nh)$. The condition for a non-zero solution is that the determinant of the coefficients of $F(nh)$ should vanish, and since each equation is of the same form as Eq. (13), the determinant is a polynomial of degree $N + 1$ in c. But the differential equation has usually (for sub-critical wavelengths) only one pair of eigen values whereas the difference equation has $N + 1$ roots. Now if the

truncation errors are to be reduced, h must be made small, therefore N large, and the equation that has to be solved will have $N - 1$ spurious roots. The interference of these spurious solutions was reduced by solving with successively graded nets. A coarse network was used to locate c, the mesh-length was halved and this value used as a first approximation to the more accurate c. This procedure was continued until a sufficiently accurate value had been obtained, the parameters of the wave (e.g., p) were then changed and the process repeated.

It is a general characteristic of unspecialized numerical methods that these work best when the functions are smoothest : in this case when the growth-rate is large. On the other hand, analytic methods are usually most powerful when the function behaves violently in a limited region, i.e., near a singularity. Thus for regions of very small growth-rate (at the critical points, and for very short waves), the analytic methods were used. A judicious combination of both had to be used in order to obtain the complete spectrum.

A major source of error is the inability of the finite-difference approximations to cope with the exponential-type functions involved. For short wavelengths it is obvious that the functions involved are nearly linear combinations of exp $\pm pz$, but the 3-point finite-difference operation always overestimates the magnitude of the curvature of such a function. Even at $ph = 1$ the overestimation is 9 per cent, at $ph = 2$ it is 38 per cent. (The errors in the wave velocity are of the same order). It was found that for a given mesh size, the technique gave stable pairs of solutions for sufficiently short waves. Decreasing the mesh size results in an extension of the correct (i.e., unstable) solution.

A second, rather more spectacular error is associated with the nature of the singularity of the differential Eq. (12) where c is a complex number, and effectively we integrate this equation along the real axis from $z = 0$ to $z = 1$. Part of this integration involves $\int \frac{dz}{z - c}$ and if the imaginary part of c is small, this integral consists of a sum of large real values which cancel, plus a sum of small imaginary values which add to give a contribution $\pm i\pi$.

If this integration is approximated by using point values then the real part does not, in general, cancel, and unless a grid-point happens to be close to $z = c$ the imaginary part will be hardly noticed. Thus, not only is the integral badly represented near the steering level, but its value fluctuates according to the position of $z_1 =$ Real part c relative to the nearest grid-point. A rough analysis suggests that if the imaginary part of $c < h/4$ then this effect will be important. The effect was most interesting near the critical wavelengths, where the growth-rate is small, and where a small change in the wave-number results in a large change in the wave velocity.

Acknowledgments

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The compensation of the effects of potential-gradient variations in the measurement of the atmospheric air-earth current

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(Manuscript received 23 April 1959; communicated by J. A. Chalmers)

Summary

A well-known method of determining the atmospheric air-earth current consists of collecting and measuring the current flowing into an insulated receiver at ground level. A difficulty encountered, however, is that any variation of the potential gradient produces a displacement of the bound charge on the collector, leading to a spurious ' displacement ' current. In the simplest form of the method, this current can occasionally be so large as to mask completely the true-earth current.

This article describes a development in which the sum of the air-earth and displacement currents is measured at one terminal of a differential amplifier, while a field machine provides automatic and continuous compensation for the displacement current at the other terminal. The apparatus thus enables a continuous true recording of the air-earth current to be made, and is an improvement on previous methods of eliminating the displacement current which necessitated intermittent recording of the integrated charge collected over a period of time. Satisfactory results are obtained with this method in all weather conditions except thunderstorms.

1. Introduction

For the direct measurement of the air-earth current, a plate is placed in the plane of the earth's surface and connected to earth through some form of measuring instrument; the same applies to the measurement of the precipitation current to an exposed plate. However, if the potential gradient is changing, there is a current through the measuring instrument additional to the true air-earth current, since charge must pass to or from the plate so that the bound charge on it shall always correspond to the potential gradient. This additional current may be termed a ' displacement ' current and if it is desired to measure the true air-earth current, it is necessary to correct or compensate for the displacement current.

Simpson (1910) measured the charge received from the conduction current to a plate over a period of time and corrected for the displacement current by measurements of the potential gradient at the start and end of the period. Chalmers (1956) has done the same for measurements of the total current (conduction and precipitation). These methods are satisfactory only if one is interested in the average current over a period of time and they cannot be applied for continuous measurements. Scrase (1933) attempted to compensate for potential gradient changes by using a quadrant electrometer with one pair of quadrants connected to the collecting plate and the other pair through a condenser to a radioactive potential equalizer; the current to the second quadrants should then be proportional to the rate of change of potential gradient, i.e., to the displacement current. Scrase's method, however, failed because the radioactive equalizer was too sluggish in its action. Kasemir (1955) eliminated displacement currents that depend upon changes of potential at a high altitude by using a *C-R* circuit in his measuring apparatus with the same time constant as the relaxation time of the air; but his method does not operate for effects due to changes of conductivity and would be of no value with precipitation currents.

The purpose of the present work is to describe apparatus based on the same principle as Scrase's, but using electronic methods of measurement and avoiding the sluggishness of Scrase's equalizer by the use of a field machine to give the potential gradient.

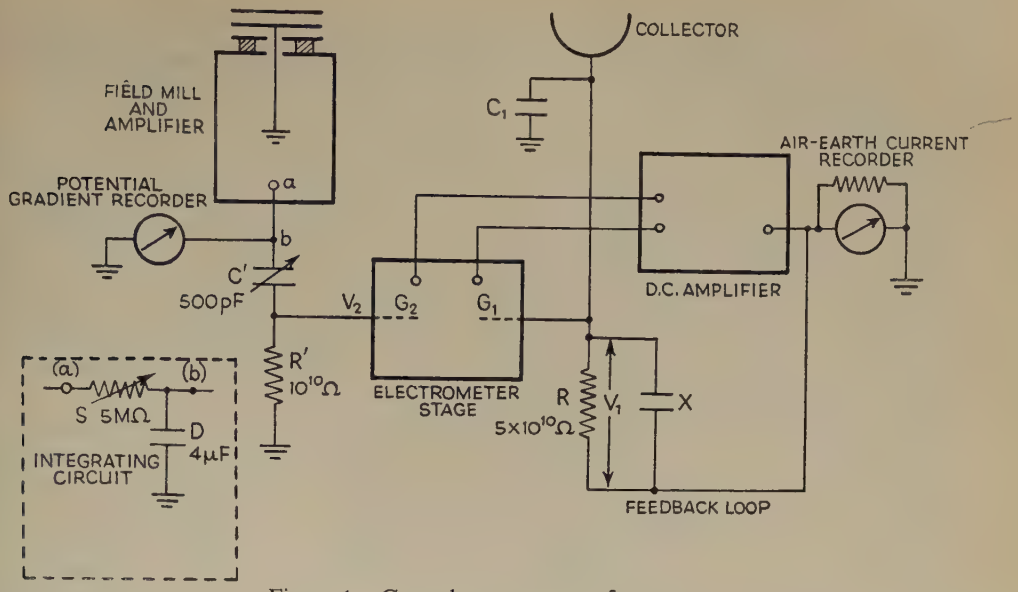

Figure 1. General arrangement of apparatus.

2. Principle of the apparatus

The general arrangement of the apparatus is shown in Fig. 1. The detailed mode of behaviour is of no concern, except for one property which can be understood by following through the complete circuit diagram (Fig. 3) of the electrometer stage and d.c. amplifier. This is that the d.c. amplifier measures the difference between the potential drop V_1 across the input resistor R for the air-earth current, and the potential V_2 relative to earth on the compensating grid G_2. The gain with feedback for $(V_1 - V_2)$ is very close to -1.

The simplest case to envisage is that of a steady rate of change of potential gradient, dF/dt, which has been going on long enough so that all transients arising from the initiation of the change have died away. If the collector, of area A, receives a current density i from conduction and precipitation, there will be in addition to the current iA, a displacement current $A\,\epsilon_0\,dF/dt$. This current produces a potential drop V_1 across R equal to

$$R\,(i + \epsilon_0\,dF/dt)\,A.$$

If the field machine output after amplification gives the voltage BF, the circuit C', R' differentiates this so that the grid G_2 has on it the potential $V_2 = BC'\,R'\,dF/dt$. This will compensate the effect of the displacement current on the collector side if

$$R\,\epsilon_0\,A = BC'\,R'.$$

In practice, R, A and B are fixed by consideration of the sensitivity of the d.c. amplifier and field machine as separate units, so the compensation is achieved by the appropriate choice of C' and R'.

3. Time constants

The condition expressed in the previous section is sufficient to ensure that there shall be compensation of the displacement current when this is constant. However, if the displacement current itself changes, i.e., $d^2\,F/dt^2 \neq 0$, the circuit $C'\,R'$, does not behave as a linear differentiating circuit, and thus the further condition is imposed that the time constants on either side must be the same. It was the absence of this condition that led to the failure of Scrase's apparatus as originally intended.

Since the field machine and its amplifier have for practical purposes an instantaneous response (approximately 0·1 sec), it is clear that the time constant on the compensating side is simply $R'\,C'$. This product is fixed at about 5 sec by the other design considerations of the apparatus and the condition for compensating steady displacement currents.

It would seem that it should be possible to achieve the same time constant on the collector side by connecting a capacity X across the input resistor R such that $RX = R'\,C'$. This simple picture is not, however, correct, in that no account has been taken of other capacities in the circuit. Stray capacities across the valve electrodes are of course responsible for producing some spurious effects, but these are negligible compared with the influence of the capacity between the air-earth current collector and earth. This capacity, shown in Fig. 1 as C, has the effect of decoupling grid G_1 from the voltage being transmitted to it from the feedback loop via R. Therefore, at the beginning of the application of V_2, the full unfed-back gain of the amplifier operates on V_2 until C becomes charged, producing a transient many times larger than the signal.

This difficulty has been overcome in the following way. Normally, capacity C would comprise the capacity between the collector itself and the shield which surrounds it in its outdoor location (about 260 pF), and that of the concentric cable carrying the current to the electrometer stage (about 440 pF). However, by connecting the sheath of the input cable to the feedback loop instead of to earth, about 440 pF is removed from C and transferred to X. The remaining capacity C is thus only 260 pF, contributed by the earthed shield round the collector. With X fixed at 440 pF, the time constant RX is balanced by the introduction of an integrating circuit S, D (see Fig. 1) between (a) and (b) to delay the response of the field machine.

With this arrangement, X is not sufficiently greater than C for the complete elimination of the transients, but the improvement is sufficient to reduce these to less than 20 per cent of the true output for most types of potential gradient changes. An adverse consequence of using the capacity X is the increase of the overall time constant to about 20 sec, but this is still sufficiently small for satisfactory recording in a wide range of weather conditions.

4. The field machine

The field machine provides compensation for the displacement current and is also used to obtain a recording of the potential gradient. It is a variation of the sector-plate field mill so often used for the measurement of potential gradient (Mapleson and Whitlock 1955). Briefly, the main advantages of this type are its rapid and accurate response to the potential gradient, and the readiness with which simple types of a.c. amplifier can be used in conjunction with it.

The present machine has a novel feature in that it incorporates negative feedback over the whole system; not, as is usually the case, over the associated electronic equipment only. This was to meet the stringent requirement for linearity of response needed to obtain accurate compensation.

The important parts of the machine are shown in Fig. 2. Induction of the signal takes place on an insulated collecting electrode in the form of a disc. A vane, consisting of four 45° sectors of a disc of the same size, is supported about 2·5 cm away from the collecting plate. Between these a similar vane, which is earthed, rotates at 3,000 r.p.m. Considering for the moment the upper vane to be at earth potential also, it can be seen that the rotating vane alternately screens and exposes the collecting electrode so that the bound charge induced by the atmospheric potential gradient is modulated.

The signal is passed through a cathode follower to reduce its associated impedance and is then amplified by three stages of conventional R-C coupling. Phase-sensitive rectification is then required to realize discrimination between positive and negative potential gradients. To effect this, the amplifier output is led to a commutator mounted on the shaft carrying the rotating vane.

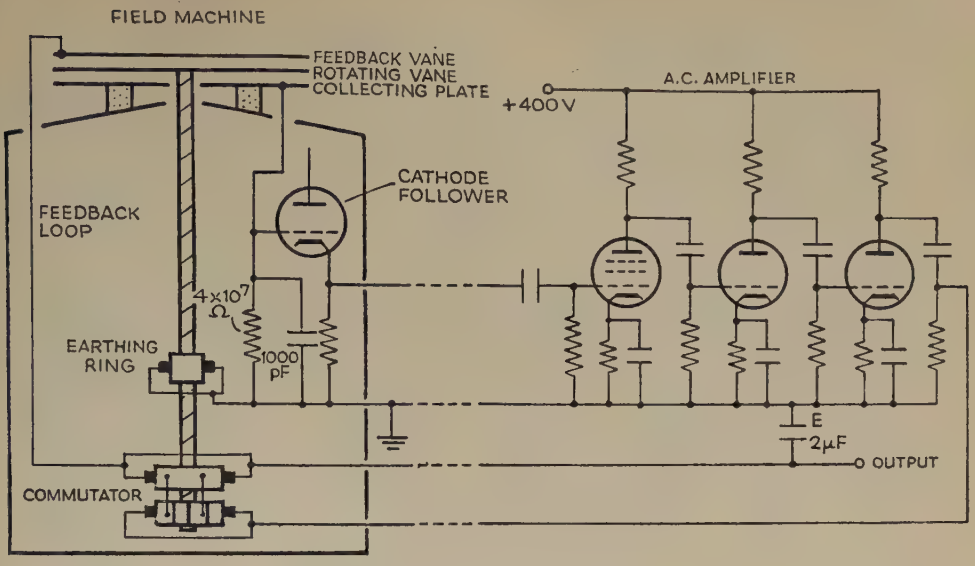

Figure 2. The field machine.

The negative feedback is provided by connecting the rectified output, which has the same sign as the potential gradient, to the upper vane of the machine, previously considered as earthed. This is a development of the method first used by Harnwell and van Voorhis (1933) for measuring the voltage generated by a Van de Graaff machine. The collecting electrode is thus exposed alternately to the atmospheric potential gradient and the electric field set up by the voltage on the upper vane, which is also the output voltage. This increases until the signal induced by it has almost the same magnitude as that induced by the potential gradient, and, since the two are out of phase, the result is the nominally zero input which is a characteristic of feedback systems. The output is then equal to the potential of the atmosphere in the plane of the upper vane, so that it is as if this vane were an unbroken disc all at the same potential.

The rotating vane is keyed to a steel shaft which is earthed through a copper slip-ring and a pair of graphite brushes connected together to earth. An earthing resistance of about 0·1 ohm is attained. After amplification, the output is returned to the commutator, also keyed to the shaft. Contact to the commutator is made by pairs of brushes connected in parallel, the duplication eliminating intermittent contact failure due to brush chatter.

A one-sixth h.p. induction motor drives the mill. The motor is separately housed and drives the machine through a V-belt, as it was found that internal mounting resulted in a small spurious output of low frequency.

An abbreviated diagram of the associated electronics is shown in Fig. 2. The cathode follower, which is installed within the machine casing, is run at reduced heater and anode voltages to keep grid current to a minimum. The amplifier is of conventional a.c. design. However, because the whole circuit incorporates rather a large number of *R-C* couplings, there would normally be a tendency to self-oscillation. The difficulty has been overcome by connecting a capacity *E* across the *rectified* output to short-circuit incipient oscillations, while not interfering with the d.c. feedback to the upper vane of the mill.

5. The electrometer stage and d.c. amplifier

A circuit diagram of the electrometer stage and d.c. amplifier is shown in Fig. 3. Although the circuit responds both to the air-earth current and the compensating signal, it is consideration of the former which largely dictates the design. On passing through the

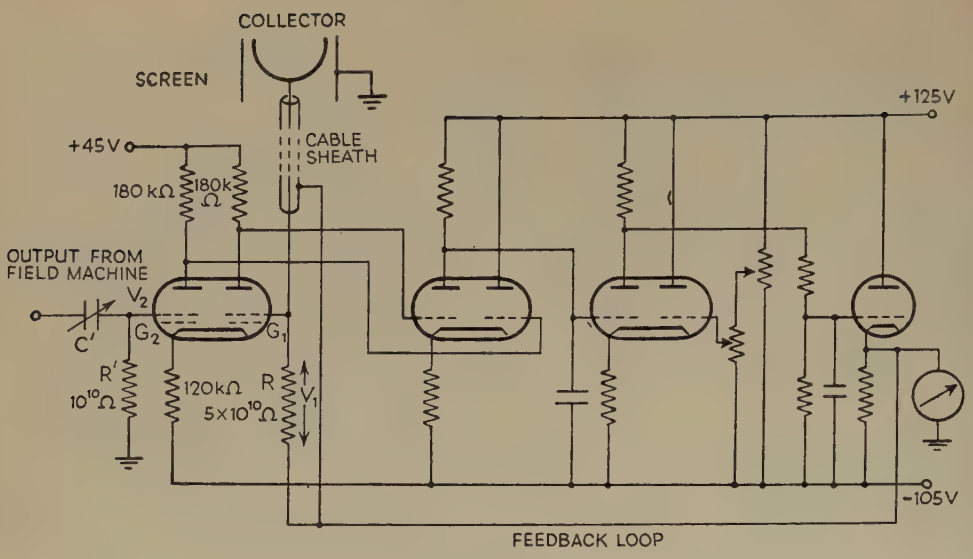

Figure 3. Electrometer stage and d.c. amplifier.

high resistor R, the current in the collector develops the potential drop V_1. The electrometer stage and amplifier can be regarded as an impedance transducer which reduces the high impedance associated with V_1 and presents the same voltage (inverted) across a much lower impedance so that it can be measured with an ordinary galvanometer. The electrometer stage, (Ferranti DBM4A twin tetrode), presents to the signal a very high input impedance, and is followed by the d.c. amplifier incorporating most of the loop gain of 320. Stability and linearity of response are achieved by returning the amplifier output to the resistor R. The same feedback loop also controls the gain for the compensating signals applied to the other grid. In fact, over the whole circuit, the gains for the voltage developed across R and the compensating voltage are respectively -1 and $+1$, so that the required differential response is obtained.

The collector is a hemispherical bowl of aperture 0·2 m^2 housed in a pit adjacent to the field machine, with its rim at ground level. It is supported on a polystyrene insulator kept dry by an electric heater. A cylindrical earthed screen round it prevents spurious currents due to contact potential differences between the collector and the ground. The current entering the collector is transmitted to the electrometer stage indoors by a concentric cable, whose sheath is connected, not to earth, but to the feedback loop. As described above, this reduces the transients resulting from sudden potential gradient variations. The collector has to be fitted with a tray, usually containing earth, lying just below the plane of the rim; otherwise its effective aperture is different for precipitation and conduction (or displacement) currents.

6. Recording procedure

Simultaneous recording of the air-earth current and potential gradient is made using mirror galvanometers and a rotating-drum camera. The cylindrical lens of the camera has a graticule behind it and the operation of a fogging lamp provides a reference scale on the developed film. Time indication is provided by clock-operated relays that switch the lamp off periodically. To account for the small drift in the d.c. amplifier zero, an earthed plate is placed over the collector and field machine every few hours during a recording to establish the zero.

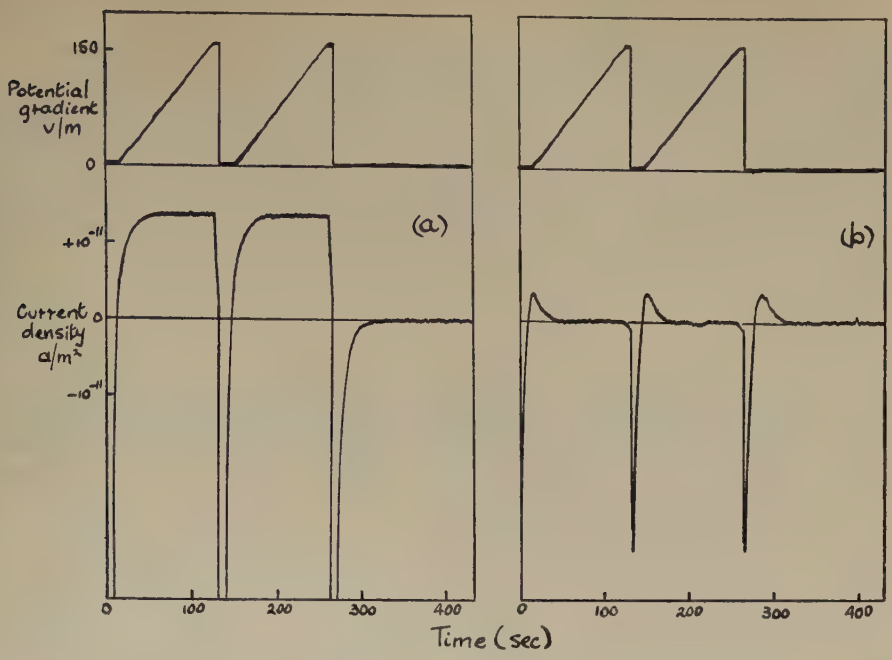

Figure 4. Recordings: (a) uncompensated; (b) compensated.

A small adjustment of the differentiating condenser is required from time to time to restore accurate compensation. This adjustment is usually carried out daily, as follows. A horizontal test plate, insulated from the ground, is supported about 50 cm above the collector and the field machine, and generates an artificial potential gradient over the instruments. This potential gradient has a ' sawtooth ' waveform produced by connecting the plate to the tapping point of a circular potentiometer with 70 v across it, the wiper arm being turned through gears by a synchronous motor. The cycle period is about 110 sec. First, the test plate is held at earth to give a zero indication at the d.c. amplifier output, and then the synchronous motor is started. During the period when the test plate voltage is rising linearly, corresponding to a constant displacement current, the differentiating condenser is adjusted to restore the zero output. It usually takes about three cycles to effect this properly. Fig. 4 compares two records taken when the sawtooth waveform was being applied. In (a), the compensating grid was held at earth so that the recording shows the displacement current. In (b), the field machine applied compensation, and apart from the transients, the zero is restored in each cycle.

The transients, which have been described previously, are large because at the end of the linear rise in potential the plate is suddenly earthed. In operation exposed to the atmosphere, such violent changes seldom occur, and the transients observed are at least an order of magnitude smaller. The cycle period is sufficiently long to allow the transients to die away and enable the differentiating condenser to be adjusted.

No adjustment is necessary to the integrating circuit (Fig. 1) smoothing the field machine output before this is applied to the differentiating condenser. The components of this are not critical and are pre-set to minimize the transients occurring during the application of the test potential gradient.

The short- and long-term drift of the d.c. amplifier is equivalent to about one-tenth of the normal fine-weather conduction current of 2×10^{-12} a/m^2 and is generally of negligible effect when there is precipitation current as well. The time-constant of response is about 20 sec and current densities up to 5×10^{-10} a/m^2 can be measured before the amplifier saturates. The field machine gives accurate recording of the potential gradient (linear to 2 per cent) up to $\pm$ 4,500 v/m when its associated a.c. amplifier

saturates. Because of the transients in the current recording discussed previously, a limit on the rate of rise of potential gradient for satisfactory recording is set at a few tens of v/m per sec.

Summarizing, the equipment is suitable for use in most weather conditions except violent thunderstorms, which are likely to produce excessive transients in the current recording. The time constant of response is very long compared with what is usually attainable in electronic equipment, but the method is considered an improvement on earlier methods of discontinuous recording, in which current is collected over discrete periods of a few minutes and the integrated charge measured.

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A direct measurement of the humidity in the stratosphere using a cooled-vapour trap

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Summary

A vapour trap designed for sampling the water vapour and carbon dioxide of the stratosphere is described. The results of one flight at about 27 km over south-east England are given and discussed.

1. Introduction

In the course of investigations into the composition of the air in the stratosphere, there was a requirement to collect about 1 mg of water from an altitude around 30 km. A consideration of the limited information available on the water content of the stratosphere, indicated that over 10^4 litres of air might have to be sampled to yield this quantity of water. The method adopted was to freeze out the water vapour and carbon dioxide from the air directly, by drawing the air through a vapour trap cooled in liquid nitrogen. The amount of carbon dioxide collected was then used as a measure of the total amount of air sampled, assuming the ratio of carbon dioxide to air to be constant. (Glueckauf 1944).

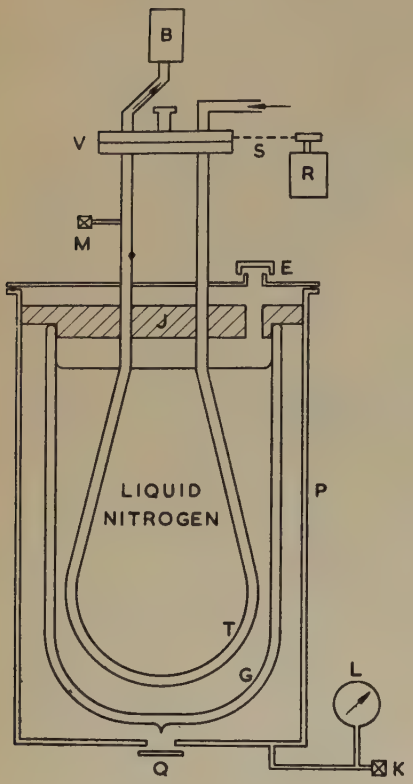
The experimental apparatus described in this paper has been successfully flown on a large polythene balloon for a period of $3\frac{1}{2}$ hr at a mean height of 27 km. The results of this flight are presented and discussed.

2. Description and operation of the apparatus

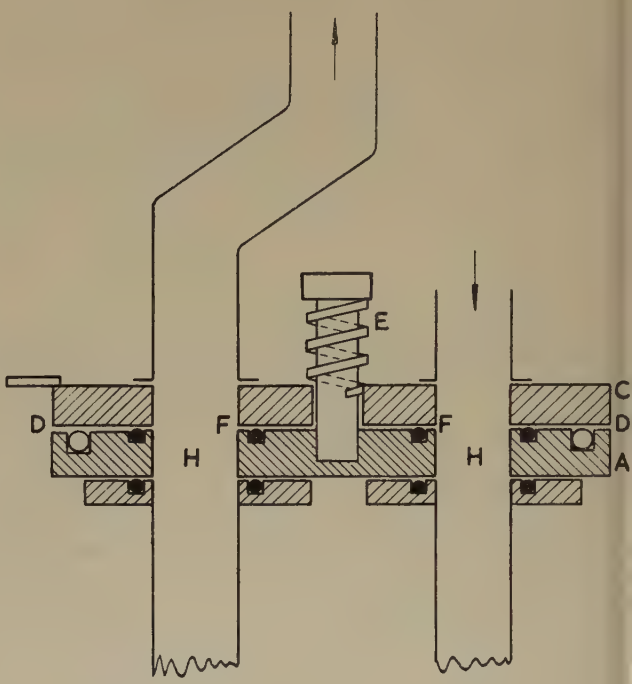
The apparatus consists of a simple U-tube vapour trap, a Dewar flask containing liquid nitrogen, a centrifugal blower, and a disc-type rotary valve, see Fig. 1 (a). The air to be sampled enters the intake pipe connected to the upper movable disc C of the rotary valve V, see Fig. 1 (b), and is drawn through the stainless steel vapour trap T via the outlet port of the valve by the blower B. The valve V consists of two polished aluminium plates pressed together by a spring E and slightly separated by a ring of steel balls D located in a channel in the lower plate A. The vacuum is maintained by silicone rubber-ring seals F, likewise located in channels in the lower plate A, one around each port H.

A wide-bore valve of this type was chosen for three reasons : to avoid the constrictions inherent in commercially available vacuum taps; to minimize the use of grease, and to facilitate the baking out of as much of the inner surfaces as possible. The leakage rate is not good by vacuum standards, being such that a pressure of about one centimetre of mercury developed in the evacuated tube in 24 hours. However, the amount of water so entering is quite small, being less than 0·1 mg. The valve is turned by means of a rotary actuator *R* via chain and sprockets S. The baking and pumping out of the trap prior to flight and the recovery of the sample is carried out through the all-metal tap M.

The liquid nitrogen is loaded through the filler plug E into the glass Dewar flask G which is located by the anti-splash cork J in the aluminium pressure vessel P. In order to prevent solidification of the liquid nitrogen when boiling under reduced pressure, and to prevent the collection of oxygen as well as carbon dioxide and water vapour, the liquid nitrogen is kept at a temperature above the oxygen condensation point, by maintaining the pressure inside P at about two atmospheres absolute before take-off on the ground.



(a)



(b)

Figure 1. (a) The vapour trap; (b) details of rotary valve V.

At full altitude the absolute pressure inside P will then be at least one atmosphere. This excess pressure is first provided by compressed nitrogen, and the ball-release valve K adjusted until gauge L indicates the correct pressure. The internal pressure is then maintained by the quiescent but adequate simmering of the nitrogen.

In the event of a bad landing or launch causing the Dewar flask to break, the pressure in P would build up too fast to escape through K and an explosion could result. To obviate this danger, a spring-loaded safety valve Q of large-port area is included. As a further precaution, Q is opened on closure of the valve V, by melting nylon thread, using a suitable heater circuit. Also, should the balloon not reach ceiling, Q will then be tripped by a simple one-way pressure capsule set for a height of about 5,000 ft.

The equipment is powered by a 24 volt silver-zinc accumulator which is contained in a sealed housing. The opening and closing of the valve at altitude are determined by simple alarm clocks, fitted with contacts.

The apparatus is hung in a barrel-shaped cane basket to prevent damage on landing. Every effort is made to minimize the amount of water carried up on the equipment. The cane of the basket is painted with shellac, and the thermal insulation is such as to ensure that no outside surfaces of the apparatus are at temperatures below the dewpoint of the ambient air. Flights are not made when there is cloud. The upper parts of the apparatus are painted with an acetone based, matt-black paint to increase heating by solar radiation. The present version of the equipment weighs 90 lb with liquid nitrogen, and the basket 25 lb; this, however, can be considerably reduced in any further design.

Evidence that the blower is working, that the valve has operated satisfactorily and that pressure over the nitrogen is at the required value is provided during flight by means of a modified radiosonde carried with the apparatus.

3. Tests and calibration

The necessary length of tube in the cold trap was deduced from the well-known solutions of the analogous heat-conduction equations (Carslaw and Jaeger 1947). The cold trap was made longer than these solutions suggested for the type of blower chosen, because of the uncertainty of the collection efficiency for molecules of carbon dioxide and water at the cold surface.

The amount of carbon dioxide collected during a flight is used to deduce the total amount of air sampled, by taking the carbon-dioxide concentration of stratospheric air to be 0·025 per cent by volume (Glueckauf 1944). The amount of water collected then enables the humidity-mixing ratio of the sampled air to be deduced by assuming equal collection efficiencies for both carbon dioxide and water vapour. It is also assumed that no crystals are formed in the air as it passes through the trap. The air was drawn through the apparatus in order to avoid possible contamination of the air by the blower.

As a check on these assumptions, measurements of the flow rate were made in a large decompression chamber at the Royal Aircraft Establishment, Farnborough, using a hot-wire anemometer. The results of these measurements agreed with the flow rate found during a flight, using the carbon-dioxide calibration, to within 10 per cent.

Several tests were made to determine how much water was left in the trap after the 'baking-out' process, and whether any water was added in the subsequent analysis. This was done by pumping out the trap to a pressure of 10^{-5} mm of mercury, 'baking-out' at a temperature of at least 400°C until appreciable out-gassing ceased, and then allowing air of known humidity to enter the trap. This was then pumped out slowly through a pyrex U-tube immersed in liquid nitrogen which collected the water. This U-tube was sealed by melting both the inlet and outlet pipes. The amount of water in the tube was then determined by immersing the sealed tube in a cold bath of controllable temperature and so finding the dewpoint of the vapour enclosed. Knowing the volume of the tube, the water content was then deduced. The contents of the tube were also subjected to a gas-analysis method, which involved converting the water vapour to hydrogen by passing the gaseous contents over heated zinc. The pressure and volume of this hydrogen were then measured, and so the amount of water vapour, originally present, could be deduced. Excellent results were obtained by the dew-point method and it was found that the gas analysis was liable to add less than 0·5 mg of water.

4. The flight trials

The balloon was launched from Cardington, near Bedford, at 0640 GMT, 2 May 1958. The general wind direction in the troposphere was NW'ly. A plastic-sounding balloon launched the previous day indicated that the wind at 27 km was easterly. On the actual flight, however, the balloon never reached the E'ly winds but continued, in a SE'ly direction, at heights between 29 km and 25 km, as given by ground-based radar. It should perhaps be mentioned here that flights are only made on those days when the upper winds are such as to reduce the possibility of losing the equipment in the sea. This limitation means that days with light E'ly winds, at the height at which the balloon floats, are generally preferred.

The sampling apparatus opened at 0915 GMT when at a height of 28 km and closed 3½ hr later when at 25 km. The equipment was released from the balloon at 1300 hr and landed in a wooded area in Kent at about 1330 hr. Unfortunately, it was necessary to make an extensive search for the equipment, which resulted in some delay, but the apparatus was finally returned to Cardington at 2300 hr when it was pumped out (as described in § 3).

5. Results

Weight of water collected (by gas analysis)	21·8	mg
Estimated contribution due to leak and analysis	0·5	mg
Volume of carbon dioxide collected	118·3	cc N.T.P.
Volume of air sampled (from the CO_2)	473	l N.T.P.
Humidity mixing ratio	0·037 g kg^{-1}	

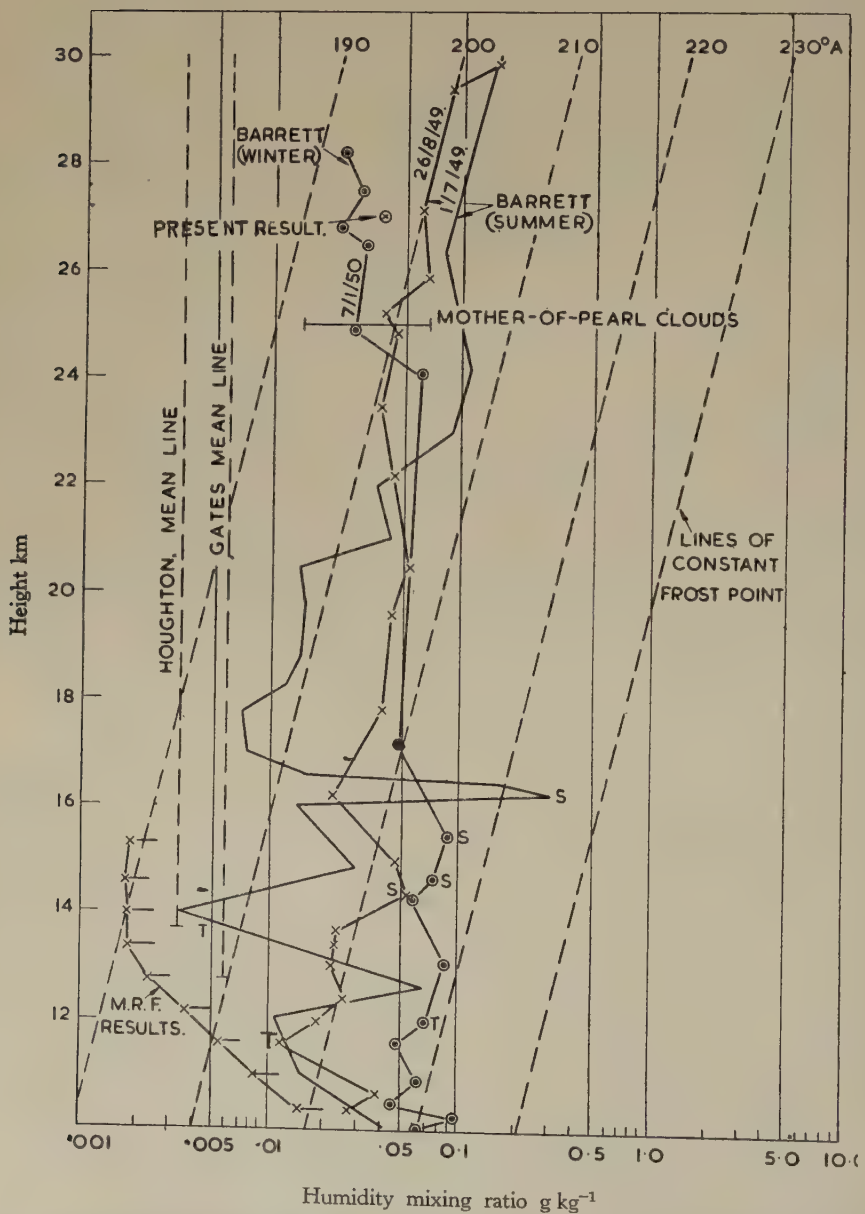


Figure 2. Height-humidity mixing ratio distribution derived from the reported results of various observers.

6. Comparison with other measurements (See Fig. 2)

Most of the quantitative data available on the humidity of the air in the stratosphere are presented in Fig. 2, which shows the height-humidity mixing ratio distribution derived from the reported results of various observers. Lines of constant frost point are also drawn. An I.C.A.N. atmosphere has been assumed except in the case of the results of Barrett, Herndon and Carter (1950) where the actual temperature and pressure measurements have been used. These results, which were obtained by flying an automatic frost-point hygrometer on a large polythene balloon to heights around 30 km over Minnesota, U.S.A., are reproduced in full. The date of each of their three soundings is given on the plot. The height of the tropopause is indicated by the letter T, and regions of saturation by the letter S. It is seen that all three flights give saturated layers between the tropopause and 17 km. The two summer flights also show the mixing ratio to increase between 20 km and 30 km whilst that of the winter indicates the opposite trend. In general, the humidity mixing ratio is between 0·01 and 0·1 g kg^{-1}, at heights between 10 km and 30 km on all three occasions.

The Meteorological Research Flight (M.R.F.), of the British Meteorological Office, has been making humidity soundings up to heights of about 15 km since 1953. A Canberra aircraft, fitted with a pressurised aircraft hygrometer of the Dobson-Brewer type (Brewer, Cwilong and Dobson 1948) or one modified for use with jet aircraft (Goldsmith 1955), is used in this work. These measurements are presented as a plot of the mean humidity mixing ratio curve, extracted from the first 35 soundings (Murgatroyd, Goldsmith and Hollings 1955). Subsequent results show no significant variation from this mean (Helliwell, Mackenzie and Kerley 1957).

These aircraft measurements show the lower stratosphere over England to be very dry, the humidity mixing ratio being about 0·002 g kg^{-1}, with little seasonal variation. However, there is some evidence, from measurements about 20 km in cirrus cloud (Murgatroyd and Goldsmith 1956), that the frost point as measured by the Dobson-Brewer hygrometer may be about 2°C too low. This possibility is indicated by the horizontal lines on the plot of the M.R.F. results.

The occurrence of mother-of-pearl clouds over southern Norway (Störmer 1940) at heights between 20 km and 30 km suggests the frost point there may be between — 70°C and — 80°C. This evidence of the stratospheric humidity is indicated by the horizontal line at 25 km.

Measurements of the infra-red solar spectrum from high-flying aircraft (Houghton, Moss, Seely and Hawkins 1957), and from high-altitude balloons (Gates, Murcray and Shaw 1958) enable the total amount of precipitable water above the spectrometer to be deduced. Houghton's measurements give 5 μ of water above 13·5 km over southern England, whilst those of Gates give 10 ($\pm$ 10) μ above 12·8 km over New Mexico. The mean humidity mixing ratio of air above the spectrometer has been derived from these results, and in each case is represented by a broken vertical line on Fig. 2.

The mixing ratio at about 27 km found by the experiment described in this paper is also plotted. This result is in reasonable agreement with the measurements of Barrett at that height, and with the evidence of the mother-of-pearl clouds. The Barrett work, however, indicates a much moister stratosphere, at all levels, than that suggested by the infra-red spectrometer data and that given by the direct aircraft measurements. However, since most of the observations were made at vastly different times and places, too detailed a comparison is not justified.

It is apparent from this discussion that our knowledge of the water-vapour distribution in the stratosphere is still rather sparse, and that there is a real requirement for more measurements at these heights.

Acknowledgments

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551.509.313

On the grid length to be adopted in numerical weather prediction

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SUMMARY

The predictive equations of numerical forecasting are cast in differential form and do not have simple analytic solutions. In practice, the finite-difference analogues of the differential equations are solved using a grid and the grid length has been chosen so that significant features of the synoptic chart are reasonably defined by values at the grid points of contour height or pressure. The purpose of this note is to enquire into the grid lengths that should be chosen so as to make the best estimate of the derivatives which appear in the equations.

1. THE ERRORS IN THE DERIVATIVE

When the values of a function are given at isolated points, the derivatives are expressed through these values by means of the usual interpolation formulae, e.g., in terms of central differences we have

$$hf_0' = (\mu\delta - \tfrac{1}{6}\mu\delta^3 + \tfrac{1}{30}\mu\delta^5) f_0 + O(\delta^5 f_0)$$

where the customary notation is used, f_i being the value of the function at the i-th point, μ and δ being the averaging and central difference operator respectively and h the grid length. In numerical computations the series must be truncated and the difference between the true value of the derivative and that obtained from the truncated series is called the truncation error. Often only the first term is retained on the right-hand side and we than have the estimate

$$f_0' = \frac{f_1 - f_{-1}}{2h} \qquad . \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (1)$$

In the meteorological problem, in which the values of the contour heights are required at the grid points, errors are made in the process of estimating grid-point values from the contour chart, assuming that the latter is correct; even if the process of estimating grid-point values were exact, the contour chart itself contains errors owing to the errors in observation and analysis and there would be errors in the grid-point values. In estimating the error made in computing the derivative it is necessary to consider both the truncation and interpolation errors; clearly if in the above formulae the difference $(f_1 - f_{-1})$ is large compared with the maximum interpolation error, then the truncation error is likely to be the more important and vice versa.

2. THE ERRORS FOR A SINE WAVE

The contour heights at (say) 500 mb tend to be sinusoidal in pattern, so that, in general, Fourier analysis is more revealing of the structure than is polynomial analysis. Consider the pattern given by the simple wave

$$y = C + A \sin kx \qquad . \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (2)$$

where C and A are both constant and are heights.

By comparing the analytical and finite-difference forms of the first derivative, the truncation error in Eq. (1) is easily seen to be

$$T = Ak \cos kx \left(1 - \frac{\sin kh}{kh}\right) \qquad . \qquad . \qquad . \qquad . \qquad (3)$$

If the error made in assigning a value at a grid point is ϵ_i then the interpolation error in the derivative is

$$E = \frac{\epsilon_{i+1} - \epsilon_{i-1}}{2h} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The relative values of T and E vary over the range of grid points. It seems reasonable to attempt to choose h so that $T^2 + E^2$ is a minimum when summed over one whole wave. The mean value of E^2 is given by $\overline{\epsilon_i^2}/2h^2$ and we may assume that ϵ_i may take any value between $\pm \epsilon$ with equal probability, so that $\overline{\epsilon_i^2} = \epsilon^2/3$. Writing $z = kh$, the equation for $T^2 + E^2$ is

$$T^2 + E^2 = \tfrac{1}{2} A^2 k^2 \left(1 - \frac{\sin z}{z}\right)^2 + \frac{\epsilon^2}{6h^2} \quad . \quad . \quad . \quad (5)$$

and the minimum value is given by

$$(z - \sin z)(\sin z - z \cos z) = \frac{\epsilon^2}{3A^2} \quad . \quad . \quad . \quad (6)$$

Expansion of the function on the left in powers of z shows that for small z it varies as z^6, so that it is a rapidly increasing function. Eqs. (5) and (6) show how kh varies with the ratio ϵ/A showing that as $\epsilon \to 0$, h must also tend to zero to obtain the best estimate of the derivative .

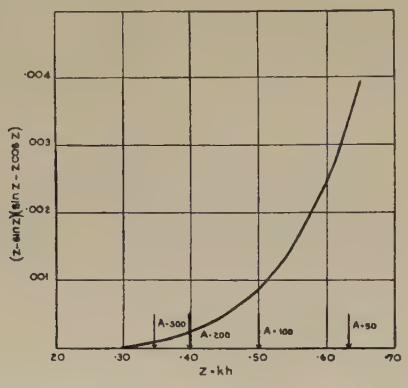

Figure 1. Values of kh giving minimum error, using a simple derivative, corresponding to $\epsilon = 5$ m.

The function on the left of Eq. (6) is shown in Fig. 1. It remains to evaluate ϵ^2/A^2 to determine kh. In reading from a chart, the value of ϵ in this expression is about 5 m, i.e., interpolated values are read to the nearest 5 m. Values of A vary from 50 m upwards, to about 200-300 m in strong meridional flow. The values of ϵ^2/A^2 thus vary from about ·01 to ·0003. From Fig. 1 the corresponding values of kh are

$$kh \sim 0{\cdot}45 \text{ for large values of } A$$

$$kh \sim 0{\cdot}75 \text{ for small values of } A$$

A large-scale sinusoidal pattern at about 50°N has a wavelength of about 3000 mi, the smaller features having wavelengths of about one-third of this. If L is the wavelength, $kL = 2\pi$ and the corresponding grid length is given by

$$h = \frac{\alpha L}{2\pi}$$

so that h varies directly with L and α where α varies with A. Table 1 shows how h varies with A for $L = 1{,}000$ mi.

It appears from Table 1 that the present grid lengths used in numerical prediction, which are of the order of 200 mi, are most appropriate to those features with wavelength of about 2,000 mi. If ϵ is doubled, the grid-length values are as shown in the second line of the table.

TABLE 1. OPTIMUM GRID-LENGTH VALUES IN MILES CORRESPONDING TO A WAVELENGTH OF 1,000 MI

		Amplitude (A)			
		50 m	100 m	200 m	300 m
Simple derivative	ϵ = 5 m	130	80	65	55
	ϵ = 10 m	140	100	80	70
Five-point derivative	ϵ = 5 m	170	150	130	120
	ϵ = 10 m	200	170	150	135

We may evaluate the mean error from Eq. (5) and this has been done for the four values of the amplitude, and for $\epsilon = 5$, 10 m. The results are similar to one another and Fig. 2 shows the variation for $\epsilon = 5$ m, $A = 200$ m.

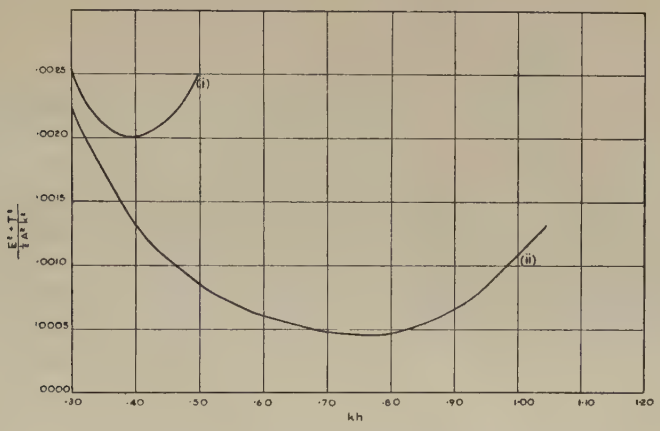

Figure 2. Mean error as a function of kh corresponding to $A = 200$ m, $\epsilon = 5$ m.
(i) Three-point derivative. (ii) Five-point derivative.

3. USE OF A HIGHER APPROXIMATION TO THE DERIVATIVE

The truncation error may be reduced by using the finite-difference formula

$$f_0' = \frac{1}{12h}(-f_{+2} + 8f_{+1} - 8f_{-1} + f_{-2}) \quad . \quad . \quad . \quad (7)$$

where the successive points, distant h apart, are labelled from -2 to $+2$. Then

$$T = Ak \cos kx \left(1 - \frac{4 \sin kh}{3\ kh} + \frac{\sin 2\ kh}{6\ kh}\right)$$

and the corresponding equation for the minimum value is

$$\frac{1}{6}\left(z - \frac{4 \sin z}{3} + \frac{\sin 2z}{6}\right)(8 \sin z - 8z \cos z - 2 \sin 2z + 2z \cos 2z) = \frac{65\,\epsilon^2}{108\,A^2} \quad . \quad (8)$$

where $z = kh$. Fig. 3 shows the variation of the left-hand side of Eq. (8), and the values of kh corresponding to $A = 50$, 100, 200, 300 m for $\epsilon = 5$ m. Table 1 gives the grid-length values corresponding to the minimum mean error. As was to be expected the refinement of the finite difference approximation to the derivative has to be matched by an increase in grid length to reduce correspondingly the error introduced by the presence of ϵ, and the optimum grid length is approximately doubled. The mean error has been computed and is shown in Fig. 2. We note that the curve is flat within the region of the minimum error so that the precise choice of kh is not important.

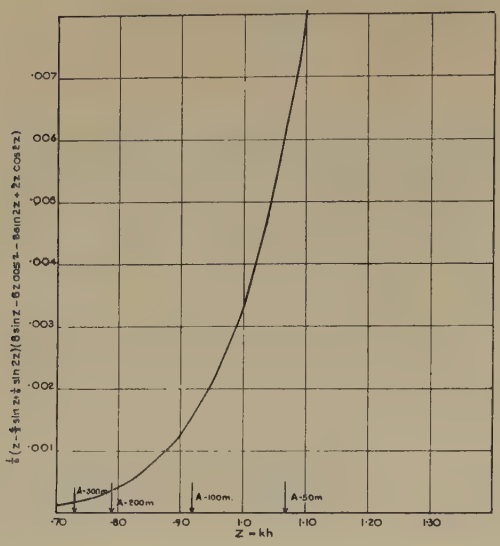

Figure 3. Values of kh giving minimum error using a five-point derivative, corresponding to $\epsilon = 5$ m.

The ratio of the error made with use of a five-point derivative to that made with use of a three-point derivative is shown in Fig. 4. The upward arrows show the values of kh which give a minimum error for various values of A using a three-point formula for the derivative, and the downward arrows similar values of kh using a five-point formula. The five-point formula for the derivative is preferable for all values of kh in the interval shown and shows errors of about 1/20th or less of those for the three-point formula near the optimum values.

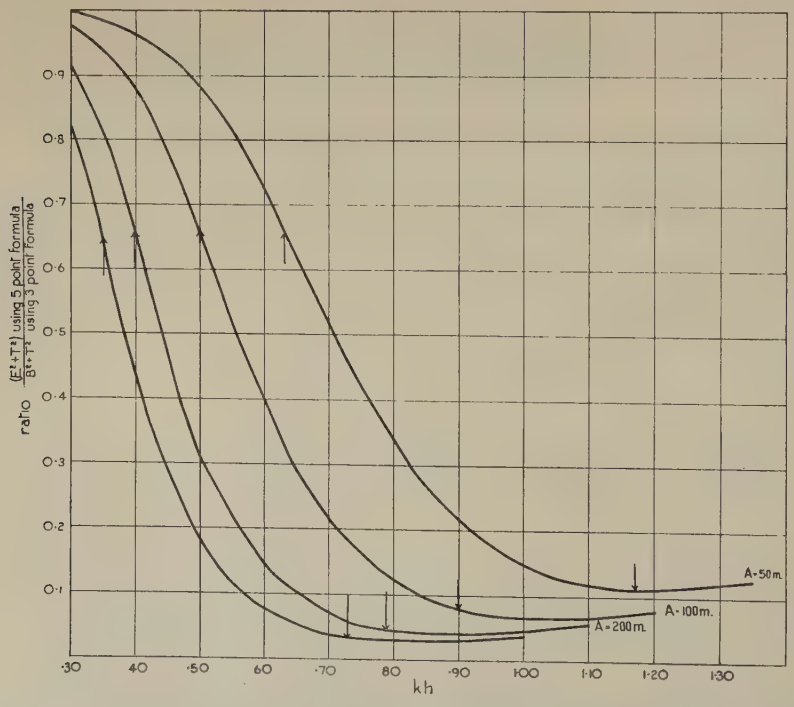

Figure 4. Ratio of mean error using five-point derivative to that using three-point derivative, corresponding to $\epsilon = 5$ m.

4. The second-order derivative

It is necessary in numerical prediction to compute first derivatives to yield velocities, and second derivatives to yield vorticities. The second derivative may be given by either of the two expressions

$$f_0'' = \frac{1}{h^2}\,(f_{-1} + f_1 - 2f_0) \quad . \quad . \quad . \quad . \quad . \quad . \quad (9a)$$

$$f_0'' = \frac{1}{4h^2}(f_{-2} + f_2 - 2f_0) \quad . \quad . \quad . \quad . \quad . \quad . \quad (9b)$$

where the first is obtained from first derivatives at half-way between the grid points, and the second is obtained from the first derivatives at the grid points themselves. It is not immediately clear which of these forms should be taken. Proceeding from the sine wave we have

$$T^2 = A^2 \sin^2 kx \left\{ -k^2 + \frac{2\,(1 - \cos kh)^2}{h^2} \right\}^2 \quad . \quad . \quad . \quad (10)$$

$$E = \frac{2\epsilon^2}{h^4} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

and writing $z = kh$, the equation for the minimum mean error is

$$\{z^2 - 2\,(1 - \cos z)\}\,\{2\,(1 - \cos z) - z \sin z\} = \frac{2\epsilon^2}{A^2} \quad . \quad . \quad (12)$$

The left-hand side of Eq. (12) is shown in Fig. 5 with the corresponding solution for $A = 50$, 100, 200, 300 m. Comparing the values with those found in Fig. 1, it is seen that the minimum mean error occurs when the value of kh is double that for a simple estimate of the first derivative so that the second form, Eq. (9b), is preferable to the first. The values of kh found in Figs. 3 and 5 are about equal, so that Eq. (9a) should be used when the derivative is computed by Eq. (7) and this computational scheme is preferable since it reduces the mean error.

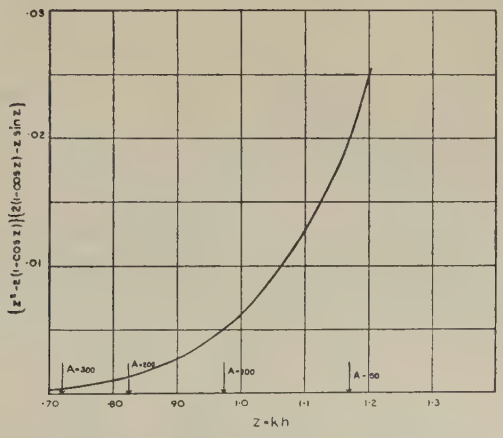

Figure 5. Values of the kh giving minimum error for Laplacian, corresponding to $\epsilon = 5$ m.

Conclusions

It is not possible to separate the truncation error from the experimental error made in obtaining the values at the grid points by some system of interpolation.

The optimum grid length for a given wavelength is a function of the ratio of the magnitude of the experimental error to the amplitude of the waves. The mean error made in the estimation of a derivative may vary slowly with grid length for a given wave number in the vicinity of the optimum value, so that a precise value of the optimum grid length is not important in reducing the error.

The errors made in first derivatives can be reduced by using a five-point formula in place of a three-point formula; to obtain the maximum reduction, the optimum grid length found for a simple estimate of the first derivative must be increased by a factor of about two.

In estimating second derivatives, the grid length used should be about twice that used in estimating first derivatives by a three-point formula, and the same as that used in estimating them by a five-point formula. The latter computational scheme gives lesser errors than the former.

For meteorological applications, wind speeds should be computed by a five-point formula and vorticities computed by the ordinary simple formula, both computations using the same grid length.

In numerical predictions we cannot say that one range of amplitudes is more important than any other range, although if we wish to predict rapid changes associated with large-amplitude motions we should choose a grid length corresponding to an amplitude of 200-300 m or 20-30 mb in surface pressure.

551.508.21 : 551.508.23

The use of the Campbell-Stokes sunshine recorder as an integrating actinometer

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(Manuscript received 8 October 1959)

SUMMARY

A simple method is described to evaluate the daily total direct solar radiation, by measuring the weight of paper burnt in the Campbell-Stokes sunshine recorder.

1. INTRODUCTION

Meteorologists, climatologists, biologists and others, have repeatedly expressed the need for an instrument to measure direct solar radiation which should offer simultaneously simplicity of operation and maintenance, ruggedness and low cost, together with acceptable accuracy. This has recently been further emphasized at the February 1959 meeting of W.M.O. and U.N.E.S.C.O. experts on solar radiation, in connexion with the U.N.E.S.C.O. Arid Lands programme. We have, therefore, explored the possibilities offered by the cheapest and most widely distributed instrument in present use in this field : the Campbell-Stokes sunshine recorder. A preliminary analysis shows that the weight lost by the paper after a day's exposure is a simple function of the corresponding daily total of direct incident solar radiation.

2. METHOD

The instrument used was a standard Campbell-Stokes sunshine recorder (tropical model), installed unprotected on the roof of the Instituto de Ciencia Aplicada in the University grounds on the outskirts of Mexico City (19° 20'N., 2,268 metres a.s.l.). On the same roof was installed an Eppley actinometer on a heliostatic mounting, connected to a Kipp recording millivoltmeter. The actinometer was calibrated in the Institute against an Ångström absolute pyrheliometer.

In the course of 12 months, 110 comparisons were made between the weight lost during a day by a paper strip in the sunshine recorder and the corresponding daily total of direct solar radiation as estimated by planimetric integration of the record of the Eppley instrument.

The series of observations covers a wide variety of meteorological conditions including some days with heavy rainfall and days of both exceptionally high and exceptionally low atmospheric turbidity.

* Member of U.N.E.S.C.O. Technical Assistance Mission.

Various types of paper were tested but we refer here only to the 110 results obtained with the standard paper of local manufacture (undressed cardboard), used by the Mexican Meteorological Service in its routine observations. This paper appeared to give results as good as any other type of paper tested.

The paper strip to be used is first thoroughly dried in a laboratory oven at 125°C for at least one hour. It is then quickly transferred to a calcium-chloride desiccator and allowed to cool to ambient temperature. The next step is to weigh the strip on an analytical balance of accuracy ± 0·1 mg. Experience has shown that this operation can normally be completed before the weight of the paper strip becomes affected by the ambient humidity. The paper strip is now ready to be used, and it is exposed in the sunshine recorder in the usual way – from before sunrise until after sunset. After removal from the instrument the strip is thoroughly dried and weighed in the same manner as before.

3. Results

The results of the 110 comparisons between the weight of paper burned and the corresponding daily total of direct solar radiation, as measured by the actinometer, are represented graphically in Fig. 1.

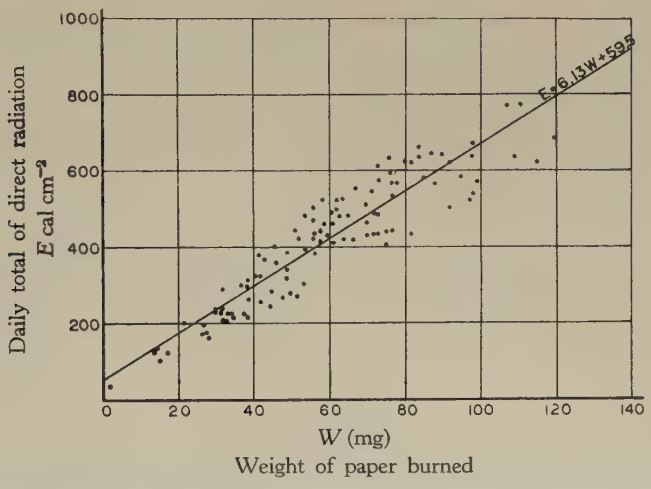

Figure 1. Relation between daily totals of direct radiation as indicated by Eppley actinometer and corresponding daily loss of weight of paper strip in Campbell-Stokes sunshine recorder.

These results show a high positive correlation ($r_{we} = + 0{\cdot}93$) between the two variables, which is significant at the 0·01 per cent level.

On the assumption of a linear relation between the variables, the method of least squares gives the following equation :

$$E = 6{\cdot}13\ W + 59{\cdot}5$$

E being the daily total of direct radiation expressed in g cal cm^{-2}, and W the weight of paper burned in mg.

In order to determine the accuracy of individual estimates of the daily radiation total from the weight of paper burned, we calculated the residual error (σ_{ew}) from the equation :

$$\sigma_{ew} = \sigma_e\,(1 - r^2_{we})^{\frac{1}{2}}$$

where σ_e is the standard deviation of the series of 110 values of daily radiation and r_{we} the correlation coefficient between the two variables. The value of σ_{ew} obtained was ± 61·0 cal cm^{-2}.

A plot of the individual errors against the corresponding observed values of E (Fig. 2) reveals a tendency for the error to increase with increasing magnitude of E. One-half of the points lie below a line which corresponds to a error of ± 11½ per cent in the estimate, and two-thirds of the points lie below a line corresponding to ± 15½ per cent.

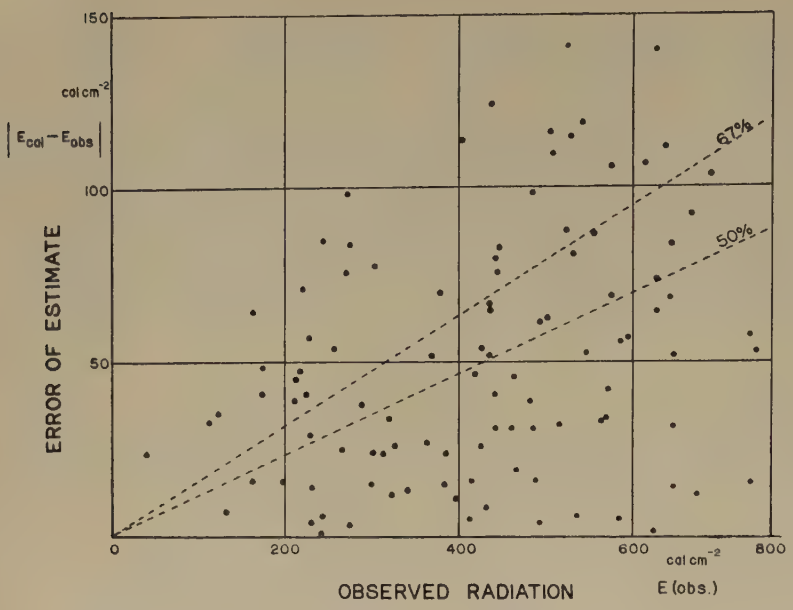

Figure 2. Relation between daily totals of direct radiation as indicated by Eppley actinometer and the corresponding errors of the estimates based on the weight of paper burned in the Campbell-Stokes sunshine recorder.

These figures include of course the errors inherent in the actinometer, its millivoltmeter recorder, and in the planimetric integration of the records. The probable interdiurnal error from these sources is estimated to be of the order of ± 3-4 per cent so that the resultant probable error of individual daily estimates of radiation from the sunshine recorder may well lie within ± 11 per cent

Conclusions

The Campbell-Stokes sunshine recorder appears to offer the possibility of measuring not only the duration of sunshine but also its intensity with an accuracy of the order of ± 11 per cent.

The estimated error includes all those due to astronomical and meteorological factors (solar altitude, cloudiness, humidity, rainfall, etc.).

This extension of the function of the sunshine recorder requires only that the paper strips be carefully dried and weighed before and after exposure. It is possible that in regional studies a central observatory could undertake this task for its dependent second-order stations.

In view of the relative cheapness of the sunshine recorder, and the large numbers of this instrument already in service in many countries, its possible use as an integrating actinometer appears to be worthy of study by the meteorological services.

Acknowledgments

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We are indebted to Ing. Federico Peña Aguirre, Director of the Geographical and Meteorological Department of the Ministry of Agriculture in Mexico, for the generous loan of the Eppley actinometer and heliostatic mounting, and for the supply of the paper strips used in the sunshine recorder.

551.578.46 : 311.214

Snowfall measurements in northern Canada

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Summary

Sources of error in Canadian snowfall records are discussed, with particular reference to the six-hour interval between measurements. Very slow rates of fall are usual at high latitudes and these cause ' traces ' of snow to be recorded frequently which are not included in precipitation totals. Experience on Ellesmere Island suggests that the contribution from these traces may form a high proportion of total winter snowfall, and a longer interval between measurements seems desirable.

1. Introduction

The problem of the accurate measurement of solid precipitation has so far evaded solution. In an exhaustive study of the many types of gauge available, Kurtyka (1953) was unable to find one which gave satisfactory results. He cited the results of other workers who had found that, compared to the fall measured in snow sections, the average raingauge catches approximately 80 per cent of the snow in normal conditions. With a wind shield the proportion may rise to 90 per cent. These ' normal conditions,' however, include winds of 8-15 mi hr^{-1}. As soon as winds increase above these values, it was found that reliability decreases very rapidly, and a wind shield makes little difference.

An alternative method of snow measurement avoids the use of a gauge by measuring directly the depth of newly-fallen snow. The precipitation (water-equivalent) is then calculated by means of an assumption concerning the specific gravity ratio between snow and liquid water. This is the method used at most Canadian stations and it can best be described by quoting from the instructions given to observers :

> 5.8.2. Snowfall. The amount of snow which has fallen in a given period shall be determined by measuring and averaging the depth of snow in several places. Snow depth shall be measured in tenths of inches. In so far as possible, the depth of new snow shall be measured in spots where the snow has fallen undisturbed by the wind. When the snow has been drifted by the wind, the depth of new snow in drifts and in exposed areas shall be measured, and the observer shall then estimate the depth of snow that would have accumulated if the fall had been undisturbed by the wind, making due allowance for the relative size of the drifts and exposed areas. (Canada, Meteorological Division, 1957).

It is assumed in Canadian measurements that 10 in. of newly-fallen snow have a specific gravity equal to 1 in. of rainfall. The average depth of new snow is therefore multiplied by a factor of 0·1 to calculate the precipitation for the period concerned.

2. Sources of error in the Canadian technique

This method of measurement is unsatisfactory for several reasons. Bruce and Potter (1957) have pointed out the great difficulty in obtaining representative measurements of snow depth, particularly after any drifting has taken place. It is not sufficient to consider ' the relative size of the drifts and exposed areas,' because the degree of compaction in drifts varies considerably and it is impossible for an observer to estimate the compaction accurately and then to visualize the equivalent depth of newly-fallen snow, evenly distributed over the landscape.

In addition to the difficulties involved in the accurate measurement of the depth of new snow, the validity of the 10 : 1 ratio between snow and water is open to question. Currie (1947) has shown from measurements at Saskatoon that the specific gravity of snow which falls at temperatures well below freezing-point appears to be less than 0·1; he found that a 12 : 1 ratio was

more accurate. In contrast, Rae (1951) quoted measurements at Resolute, Cornwallis Island, which indicated that there the specific gravity was about 0·2, or a ratio of 5 : 1. A similar value was indicated by sample measurements at Lake Hazen, Ellesmere Island. It is, indeed, unlikely that any single value of the ratio can be acceptable for continuous measurements, even in one locality. The application of the 10 : 1 ratio throughout Canada must lead to substantial inaccuracies. Bruce and Potter (1957) suggest the error in some monthly and seasonal precipitation totals to be as much as 20 per cent. For both these reasons, therefore, the snow-depth method as practised in Canada is almost certainly inferior to the gauge method : at least the latter has the advantage of greater objectivity.

The more widespread use of a new type of snow gauge, at present being tested at a few stations, is contemplated by the Meteorological Branch (see Bruce and Potter 1957). It seems likely to lead to a considerable improvement in the accuracy of measurements.

3. The importance of traces of precipitation

A further source of error exists which has received much less attention in the past. It arises from the omission from precipitation totals of falls which do not reach measurable amount. This is particularly important in the Arctic and occurs with either method of measurement. In high latitudes the error may be quantitatively greater than those already discussed. One factor which seems particularly important is the relatively short interval of six hours between precipitation measurements. Snowfall at such latitudes is normally very light; a fall of 6 in. of new snow in one day is recorded only rarely at any of the permanent stations in the Queen Elizabeth Islands. Daily totals are usually very much less than this, and the six-hourly totals are smaller still. It is quite common for a station to record a succession of ' traces ' (i.e., precipitation less than 0·01 in.) at all four observations of a day.

The importance of this source of error can be demonstrated from measurements made on Ellesmere Island during 1957-58. In addition to the permanent Canadian-United States Joint Weather Stations at Eureka and Alert, a continuous surface weather record was maintained as part of the I.G.Y. programme at an inland site on the shores of Lake Hazen (Jackson 1959). At this station the snow-depth method was not employed; instead measurements were made of the water equivalent of snow caught in the standard raingauge (diameter 3·57 in.) with its funnel removed.

The inland site was remarkable for the low, surface wind speeds it experienced, in marked contrast to the permanent stations on the coasts. A summary of the wind records during the snow season is contained in Table 1. Because of the relatively calm conditions for much of the winter only small losses can be attributed to eddying round the gauge.

TABLE 1. Wind speeds at Lake Hazen, N.W.T. 1957-58. (number of observations in various groups based on eight observations each day)

Month	Calm	mi hr^{-1} 1-5	6-10	11 +
1957				
September	20	168	44	8
October	83	149	14	2
November	149	91	0	0
December	173	73	2	0
1958				
January	85	138	12	13
February	138	80	6	0
March	178	69	1	0
April	96	129	13	2
May	90	144	14	0
June	30	170	37	3

The station at Lake Hazen maintained continuous observations between 21 August 1957 and 10 August 1958 and the total measured precipitation during that period was 0·98 in., of which 0·07 in. fell as rain. A comparison with snow profiles on the lake suggests that 0·91 in. water-equivalent of the snow was about half the amount that should have been recorded. It is most unlikely that eddying in such favourable conditions could account for this large deficiency, and the major portion of the unrecorded snow appears to have fallen as traces. During the period of below-freezing temperatures (early September to mid-June), 187 traces were recorded. If it is assumed that two such traces are equivalent to 0·01 in., the measured precipitation is more than doubled. Even if such a ratio is too high, the significance of traces of snowfall is clearly too great for them to be ignored in an area where absolute totals are small. This error is equally likely to occur when the snow-depth technique is used. In the same period at Alert, 212 such traces were reported and at Eureka the total was 302. The frequency of traces at these stations may also be due, in part, to the difficulty of obtaining a reliable direct measurement of one-tenth of an inch of snow.

It is probable that many of these traces could have been included in measurable amounts if the snow had been allowed to accumulate for a longer period before measurements were taken. At Lake Hazen only 58 of the 187 traces were not preceded or followed by traces in adjoining six-hour periods. The principal objection to a longer interval is that the snow might undergo changes before measurement which would affect totals based on the snow-depth technique. Bruce and Potter (1957) pointed out that :

> The depth of snow . . . . . changes rapidly with time due to melting or to the breakdown of the crystal structures due to wind action, gravity, etc. Thus two sets of equally conscientious observations made at the same site, one at six hourly intervals and one at 24 hourly intervals will yield quite different data.

It seems, however, that the loss caused by the frequent recording of traces is considerably greater than the errors which may be expected from wind action or gravity.

4. Acknowledgments

The Lake Hazen data were collected during the I.G.Y. Expedition sponsored by the Defence Research Board of Canada. I am grateful to the Director of the Meteorological Branch for permission to quote from the instruction manual and for making available a microfilm copy of the observations at Alert and Eureka.

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551.515.21

Exchange of heat, moisture and momentum between *Hurricane Ella (1958)* and its environment

By M. GANGOPADHYAYA and H. RIEHL

(Read 18 November 1959. Presented by Professor H. Riehl. See *Q.J.*, **85**, p. 278)

Mr. J. S. SAWYER : It is very encouraging to see from Prof. Riehl's paper and the work which he has described to us that a consistent picture of hurricane structure aloft is beginning to emerge from the mass of observations which are being collected, at great expense, in the Caribbean.

Perhaps the most difficult thing to explain about a hurricane is the continued eviction of air without the building-up of a circulation and pressure field which preclude further eviction. There are, however, some flow patterns which permit such continued divergence, but they are not very common. Can we not regard the fact that the outflow is confined to a narrow range of height as an indication that rather special dynamical conditions are required for the outflow and that they do not normally occur at more than a narrow range of altitude in the upper troposphere ?

Dr. R. C. SUTCLIFFE, F.R.S. : In the light of this paper would the author care to comment on the reason why hurricanes quickly lose intensity on passing over land ? Whether this is the increase in friction or the cutting-off of the supply of air of sufficiently high equivalent temperature is an old controversy on which Professor Riehl's opinion would be of much interest.

Professor H. RIEHL (*in reply*) : The outflow layer from hurricanes, as Mr. Sawyer remarks, is very narrow – only about 150 mb in depth. This may be one distinguishing mark of such storms compared to other disturbances of the tropics. In the early stages, prior to the appearance of a hurricane, one often finds that the cloud system of the lower troposphere extends upward into a layer of nearly zero absolute vorticity; perhaps even with slightly negative vorticity, centred near 200 mb. The existence of such a layer initially accords with an earlier hypothesis of Mr. Sawyer's on the subject, and it can now be said that the structure of the upper-anticyclonic vortex postulated by him can frequently be detected from the wind reports.

To me, the first formation is the knottiest problem we are still facing; even although many necessary conditions are known, those necessary and sufficient have not yet been found. After formation, the circulation tends to become self-supporting in that the ocean in the core of the storm acts as a local heat source. This raises the equivalent potential temperature and permits the air to lay down a very warm column during ascent. In consequence, the height of high-tropospheric isobaric surfaces will be higher than they would be without such a source. In this way the air reaching the high troposphere has a highly supergradient wind which causes withdrawal from the centre and acts to avoid the development of a radius of stagnation for the air.

This leads to Dr. Sutcliffe's question. It is very doubtful whether friction is greatly enhanced when a storm passes over land. I hope that relevant measurements can be made in the next few years; I think they are needed to determine whether land friction as found over flat country as in Florida and Louisiana is really more than that prevailing in an ocean agitated by hurricane-force winds. It would also be of value to see whether the intensity of hurricanes decreases much more rapidly over very rough or mountainous terrain than over flatlands; here a detailed study is still lacking although it could be undertaken with presently available data.

In my opinion the main cause for the decrease in central intensity is likely to be another one. The ocean surface acts as a potent *local* source of heat and moisture in hurricanes. Central pressures much below 1,000 mb cannot be obtained hydrostatically from a vertical temperature field produced by moist-adiabatic ascent of the surface air with properties as found when passing into a storm well from the outside. A higher mean tropospheric temperature is needed, which, outside the subsidence zone in the eye, can be produced only by local heat flow from sea to air, as already described. In my opinion the weakening of hurricanes over land occurs because this heat source is suddenly reduced or nearly eliminated.

551.513 : 551.552 : 551.557

Mean meridional circulations in the atmosphere

By G. B. TUCKER

(Read 18 November 1959. See *Q.J.*, **85**, p. 209)

Prof. P. A. SHEPPARD : This work was conceived about 1951 when there was, to me, much rather profitless debate in literature on the existence and theoretical necessity of meridional circulations in the atmosphere. As background to that debate we had little more by way of data than Riehl and Yeh's valuable analysis of surface winds in latitudes between $\pm 40°$, but for the oceans only. The depths of the lower limbs of the circulations there divulged were unknown and so, therefore, were the intensities and heights of the upper limbs. One knew at that time that the amount of upper-air information would not be sufficient to provide a definitive, quantitative description of meridional circulations on a semi-global (Northern Hemisphere) basis, but it appeared worth while to make a first attempt at their evaluation, using the checks that Dr. Tucker has in fact applied. As a result it now appears that meridional circulations of order 1 m sec^{-1} are present over substantial depths of the atmosphere in lower and middle latitudes, and it is relevant to remember that a meridional velocity of this amount creates or destroys zonal momentum at the rate of about 10 m sec^{-1} day^{-1}. Such a rate can hardly fail to be significant in the budget of zonal momentum at any level to which it applies; Dr. Tucker has stressed the thermodynamical significance of the result in this paper but I hope we shall see some of the dynamical consequences he has evaluated in due course.

Dr. Tucker's results are essentially statistical averages and leave unresolved the question of the nature of the circulations disclosed. The association of descent in the horse latitudes with the persistent sub-tropical highs gives one the impression that the statistical average is there realised almost on a day-to-day basis, as a more or less steady circulation, though no doubt with longitudinal dependence such as has been proposed by J. Bjerknes. But what of the ascending motion, of order 1 mm sec^{-1}, between the tropical cells – in the doldrums ? Is this also a quasi-steady circulation or is it a residual of cumulonimbus activity ? The widespread occurrence of high cirrus in equatorial regions may suggest the former, at upper levels at least, but the lower levels then remain a problem.

Finally I would like to know whether the investigators of ozone and water-vapour distribution in the high troposphere and lower stratosphere find anything inconsistent between the interpretation of their results and Dr. Tucker's paper.

Mr. A. H. GORDON : I was particularly interested in this paper because I did some work in this field myself about 10 years ago whilst in the Marine Division. Surface fields of motion and divergence were calculated from ships' observations by means of machine methods. At that time upper-air observations were sparse and little was known about the upper levels. As Professor Sheppard has so aptly remarked, there was much conjecture about what the atmosphere ought do but very little evidence of what it really did do. I myself was guilty of conjecture of this sort. It is therefore most reassuring to see that this very valuable work by Dr. Tucker – based on actual observations, confirms my own ideas as to the existence of the return circulation aloft, and also of the upper-level convergence in the sub-tropics, so well illustrated by the divergence profiles.

There are three points I would like to question. Is the upper-level convergence belt which is found above the surface sub-tropical anticyclone significant in producing the sub-tropical anticyclone ? I have thought so for a long time. Could Dr. Tucker say anything about the order of the mean meridional velocity across the Equator ? This certainly must exist as there is a large mass transfer of air between the hemispheres, it is a maximum from N. to S. about May and S. to N. about October. Finally, could the work done by Dr. Tucker be reduced in time scale ? Could it be done, for example, for the 30-day period of the CLIMAT charts, or perhaps for the summer of 1959, in order to see whether anomalies could be related to the abnormal summer just experienced ?

Professor H. RIEHL : Dr. Tucker's paper represents considerable progress because the calculations of net meridional mass flow have been carried to high altitudes over a wide latitude belt for the first time. I am pleased to see that the results are in agreement with the computations I have made on the tropical general circulation. I, too, have had considerable reservations about the conclusions of Starr and White at latitude 15°N which, because of the integration over the entire year, cancel the effect of the seasonally-reversing net mass circulation.

I believe, in spite of Dr. Tucker's fine demonstration, that further computations will have to be performed before the sceptics will be convinced. Upper-wind data from the last few years should be sufficient to carry the calculations to the top of the troposphere. Moreover, the large gap in the Pacific and SE. Asia can certainly be filled, at least in the tropics and sub-tropics. Does Dr. Tucker have any plans for extending his study in this direction – perhaps with the of I.G.Y. data ?

Mr. A. D. H. Pennant (*communicated*) : The paper suggests an indirect circulation between Lat. 30° and 55° as a separate entity. I find this very difficult to accept. It would mean that heat energy is absorbed in the ascending limb in higher latitudes, and given out again in lower latitudes. Insolation is greater and the temperature is higher in the lower latitudes, and a circulation involving, as this does, a transportation of heat from where it is less abundant to where it is more abundant, seems to me to be contrary to the second law of thermodynamics.

A possible way out of the difficulty would seem to be as follows : the indirect circulation should really be regarded as an integral part of the general circulation where the stream of air moving away from the Equator has temporarily got underneath the return stream instead of remaining all the time on top of it.

Some part of the explanation for such behaviour could be that the air which ascended in the tropics lost its moisture by condensation and precipitation, and was cooled by expansion. It will therefore be dry, and while it is at a high level it will not lose much heat by radiation because its temperature is low. If it is now brought down to approximately sea-level, it will have its temperature raised by compression, more than it was reduced when it ascended, because the ascent released a lot of latent heat by condensation of water vapour. It will therefore be relatively very hot and very dry. If this happens over the sea, and there is sufficient wind to make white-crested waves and spray, evaporation of this spray will be rapid, in the very hot dry air, and the air will soon become just warm and damp, and it will be much less unstable at this low level than it would have been if it had not been able to reduce its temperature by means of this copious evaporation of sea-spray. If it subsequently ascends again the latent-heat energy in the water vapour will again be released by condensation and precipitation, to again strengthen the circulation.

In the Southern Hemisphere there is much more water surface available for this process in temperate latitudes so that it is natural that the circulation should be much stronger in that hemisphere.

Does Dr. Tucker feel that there is a real difficulty about the second law of thermodynamics and an indirect circulation ?

Mr. A. J. Troup (*communicated*) : While Dr. Tucker has attempted to allow for the omission of data, particularly that due to high winds, in his computation of the meridional circulation, I am doubtful whether what he has done is adequate, in view of the fact that the omission of only a few occasions of very strong wind will make a marked difference to the mean meridional component at any station. As an illustration, I have taken 300 mb winds in winter (November-February in the Northern Hemisphere, May-August in the Southern Hemisphere) and computed the effect of omitting the high wind speeds on the value of the meridional component. The stations used were Habbaniya, Auckland, Benina, Port Stanley, Larkhill and Gibraltar; the data for the first three stations were for the periods used by Priestley and Troup (*C.S.I.R.O. Div. Met. Phys., Tech. Pap.* No. 1, 1954) for Port Stanley and Larkhill for 1951-55 (*Met. Office* No. 628, Pts. 1 and 2, H.M.S.O.) and for Gibraltar 1948-50 (*Met. Office* No. 555, Pt. 8, H.M.S.O.). Apart from Gibraltar, these have very few missing observations at 300 mb; for the first three, missing values have been interpolated. The Table below sets out the mean meridional components obtained by successively omitting winds over 120 kt, 100 kt and 80 kt, and the percentages of flights so included.

Mean meridional component (poleward components positive in kt.)

Station	All observations	Omitting Winds ⩾ 120 kt	⩾ 100 kt	⩾ 80 kt
Habbaniya	+ 2·15 (100)	+ 0·39 (93)	− 0·38 (83)	− 3·54 (68)
Auckland	+ 1·50 (100)	+ 1·55 (99)	+ 0·63 (95)	+ 0·92 (85)
Benina	+ 1·06 (100)	− 0·30 (96)	− 1·52 (88)	− 4·21 (77)
Stanley	− 2·07 (99)	− 2·78 (97)	− 2·93 (91)	− 3·36 (80)
Larkhill	− 16·75 (99·5)	− 14·83 (97)	− 12·44 (90)	− 8·33 (80)
Gibraltar	− 9·77 (93·5)	− 9·85 (93)	− 9·19 (92)	− 8·09 (88)

It will be seen that at many stations the loss of only a few observations can altogether change the value of the mean. The figures show that at stations to the west of a marked mean trough, the strongest winds occur with equatorward components, and it is inferred that the converse applies at stations to the east. These effects might be expected to compensate if there is no difference in mean speed ahead of, and behind, a mean trough, but this is not generally the case. We also observe that where the flow is nearly zonal, and in regions of strong mean wind, the highest winds occur with poleward components. A similar effect is shown in an analysis by Priestley and Troup (loc. cit. Table 13) of the contributions to poleward eddy transfer of momentum from different ranges of wind speed. This introduces a systematic bias when the strongest winds are not allowed for.

In view of these results, 20 observations per month (Dr. Tucker's criterion for acceptance) may give quite a false monthly mean. Moreover, since the missing observations are selective, it is not correct to assume that the missing observations at any level behave in the same way as those that reach this level. Finally, one must bear in mind that the figures for vertical velocities and mean trajectories among others, are based on *differentiation* of the mean meridional components.

It would therefore be useful in assessing the accuracy of this work, to know what percentage of observations was missing at each level, particularly at the higher levels. Could Dr. Tucker also indicate whether, in analysing the charts of mean meridional components, greater weight was given to stations with better records ?

Mr. L. C. W. Bonacina : I should like to ask Dr. Tucker whether his meridional circulation signifies the temporary departures from the mean latitudinal or regional circulation such as is figured, for instance, in Bartholomew's charts of wind and pressure. If so, these temporary departures would seem to be of vital importance in the general circulation as constituting the actual mechanism by which the interchange of air between the Equator and the Poles in the lower levels of the atmosphere is effected.

Of course the temporary departures must be included in the values shown on the mean wind and pressure charts if based on sufficient observations, though not specifically indicated thereon. In view, therefore, of the implied critical importance of the meridional components I should have expected them to appear with greater frequency than apparently Dr. Tucker found when he wrote the paper.

Dr. G. B. Tucker (*in reply, partly communicated*) : The day-to-day strength of the mean meridional circulation mentioned by Professor Sheppard, and the variation in strength along a line of latitude are difficult to resolve. Any single wind observation is, of course, composed of geostrophic and ageostrophic components. The geostrophic component is usually the greater even in relatively low latitudes, and the ageostrophic component may be largely due to a cyclostrophic effect; consequently, any truly 'toroidal' component will be swamped. On a day-to-day basis, therefore, I think the strength of the meridional circulation cannot be obtained by direct wind observation. The long-term variation along a line of latitude could probably be estimated from surface observations, and this brings me to Mr. Bonacina's remarks.

If Bartholomew's charts of wind and weather are correct, that is if the winds are really vector mean winds and not vague estimates based on the pressure pattern, then the mean meridional velocity is the average (along a line of latitude) deviation of the wind from the geostrophic, that is, the average horizontal wind component normal to the isobars. Accurate normal, seasonal-surface charts of wind and pressure may thus reveal the long-term longitudinal variation of mean ageostrophic velocity at the surface which could possibly throw some light on the longitudinal variation of the toroidal circulation.

Mr. Gordon's first question concerning the relation between the upper-level convergence belt and the surface sub-tropical anticyclone with its associated belt of surface divergence is a difficult one. Each is dependent on the other for its existence, and because of this mutual dependence, cause and effect probably cannot be resolved.

The order of magnitude of the mean meridional velocity near the surface at the Equator can be estimated from observations made by ships. I have not analysed these in detail, for the two months mentioned by Mr. Gordon, but they do seem to suggest a southward flow of roughly one metre per second in May (where, according to Mr. Gordon's calculations, there exists a net southward mass flow) and also in October (when a net northward flow is required). If correct, this implies a weaker upper-reverse flow in May and a stronger one in October, both directed towards the north.

Any reduction in the time scale of my analysis, as suggested by Mr. Gordon, could only be done at the expense of accuracy. Notwithstanding the fact that two, and possibly four, observations a day could be used if the summer of 1959 was examined, any estimate of the standard

error of the mean meridional velocity over a 30-day period would probably be larger than the velocity itself.

In answer to Professor Riehl, studies of the transfer properties of the wind field at various levels are continuing, mean meridional circulations will probably be obtained as one of the products of these analyses. I.G.Y. data will probably be used in the near future.

In reply to Mr. Troup, I agree that there is undoubtedly some sampling bias towards lighter winds at high levels, and precisely why I chose 300 mb was because the majority of observations missing at this level are not due to the balloon being blown out of range. The Table showing the effect of omitting high winds is interesting but does not apply to my analysis. I can illustrate this by referring to observations at Habbaniyah (one of Mr. Troup's illustrative stations) which experiences very strong winds in winter at 300 mb. Since May 1951, the number of occasions on which the range of the target exceeded the scale limit, usually 96,000 yards (i.e., the number of times the balloon was lost because of high winds) has been reported, together with the number of times the loss was due to signal fade out. The average height at which both losses occurred is also given. I have combined these for a 12-month period into the following Table, and, I have also included the number of ascents which reached 300 mb (two ascents being made in each day).

Upper-wind soundings at Habbaniyah May 1951 to April 1952

Month	No. of ascents reaching 300 mb	Ascents limited by scale limit		Ascents limited by signal fade out	
		No.	average pressure (mb)	No.	average pressure (mb)
1951					
May	62	1	130	3	66
June	60	0		3	136
July	62	0		1	130
August	62	0		0	
September	60	0		3	–
October	62	1	130	10	108
November	60	2	130	3	85
December	62	7	117	5	120
1952					
January	58	14	142	7	154
February	54	22	130	9	267
March	62	17	137	4	147
April	60	0		0	0

The Table shows that only in January and February did any ascents fail to reach 300 mb and then only four in each month; this means that 99 per cent reached 300 mb during this period. The Table also shows that : (a) because an observation is discontinued at a high level it can by no means be assumed that this was due to high winds; (b) the average height at which an ascent is discontinued because the balloon is blown out of range is well above 150 mb at Habbaniyah.

Maximum wind speeds often occur at lower levels in higher latitudes but these winds are generally less strong than at Habbaniyah. At Lerwick, for the same 12 months as in the above Table, the ratio of observations reaching 300 mb was 95 per cent.

The lower limit of 20 observations in any one month is not a characteristic of each station; of the 84 stations I used, in the standard 2 yr period there were only 14 occasions when there were less than 24 observations in any one month. The average ratio of ascents reaching 300 mb was 87 per cent, consequently I used the few stations in my network with a poor observing frequency to obtain a best estimate of the mean velocity components. The Table suggests that observations missing at 300 mb should not be assumed to be largely due to high winds. When the charts were constructed greater weight *was* given to stations with better records. One further point on this topic which Mr. Troup did not mention, but which I consider a weakness of my analysis, is the fact that I was obliged by the availability of data to consider only one observation a day. Subsequent work on more recent and frequent (four times daily) observations shows this to be inadequate for estimating eddy fluxes although a reasonable estimate of the advective or standing eddy flux and consequently of the mean velocity components is obtained.

I do not quite follow Mr. Pennant's argument. The net result of an indirect circulation does not necessarily involve an equatorward transfer of heat, and even if it did, this would not contravene the second law of thermodynamics since a far greater poleward heat transfer is brought about by the eddy flux. The net result of all atmospheric motions averaged around a latitude circle is the necessary poleward transfer of heat. When averaging, it is only to the net results of all components of atmospheric motion that the laws of thermodynamics can be applied. Finally, it is perhaps as well to re-emphasize that my results were essentially a statistical average, and if treated as a steady flow with no eddy components they will lead to many spurious results.

551.578.7 : 551.501.81

Patterns of hailstorms in Alberta

By R. H. DOUGLAS and W. HITSCHFELD

(Read 16 December 1959. Presented by Dr. W. Hitschfeld. See *Q.J.*, **85**, p. 105)

Dr. J. M. Stagg (*President*) : I, and perhaps others, would be interested to know why this part of Canada was chosen for this valuable work. Why should southern Alberta be so particularly prone to hailstorms ?

Mr. L. C. W. Bonacina : I think this is a most stimulating paper. The correlation between the size of hailstones and height of the storms as a measure of instability, and probable correlation, also, between the duration of the storms and height seems to me very significant. Duration is no doubt the deadly factor in the destructiveness of hailstorms. I would, indeed, suggest that a wood would be more effectively stripped of its leaves by hail of only walnut size lasting an hour than of golf-ball size lasting only a quarter of an hour. Ruskin, describing a protracted hailstorm in Tuscany wrote : " I tell you, I have seen the hail fall in Italy till every tree was bare."

Could the authors say how Alberta compares with such a hail-stricken country as France ? This lies some 10° latitude further south than Alberta with stronger summer convection, and more abundant supplies of maritime moisture available : a powerful factor in thunderstorm development. The Hon. Ralph Abercromby in his *Weather* quotes the case of 21 August 1879 when in an alignment of thunderstorms stretching 600 miles from Brussels to the Pyrenees and 10 to 20 miles in width, dense curtains of hail moved across France at 30 miles an hour, devastating huge tracts of vineyards in a few minutes.

Mr. F. H. Ludlam : Hail studies on rather similar lines to those described by Dr. Hitschfeld have been conducted in this country during the last three years. Although hailstorms are rather less frequent here than in Alberta we have occasional storms which are more severe : for example, the Horsham storm of 5 September 1958 (largest stones weighing nearly ½ lb. and of ' half-grapefruit size ') and the Wokingham storm of 9 July 1959 (largest stones being almost of tennis-ball size). It appears from our experience that in the massive cumulonimbus, the hail factory, the hail is produced in individual thermals, or assembly rooms, which have a typical dimension of about one mile and a typical life of about ten minutes. Unfortunately the conventional radar and voluntary observer networks which are used do not have the degree of resolution to show us clearly the workings of the cumulonimbus on this scale. Consequently I think we shall be driven to invent more refined techniques of investigation, but for the immediate future I hope that the fascination and difficulty of the studies described will encourage the members of this audience to give their active support.

Mr. R. G. Veryard : I have witnessed many hailstorms in tropical and sub-tropical regions and have noticed on many occasions that hailstones of different sizes, large and small, have fallen at one and the same time. Have your observers reported similar occasions ? If so, may not this affect some of your results ?

Dr. R. C. Sutcliffe, F.R.S. : It would be interesting to know if there are any conclusions about the geographical distribution of the storms. Is it apparently a random distribution over this nearly uniform territory ? One wonders whether pre-existing irregularities, particularly of humidity, would be found by aircraft flying across the region in the morning before convection has begun. A clue to prediction might be obtained in this way.

Dr. R. S. Scorer : In searching for orographic effects it must be realised that the direction of the wind may be all-important in determining whether an effect appears or not. A wind blowing along a valley may produce an effect, such as the absence of showers until the evening, which would be absent if the wind were as little as 15° different in direction.

The records, therefore, cannot be used in a climatological manner, and in deciding how to analyse them, if at all, it is necessary to have a theory, preferably a correct one, to direct one's attention to particular things. But even an incorrect theory is better than none at all.

Mr. J. S. Sawyer : Is a typical Alberta hailstorm accompanied by a squall ? If so, was the distribution of hail relative to the squall-line investigated ?

Mr. R. F. Jones : I would have thought that the rapid increase in probability of hail at temperatures below — 30°C was at least as significant as the relation with height on which the authors place most reliance. Fig. 19 (of the paper) is perhaps not as convincing as it looks at first sight. From Fig. 19 (a) we see that there were probably 4 cases when hail was reported with echo-top heights of 20,000 ft. or less. Fig. 3 (of the paper) gives a strong probability that the temperature at 20,000 ft was in the range — 25 to — 28°C. There were therefore probably only 4 hail occasions at temperatures at or below — 28°C for which there are 4 points in Fig. 19 (d). The left-hand part of Fig. 19 (d) can hardly therefore be significant, particularly as the authors say that the probabilities are unrepresentative for heights below 15,000 ft where the temperature (Fig. 3) was probably about — 15°C.

Dr. R. H. Douglas (*in reply, communicated*) and Dr. W. Hitschfeld (*in reply*) : As Dr. Bonancina's and Mr. Ludlam's remarks illustrate so well, hail is not confined to North America nor is the position of Alberta unique within that vast hail region there, which includes the Great Plains and the Foothills. What made Edmonton and Penhold so inviting to us as bases of operation was the keen interest and cooperation of the local authorities. Further, in answer to Dr. Stagg, one good reason for the hail climate of the region may be the one put forth by H. Appleman (1959), namely, that hail would be common where the water content between cloud base and the level where the larger drops freeze is relatively low. This would ensure some precipitation of ice, rather than the predominance of water precipitation.

Our present observational techniques do not generally allow us to study phenomena of the fineness demanded by Mr. Ludlam (dimension 1 mile, duration 10 minutes), but while it would be interesting to conduct finer-scale observations in the future, the possibilities of the available tools have by no means been exhausted. Moreover, we doubt that one is justified on the evidence of all the material available to us (and some aspects of our data approach the stated limits) to draw the boundaries of the ' hail factory ' as narrowly as Mr. Ludlam suggests.

The Alberta observations of hailstones agree with those alluded to by Mr. Veryard. There is generally in each shaft of hail a wide asssortment of sizes. A few detailed distributions have been measured in 1958 and 1959, particularly to estimate the radar scattering cross-section of the hail shaft itself. It may be noted, however, in the present paper, that where hail-size is mentioned, what is always meant is the size of the largest observed hailstone.

Dr. Sutcliffe is probably right in supposing that local irregularities, for instance of humidity, exist prior to the onset of convection. Desirable as such observations undoubtedly are, they have not been attempted by us. Further, to the same point, by examining larger bodies of data, we are turning up a few sites which seem to favour cumulus development. So far, with Dr. Scorer's excellent point in mind, we have not yet established any correlations with such orographic features as do exist. A similar point may be made with regard to some areas in the region which, especially on the basis of their hail insurance records, seem to be notably prone to hail.

In reply to Mr. Sawyer, some of the severest hail storms were associated with a front or a squall line, although many more, and especially the lesser ones, were not. The distribution of hailstorms along a squall line is a matter of great interest, as he implies, and it is hoped that some information on this point will become available through the network of micro-barographs which have recently been installed in a 40 × 50 mile area, to investigate the pressure field surrounding the storms. But obviously the number of hailstorms may one expect to corral in such an area is small.

Mr. Jones correctly explains our reluctance to make much of the striking minimum of the graph of Fig. 19 (g) in our paper, as being due to a feeling that we did not have enough data to support so odd an effect. The summer of 1957 actually had provided eleven (not four, as Mr. Jones suggests) observations of hailstorms with echo-top temperatures — 32°C or warmer. But including the summers 1958 and 1959, such observations now number more than 20. If all data are put together, the dramatic appearance of the minimum largely disappears. The average probability v. echo-top temperature curve now rises from zero at — 8°C, reaches a first maximum

of 40 per cent at — 22°C, dips to a shallow minimum of 30 per cent at — 30°C, then rises linearly to 85 per cent at — 46°C, thereafter to decline slightly at lower temperatures. The average probability v. echo-top height curve (analogous to Fig. 19 (b)) for the three seasons rises more uniformly, but shows a slight though well-defined dip near 32 kilofeet, the average height of the tropopause.

Reference

Appleman, H. 1959 *Nubila*, **2**.

551.524.7 : 551.524.32 : 551.581 : 551.587 : 551.521.32

Seasonal and other temperature changes in the antarctic atmosphere

By H. WEXLER

(Read 16 December 1959. Presented by Mr. J. MacDowall. See *Q.J.*, **85**, p. 196)

Mr. L. C. W. Bonacina : This paper, as printed, contains on p. 207 a sentence which, from a theoretical standpoint, is of great importance in support of those who lay stress upon the effects of geographical differentiation in the genesis of weather and climate. The sentence attributes the different thermal behaviour of the stratosphere in the Arctic from that in the Antarctic to the zonal contrasts of land and ocean in the Northern Hemisphere.

I agree with the author that the sharp peak in the seasonal winter minimum of mean temperature at Verkhoyansk in N.E. Siberia is primarily due to continentality, but the comparison with the South Pole is not quite comparable, since Verkhoyansk in latitude 67 degrees lies only just within the Arctic Circle, so that the return of the sun there in February would tend to prevent a flat minimum.

Mr. H. H. Lamb : I think it necessary to dispute Dr. Wexler's strange description of the V-bottomed seasonal temperature trend, found at Verkhoyansk and in parts of N. America, as ' usual ' in or ' typical ' of polar climates. I believe it was essentially due to the relatively low latitudes of the places at which it occurred. The flat-bottomed type of seasonal curve (*kernlose winter*) was much more widely found in high latitudes north and south.

The paper contained many interesting ideas which I would have liked to comment on, or ask about, but time does not permit. It seems important, however, to add something to what was said about the climatic trends of the present century.

Spitsbergen is not a typical Arctic station, being so situated as to show extremely great temperature changes whenever the ice limit in the area advanced or withdrew a little. Moreover the best-fitting curve to represent the Spitsbergen trend would rise very steeply from about 1915 to 1938 and then fall slightly to date.

The (lesser) warming trend at Little America from 1911 right to 1958 derived by Wexler from just six odd years' data is probably real; corresponding changes in the landscape about the Ross Sea are reported by explorers who have revisited the area. But it would be a mistake to assume that this trend is representative of the Antarctic in general.

Around Maudheim (10°W) it has been concluded that there was no diminution of the ice shelf for many years up to 1950. Similarly near Mawson (60-70°E) the glacier tongues and main ice front are known to have maintained their positions substantially unchanged since 1936 (M. Mellor, *Geogr. J.*, **125**, p. 230, 1959). The 56-year sequence of temperature observations at Laurie Island, S. Orkney (61°S 45°W) likewise shows no trend at all from 1903 to 1958. We may reasonably conclude that the position has remained static during this period along the periphery of the ' permanent ' ice everywhere from the Weddell Sea to much of East Antarctica.

West of Grahamland there has been no sufficiently long continuity of observing stations in one position – nor have successive expeditions re-used the earlier sites – for any proper comparison with the early part of the century. The best available series, near 65°S 64°W suggest a slight fall of temperature, since 1908 and 1958 was the coldest year on record. Ice conditions in the Bellingshausen Sea have also probably worsened since the turn of the century.

The pattern which emerges from these fragments reveals one of the ways in which the atmospheric circulation can vary. The Southern Hemisphere circumpolar whirl is normally centred away from the Pole towards the Indian Ocean sector, but this is a statistical result. The extent of the eccentricity and the sector towards which it is tilted vary from month to month and over longer periods. We may suppose that similar shifts of ' centre ' occur also in the Northern Hemisphere general circulation but are harder to determine because of the large-amplitude waves which distort its form.

The unlike trends at Little America and in the Bellingshausen Sea might perhaps be interpreted as some ridge and trough effect.

Dr. J. M. Stagg (*President*) : To me the figure which showed monthly mean temperatures at the surface and at levels up to 200 mb over Little America does not quite fit Dr. Wexler's explanation of the reduced seasonal range of temperature in the antarctic troposphere compared with the stratosphere. If meridional exchanges confined to the troposphere were the whole mechanism I would expect to see a feature like the June rise of temperature at the surface extending above the 750 mb level, but it is hardly noticeable even there. Could not an effect like this be produced by large-scale drainage from the ice plateau, similar to what happens in Greenland ?

Mr. C. I. Jackson : I would like to support Mr. Lamb's remarks on the flat winter minimum to be found in the Arctic troposphere. It seems somewhat misleading to describe Verkhoyansk as showing a ' polar continental ' winter minimum. Central Siberia is indeed a source area for what are termed ' polar continental ' air masses, but it is impossible to regard the area as polar in terms of the radiation balance, because of the relatively low latitudes involved.

A more suitable station for comparison is the Canadian I.G.Y. site at Lake Hazen, in the interior of northern Ellesmere Island (*J. Geophys. Res.*, **64**, pp. 1451-55, 1959). During the 1957-58 winter, temperatures below — 40°F were recorded on 121 days – one-third of the year – and temperatures below — 50°F occurred on 73 days. These extremes were recorded in an area which is free of snow for about six weeks in summer.

The reversals of temperature decline appear (as might be expected) to be of greater importance in such *kernlose* conditions in the Arctic than in the Antarctic. An extreme case in January 1958 may be used as an example; the associated synoptic conditions have been discussed by Thomas and Titus (*Mon. Wea. Rev.*, **86**, 1958, pp. 19-22). On 4 January no part of the troposphere above Alert (82°30′N, 62°20′), was warmer than — 39°C and the sounding resembled the ideal isothermal conditions suggested by Dr. Wexler in a previous paper (*Mon. Wea. Rev.*, **86**, pp. 122-136, 1936). Three weeks later, on 24 January, the temperature at 937 mb was + 3·7°C and surface temperatures had risen by about 40°C at all three stations on Ellesmere Island : Alert, Eureka and Lake Hazen.

In regard to the point made by Mr. Bonacina, Longley has recently discussed the annual temperature curves at Resolute, N.W.T., in relation to the return of the sun (*Quarterly Journal*, **84**, pp. 459-63, 1958) . He found that the effect of the returning sun was not noticeable for about 15 days after its first appearance, and the mean temperature did not begin to rise for about a month. Similar dates appear to be true of Lake Hazen.

Finally, I would like to ask Mr. MacDowall whether he feels that long-wave radiation from falling snow can have an appreciable effect on surface temperatures during the sunless period. Dr. Wexler has shown that cirrus cloud radiation is significant; I feel that snowfall radiation may also be of the right order of magnitude, although I am not aware of any mention of this by other workers.

Mr. R. G. Veryard : In regard to the discussion of climatic change in the Antarctic as compared with the Arctic we should bear in mind that Spitzbergen may not be representative of the whole of the Arctic; there is little evidence that the warming at Spitzbergen is general to the North Polar region.

Dr. F. Loewe (*communicated*) : In Pt. 5 of his interesting paper Dr. Wexler gives (Fig. 14) average annual temperatures that are throughout about 1·5°C lower than those in the original expedition reports which he quotes in the more comprehensive paper in *Weather*, p. 191, 1959. The averages in Wexler's paper are identical with those in an earlier paper by Flowers (I.G.Y.

Bulletin, p. 997, 1958). No explanation for the substantial discrepancy is given. Possibly the solution can be found in Wexler's statement (*Weather*, p. 193, 1959) that ' the annual average temperatures were found by plotting the monthly averages and equalizing areas above and below the annual average.' Wexler's curves of monthly averages (Figs. 9, 13) comprise only the time from mid-January to mid-December; if the above procedure has been applied to these curves, it would indeed give values that are about 1·5°C lower than the true annual means. If the same procedure, as seems likely, has also been used for the years 1956 and 1957, the ' additional evidence presented ' from the snow temperatures in *Weather* (p. 194, 1959) would become much less convincing. However, the warming trend found by Wexler in the meteorological observations is not affected; in fact the regression of the true annual temperatures on time is even more significant than that of Wexler (at the 0·1 per cent level), and moreover, every individual month except March shows a positive trend. Inclusion of the values for 1958 does not modify this result. It may also be of interest that observations at stations located on the opposite side of the Antarctic continent near 65°S, 64°W (Port Charcot, Port Circoncision, Waterboat Point, Argentine Islands) give – for 15 years since 1904 – no significant temperature trend, thus confirming an earlier result for Laurie Island in 61°S, 45°W (Prohaska, *Archiv. f. Met.*, B, III, p. **72**, 1951).

Dr. A. COURT (*communicated*) : Twenty years ago this autumn, when the Weather Bureau was packing me off to the Antarctic with 140 boxes of gear, Dr. Wexler was most interested that I obtain both radiation measurements and data on thermal structure near the snow surface. I tried hard on both these counts during my year at Little America, but the radiometer (a Smithsonian Melikeron pyranometer) was too sensitive and the thermal elements (large-mass resistance thermometers) not sensitive enough. So I came back with a set of routine surface observations, some pilot-balloon runs, and 190 radiosonde flights; Dr. Wexler was disappointed, and I was left pretty much to myself to tabulate and interpret my results.

Hence, I am quite gratified that recently so much time, energy, and attention have been given to resumption and expansion of the stratospheric observations that I made single-handed, and also to the extension and revision of my conclusions. Particularly pleasing is the apparent verification, by the newer observations (Bannon, *Quart. J. R. Met. Soc.*, **84**, pp. 434-436, 1958), of the extrapolations I made from my limited 1940 data. For example, in 1950 (*Compendium of Meteorology*, p. 931) I estimated that at 20 km over Little America ' the lowest in late winter was probably colder than — 100C, for an absolute range of more than 70C in three months.' Dr. Wexler's Fig. 1 indicates a range between the warmest and coldest monthly means in 1957 over Little America of about 50°C at 30 mb, and implies a somewhat greater range in monthly means over Maudheim and over the South Pole itself. Could Dr. Wexler indicate the absolute ranges at this height over the various stations ? Also, are the mean temperatures simple arithmetic averages of all available observations at the given level, or are they obtained by average lapse rates, as were the mean temperatures in my report (*Mon. Wea. Rev. Supp.*, **48**, 1940) ?

Gratifying also is Dr. Wexler's refinement of the explanation I offered in 1942 (*Bull. Amer. Met. Soc.*, **23**, pp. 220-238) for the thermal differences between the two polar atmospheres. Elaborating conjectures of Brunt and Bjerknes, I attributed this to ' a general lack of circulation between the Antarctic and the rest of the world.' Dr. Wexler would restrict this isolation to the stratosphere, and call on ' strong heat advection into the continents ' in the troposphere to account for the disappearance of the tropopause, shown by my soundings and those which followed. He offers monthly charts in substantiation of ' the great difference in flow pattern between the troposphere and stratosphere.'

Essentially Dr. Wexler is suggesting that motion in the stratosphere is zonal, while that in the troposphere below it has a much greater meridional component. Could these characteristics be used instead of thermal lapse rate to distinguish between the two regions ?

Dr. Wexler appears to attribute the rapid temperature rise at all levels over the South Pole after the sun's return (2°C/day in October) to absorption of radiation and also to ' the sudden onset of subsidence,' whilst the halt to this warming in November he ascribes to dynamic processes, apparently a halt of the subsidence. My original suggestion involved radiation absorption by ozone, which had increased in concentration during the polar night, until the resulting warming caused the concentration to decrease so that the absorption diminished. Could Dr. Wexler comment on this possibility, in the light of ozone measurements over the South Pole and other Antarctic stations ?

Mr. J. MACDOWALL (*in reply*) : With regard to the comparison between Verkhoyansk and the South Pole, I agree with Mr. Bonacina's comments. However, the prevalence of a flat winter minimum in the surface air temperature even at the South Pole does demonstrate a

remarkable lack of ' continentality ' even in the heart of Antarctica. In this respect Mr. Bonacina agrees with Dr. Wexler, I think, for both postulate that at points distant from the sea the surface air temperature falls until the return of the sun.

I should like to thank Mr. Lamb for his valuable added comments on the difficult subject of climatic change. I am sure he will agree that Dr. Wexler has provided us with an extra piece of the puzzle, and so contributed to the eventual solution.

I think that Dr. Stagg has an important point on which I should like Dr. Wexler's comment. I, too, would not expect the rapid attenuation of the June surface air temperature maximum, in view of Dr. Wexler's findings of the Antarctic troposphere, unless changes were more complex than Dr. Wexler indicates.

With regard to the effect of the returning sun, at Halley Bay (75° 31′S, 26° 37′ W) surface air temperatures were more susceptible than at Resolute. Mean temperature rose significantly within 15 days of the returning of the sun, and over the whole year the first harmonic (12-month period) of surface temperature lagged the total solar radiation by 13 days. I am glad that Mr. Jackson refers to the considerable variation in the troposphere temperature over Alert in January 1958. The lesser variability of the Antarctic troposphere was referred to by Dr. Wexler, but this should not be confused with the flat winter minimum of surface temperature. To a first approximation cloud is regarded as radiating as a black body. Therefore, in general, during snowfall we expect a greater downward flow of long-wave radiation compared with a clear or cirrus-covered sky.

My reply to Dr. Court is that I think we must be careful before we throw out our present definition, based on lapse rate, of the tropopause. As far as Halley Bay data were concerned, we found that the normal W.M.O. criterion was adequate, though more difficult, even during the winter. For example, in 1958 the first tropopause was ' not properly defined ' on only 14 occasions from May to September (the sun was below the horizon from 1 May to 12 August) compared with 123 occasions when the tropopause was fully defined. Moreover, 16 ozonesonde ascents in 1958 indicated that the temperature lapse tropopause was clearly associated with the start of the ozone layer.

The total amount of atmospheric ozone over Halley Bay was at a minimum during winter, but the ozone layer rose in height and the peak concentration during winter was in fact equal to that measured in early summer when total ozone was at a maximum also.

Dr. H. Wexler (*in reply, communicated*) : I appreciate the opportunity to comment on the discussion of my paper on Antarctic temperatures which was kindly read by Mr. J. MacDowall.

Messrs. Lamb and Jackson each object to comparing the monthly march of temperature at Little America (78° 30′S), with that of a station which I have chosen to call typically ' polar continental,' Verkhoyansk, and which is located at a lower latitude, 67° 33′N. In this respect, and also in answer to Mr. Bonacina's remark concerning the latitude difference between Little America and Verkhoyansk, I should like to call attention, in Fig. 9 of my paper, to the curve for the Wilkes I.G.Y. Station, just outside the Antarctic Circle; it shows a flat winter minimum, comparable to that of Little America but quite different from Verkhoyansk. Mr. Jackson suggests that a better ' continental ' station to compare with Little America would be Lake Hazen (80° 49′N) located in a sheltered hollow in the interior of northern Ellesmere Island. The proportion of low temperatures at Lake Hazen is impressive and it would be interesting to compare its mean monthly temperatures with those of the Antarctic stations when they become available.

With respect to comments by Mr. Lamb and Mr. Veryard on the warming trend at Little America and Spitzbergen, I welcome their agreement with my cautions expressed to those who might assume that these trends are representative of the Antarctic and Arctic. Mr. Lamb's comments about changes in sea and shelf-ice conditions about Antarctica are interesting but rather difficult to interpret in terms of temperature trends.

Dr. Stagg makes an interesting suggestion to explain the June temperature singularity at Little America. With the prevailing SE'ly winds at Little America, it is conceivable that large-scale air drainage from the 5,000-7,000 ft high plateau of west Antarctica could produce a föhn effect there; however, it would be just as difficult to explain why this effect apparently favours the month of June as does the hypothesis of a greater heat advection from the ocean. Accepting this latter hypothesis, the upward decrease of the intensity of the warming could be a result of a decrease of the meridional temperature gradient observed above the surface inversion layer in the coastal zone of Antarctica (see, for example, Fig. 7 of my paper). This remarkable June singularity deserves further study.

Dr. Court's comments brought back nostalgic memories of instructing him and his colleague Mr. H. G. Dorsey, Jr., in the operation of the Smithsonian Melikeron pyranometer, which, in 1936-38, had been used successfully at Fairbanks, Alaska, and Fargo, North Dakota. The absolute range of temperature at 50 and 30 mb over Little America during 1957 and 1958 was 64° and 67°C, respectively, while the South Pole values were 65° and 76°C, respectively. Monthly mean temperatures at Maudheim show a range of about 50°C at the 50 mb level. With respect to the cessation of stratospheric warming some time after return of the sun, Dr. Court suggests that a diminution of a presumed winter high-ozone content might be responsible. Firstly, we do not know whether there is a maximum of the total ozone content of the vertical column in wintertime over the polar plateau, and secondly, a decreased ozone amount in the vertical column does not necessarily mean lesser heating at 50 to 30 mb since most of the absorption is done at the top of the ozonosphere, usually well above 30 mb.

In response to Dr. Loewe's comments about the definition of the annual average temperature and the Little America warming trend, I have the following remarks : In attempting to compare average air temperatures with those measured at 10 m within the snow, I felt that the equal-area average should be the proper one to use as indicated by heat-conduction theory. However, in carrying out computations of these equal-area averages, I used only 11-monthly intervals instead of the 12-monthly intervals as Dr. Loewe has pointed out. Dr. Loewe is quite correct, therefore, in stating that the values I arrived at are on the average about 1·4°C lower than they should be. Since this applies to all of the years, it does not affect the trend, but does raise the temperatures closer to the 10 m snow temperatures and in this respect weakens somewhat the argument based on snow temperature comparisons contained in my article in the June 1959, issue of *Weather*. This neglect of the ' 12th interval ' also affects the McMurdo average temperatures published in Fig. 4, in my *Weather* article. According to Dr. Loewe, this results in temperatures about 1°C lower than they should be. This again should have no effect on the conclusion that no significant temperature trend was found at McMurdo Sound. In reviewing my original computations I found an error for the Little America value for 1940 – the average annual temperature should be 1·2°C higher than that shown in Fig. 14. This has practically no effect on the slope of the trend line but displaces it upward by 0·17°C. I am very grateful indeed to Dr. Loewe for pointing out this error in the computation of the average annual temperatures, which fortunately has practically no bearing on the trend considerations.

Proceedings at meetings of the Society

Evening Discourse, 6 January 1960

The second of the current session's series of Evening Discourses took place in the Society's Rooms : Mr. C. E. Wallington spoke on ' Mountain waves.'

A full account of this meeting appeared in March issue of *Weather*.

Ordinary Meeting, 20 January 1960

The following papers were read and discussed :

1. ' Experimental studies of wave-form clouds,' by Monsieur N. Gerbier of Paris.
2. ' Recent developments in theoretical treatment of mountain waves,' by Dr. E. Palm of the Astro-Fysisk Institutt, Oslo.

Ordinary Meeting, 17 February 1960

The following candidates were declared to have been elected that day by the Council :

Fellows

Bardon, Sqdn. Ldr. Peter John, A.R.Ae.S., R.A.F., Henlow, Herts.
Crosdale, Aubrey Lauriston, West Indies Meteorological Service, Piarco Airport, Trinidad, West Indies.
Diver, Alan Archibald, B.Sc., 17 Centre Drive, Newmarket, Suffolk.
Good, Bernard Thomas Charles, Railway Tavern, Newnham Bridge, Tenbury Wells, Worcs.
Haggett, Ronald Clifford Fred, B.Sc., 12 Mound Road, Maesycoed, Pontypridd, Glamorgan.
Hutchings, J. W., Meteorological Office, P.O. Box 722, Wellington C.I., New Zealand.
Magor, Joyce Irene, M.A., 32 Glover Road, Pinner, Middx.
Manning, Harold Leslie, M.Sc., M.I.Biol., P.O. Box 884, Kampala, Uganda.
Martin, John Hamilton Anthony, 157 Duthie Terrace, Aberdeen, Scotland (*reinstatement; elected 1955, resigned 1958*).
Pereira, Herbert Charles, B.Sc., Ph.D., East African Agriculture and Forestry Research Organization, P.O. Box 21, Kikuyu, Kenya.
Powell, John Martin, M.Sc., Forest Biology Laboratory, 102-11th Avenue, E., Calgary, Alberta, Canada.
Rogers, Peter, c/o World Meteorological Organization, 1 Avenue de la Paix, Geneva, Switzerland.
Ross, Sinclair Mowat, Officers' Mess, Royal Air Force, Jever, B.F.P.O. 25.
Smith, Edward John, 9 Rowhill Avenue, Aldershot Hants.
Sparrow, Bernard Alfred, 13 Wesley Avenue, Rhoose, Barry, Glamorgan.
Starling, Capt. Eric Allen, 43 Douglas Street, Largs, Ayrshire, Scotland.
Strachan, David Greig, Officers' Mess, Royal Air Force, Jever, B.F.P.O. 25.
Torrance, John Duncan, B.Sc., P.O. Box 8066, Causeway, Salisbury, Southern Rhodesia.

Foreign Members

Kruger, Erich Georg, House 22, Block 3, Airport Estate, Khartoum, Sudan.
Miura, Akira, M.Sc., Geophysical Institute, Tohoku University, Katahiracho, Sendai, Japan.
Vanderelst, Nérée Paul, Service Météorologique, Leopoldville, Belgian Congo.

STUDENT MEMBERS

BALCHIN, Peter Malcolm, 25 Lon Cedwyn, Cwmgwyn, Swansea, Glamorgan.
BIGNELL, Kenneth John, 26 Clitheroe Road, Stockwell, London, S.W.9.
GADD, Nicholas, 1 Hedgecroft Cottages, Newark Road, Ripley, Woking, Surrey.
LAVERACK, Barbara Mary, Christopher Hatton School House, Laystall Street, Holborn, London, E.C.1.
LEALAN, Alan Denis, 33 Redhill Drive, Fishponds, Bristol, Somerset.
MACARTHUR, Gilbert George Shaw, The Manse, Chryston, Nr. Glasgow, Scotland.
RADCLIFFE, Christopher John, 3 Tudor Hill, Sutton Coldfield, Warwicks.
RUSHFORTH, Beryl, 2 Northstead Manor Drive, Scarborough, Yorks.
TURTON, Margaret Ann, 18 Laverdene Way, Totley Rise, Sheffield, Yorks.
VOISK, Sylvia Virginia, King's College, Surrey Street, London, W.C.2.
WHEELER, David, 59 Weston Road, Balby, Doncaster, Yorks.

The meeting took the form of a joint discussion with the British Glaciological Society. Dr. H. Hoinkes of the Institute of Meteorology and Geophysics, Innsbruck, spoke on 'Studies in glacial meteorology in the Antarctic, 1957.'

An account of the meeting will appear in the May issue of *Weather*.

Evening Discourse, 2 March 1960

The third Evening Discourse, which was the last in the present session, was given by Dr. H. L. Penman, who spoke on 'Weather, soil and plant factors in hydrology.'

An account of this meeting will appear in a forthcoming issue of *Weather*.

Additional Meeting, 4 March 1960

A joint discussion meeting with the Royal Astronomical Society was held at the Royal Society of Arts, John Adam Street, London, W.C.2, the topic being 'The British Rocket Programme.' About 150 members of both Societies and many visitors were present.

Chairman and opening speaker was Prof. Sir Harrie Massey, F.R.S., and other speakers were Mr. J. F. Hazell on 'Rocket performance'; Dr. E. B. Dorling on 'Basic instrumentation'; Dr. G. V. Groves on 'Temperature and wind'; Prof. P. A. Sheppard on 'Wind'; Dr. B. L. Moiseiwitsch on 'Airglow'; Prof. J. Sayers on 'Electron and ion distribution in the ionosphere'; Dr. A. P. Willmore on 'Electron temperature'; Dr. R. L. F. Boyd on 'Solar X- and ultra-violet radiation'; Dr. S. H. Hall on 'Geomagnetic field'; and Prof. W. J. G. Beynon on 'Ionosphere sounding.'

A full account of this meeting will appear in *Weather* and a report is to be published in *Nature*, and *Geophysical Journal*.

Reviews

Introduction to theoretical meteorology. By Seymour L. Hess. New York (Henry Holt), 1959. Pp. xiv, 362; 120 Figs.; 15 Tables; 80 Problems. $8.50.

This extremely good book has been written as a text-book for new students of meteorology, but it will also serve as a useful reference book. It simply and clearly covers all important aspects of dynamic meteorology, assuming no more than an elementary background knowledge of calculus and physics. The entire presentation is in the classical physics style. The numerous applications of the results will make it easy for the student to assimilate them. The equations have been set out clearly, the Figures are excellent and the type is pleasingly large and clear.

Three chapters on thermodynamics follow the introduction and a short chapter on the equation of state. The physical meaning of entropy, usually a difficult concept for the student to grasp, is only touched upon in a heuristic non-rigorous fashion. Four thermodynamic diagrams, the emagram, the tephigram, the skew T-log p diagram and the Stüve diagram, are discussed in detail.

This section is followed by two chapters dealing with hydrostatic equilibrium, vertical stability and convection. The entrainment theory and the bubble theory of convection are both explained.

The three chapters on radiation are quite disconnected from the rest of the book and can be omitted at a first reading. The fundamental nature of radiation is first discussed, followed by a chapter on solar and terrestial radiation. This section is terminated with a chapter on applications of radiation in the earth atmosphere system. Elsasser's method is clearly stated and there are several examples. The mean annual heat balance of the Northern Hemisphere is discussed in detail.

The next ten chapters contain an exhaustive treatment of the hydrodynamics of the atmosphere. Each concept is clearly and adequately explained. Vector analysis is not used, an obvious advantage for those not familiar with that technique, and the presentation in Cartesian co-ordinates is surprisingly clear and easy to follow. The penultimate chapter on numerical weather prediction contains the basic ideas without over-elaboration into details of the many atmospheric models that have been proposed. Finally, there is an excellent chapter on the general circulation.

I think it is unfortunate that no answers are given to the problems, as it is reassuring to know when the correct answer has been obtained, but otherwise I cannot find fault in this book. Theoretical meteorology is made interesting and easy to understand and I think this book will rapidly become one of the standard text-books of modern meteorology.

F.H.B.

Turbulent transfer in the lower atmosphere. By C. H. B. Priestley. (University of Chicago Press), 1959. Pp. vii, 130; 22 Figures; 10 Tables. $3.75.

This monograph is based on a series of lectures given by Dr. Priestley (who is the Head of the Division of Meteorological Physics, C.S.I.R.O., Australia) when he was visiting research meteorologist at the University of Chicago in 1957. It might be described concisely as a ' progress report ' on micrometeorology during the last decade.

' Turbulent transfer ' is a phrase which covers many features, such as friction, convection, evaporation and diffusion of matter, which are basic in the study of the lower atmosphere. The roots of the studies are to be found in the early papers of Taylor, Schmidt and Richardson; the ' classical ' theory is summed up in the present reviewer's *Micrometeorology*, to which the author pays a generous compliment, and this volume takes the story from 1950 to the present day.

The first chapter deals with the eddy flux and its measurement, and is followed by an account of the transfer of momentum. These two chapters give a very readable account of the work of Swinbank and Deacon in Australia, and of Monin and Obukhov in Russia, on the wind profile near the ground; there is also a discussion of the approach to the geostrophic wind. Chapters 4, 5 and 6, which deal with heat transfer and buoyant motion, are perhaps the most exciting part of the book. Here there is much progress to report – one has only to recall the complete absence of a quantitative treatment of free convection in present-day text-books on heat to see how much the meteorologists, especially Dr. Priestley, have contributed in this field. Finally, there are chapters on evaporation and the evolutionary aspects of energy transfer, and the book closes with a useful bibliography.

The treatment is primarily mathematical, but never divorced from observation. The book is addressed to the expert, and some prior knowledge of the subject is essential if the reader is to feel at home in its pages. For the research worker in micrometeorology the book is timely; it gathers together and consolidates the more outstanding work of the last decade, a period in which the mathematicians have come more and more into the picture, but which has also seen a wealth of accurate measurements. In some hands the study of atmospheric turbulence tends to become an esoteric branch of mathematics, but Dr. Priestley has been too closely connected with ' field work ' to allow this to happen in his own studies, and there are plenty of practical applications to be seen in this account. Although there any many points of controversy – the nature of the subject makes this unavoidable – it can be said without hesitation that this is a balanced account, written by a leading authority, which cannot fail to stimulate other workers.

The book is well printed on good paper and the diagrams are clear.

O.G.S.

The theory of homogeneous turbulence, Students' edition. By G. K. Batchelor. Cambridge Monographs on Mechanics and Applied Mathematics. Cambridge (University Press), 1959. Pp. xi, 197; 28 Figures; 18s. 6d.

The first edition of Dr. Batchelor's book was reviewed from a meteorologist's point of view in 1953 in the pages of this *Journal*. The text is unchanged except for an occasional new footnote and the correction of a few minor misprints. There seems little to add to the previous review except to point out that the author's hope, expressed in the Introduction, that no significant work has been overlooked can hardly be fulfilled after a lapse of six years.

This students' edition has a thin cardboard binding which looks as though it will wear fairly well, and is 11s. 6d. cheaper than the cloth-bound edition. Cambridge University Press are to be applauded for going in for the wrapper-style binding, and so off-setting the alarming rise in the price of text-books, which must be putting the ownership of the standard texts beyond all except libraries.

E.K.

Arid-Zone hydrology : Recent developments. By H. Schoeller. Paris (Unesco), 1959. Pp. 125; 29 Figures; Tables. 12s. 6d.

The twelfth volume in the Arid-Zone series breaks new ground in intention and achievement. Prof. Schoeller of Bordeaux has written a short text-book of hydrology, drawing his illustrative examples from material published since 1952. There is a bias towards his own special interests – easily counter-balanced by choice of another author for the next review, if one as good can be found – a welcome bias, for it makes the subject coherent : indeed, hydrology begins to look like a science, presented this way.

The five chapters are on the formation of ground water reserves; prospecting; measurement of permeability; geochemistry; and micro-circulation. There is nothing here for the general meteorologist, but plenty for those who would like to know how engineering skill and geological knowledge can exploit scanty rainfall in regions where potential evaporation rates are high. The book is well written, in excellent English. It is cheap, too.

H.L.P.

Snow hydrology. Office of Technical Services (United States Department of Commerce), 1959. Pp. xxv, 437; 21 Tables; 72 Plates. $8.

This report gives a comprehensive summary of all snow investigations carried out in the United States, mainly to promote a more fundamental understanding of snow hydrology for the economical design and effective operation of engineering works required for river control. In 1945, a ' Co-operative Snow Investigation ' was initiated by the Corps of Engineers together with the United States Weather Bureau. Much of the report summarizes work carried out at three ' snow laboratories' set up in areas of heavy precipitation and snow accumulation in the mountains of western United States. These provided living-quarters for five to ten men and operated for periods of from five to eight years in each case. These areas, varying in size from 4 to 21 sq. miles and in altitude from 2,000 to 9,100 ft were in Upper Columbia, Middle Willamette and Central Sierra Nevada. The average annual rainfalls were 50, 105 and 70 in. respectively, and with more than half the precipitation occurring in the winter half-year, mainly in the form of snow. The climate, the extent of forest cover and the type of soil give wide variations in these three areas.

The report summarizes experiments on the instrumentation, explains the methods used for measuring snow and rain, and the procedure for determining the general precipitation from these samples. It was found that any decrease in precipitation at the crest of the mountains was ' more apparent than real.' The forest effect is regarded as that of interception of precipitation and of forest transpiration. Measurements are given of the ' throughfall ' (including drips), and the stem flow. A canopy-density meter was used and equations are given connecting canopy-density and the snowpack water equivalent. For these three ' snow laboratory ' areas, monthly values are given for the total precipitation, interception loss and net precipitation (rain and snow being shown separately) together with changes in the snowpack water equivalent, melt, potential and actual evapotranspiration (after Thornthwaite), initial and change in the available soil moisture, and finally the computed and actual run-off. The reasons for the differences in the two final sets of values are discussed.

Chapters 5 and 6 consider the melting of the snowpack and the relative importance of snowpack indexes. Temperature is found to be the best single index of snow melt. Chapter 7 explains the various methods used for observing the area of snow cover, and with winter accumulation altitude becomes the important factor. Chapter 8 is especially interesting and deals with the metamorphism of the snowpack, with its changes in liquid water holding capacity.

Chapters 9 to 12 are concerned mainly with the resultant hydrographs and with engineering problems. Chapter 9 discusses the time distribution of the snow melt run-off; Chapter 10 – design flood determinations; Chapter 11 – seasonal run-off forecasting, with the object of making the fullest use of water resources; Chapter 12 – reservoir regulation, in order to make flood control and the conservative use of storage compatible on a seasonal basis.

Each chapter contains numerous tables, useful maps and diagrams, together with a bibliography of related papers restricted mainly to those published in the United States of America.

J.G.

Cockle Park Farm. By H. Cecil Pawson. London (Oxford University Press), 1960. Pp. xiv, 261; 19 Illustrations; 13 Plates and Graphs; 25 Tables. 35s.

This book gives a connected account of the work of the Cockle Park Experimental Station from 1896 to 1956. The main purpose of the Station was to elucidate and demonstrate the principles of good husbandry in Northern England by manuring trials, by comparison of seed varieties, and by the feeding and breeding of livestock, etc. Little agricultural use was made of the meteorological records although a climatological station was established in 1898 and observations have been made twice daily ever since. However, in the early years, an attempt was made to assess the effects of weather on crop yield but results were inconclusive, no doubt because total values of rainfall and sunshine for the growing season were used as weather parameters.

An interesting experiment is recorded relating to soil moisture content. Two plots in the instrument enclosure were kept free from herbage. One was allowed to set as a consolidated surface, the other being kept in good tilth to a depth of about 2 inches. Samples of soil, after periods of dry weather, were taken in four different years; on each occasion the moisture content of the soil under the tilth was appreciably higher than under the compacted surface – the average increase was over 20 per cent. The book is well produced and contains a mass of information for the practical farmer, especially in the north of England, but it is not of much direct interest to the meteorologist.

M.T.S.

Radar meteorology. By L. J. Battan. (University of Chicago Press), 1959. Pp. xi, 161; 77 Figs.; 16 Tables. 45s.

This small volume is by an author who has been in the forefront of research into the applications of radar to meteorology for many years. Its avowed aim is to bring radar meteorology to the attention of meteorologists outside the field of physical meteorology, in the hope that its better understanding ' may act as a stimulus for radar weather research in the future by all meteorologists.' With that in mind it is unfortunate that the accent tends to be rather on the difficulties of interpreting quantitative measurements of echo intensity than on the ability to see at any given time the full precipitation pattern of the surrounding area in all its detail. The snags are only too well known to the specialist but they should not be allowed to swamp the undoubted advantages which the use of radar can give.

The first 70 pages, comprising eight short chapters, deal rather sketchily with the principles of radar and the propagation of electromagnetic waves but adequately with the scattering and attenuation by airborne particles and the application to quantitative measurements of precipitation.

The following five chapters, 63 pages, are devoted to the meteorological uses of radar in hydrologic problems, cloud physics research, mesometeorology and in the study of larger-scale weather systems with a chapter also on the uses of airborne radar for flight operations. Since the book is addressed to the non-specialist it would have been better to have expanded these chapters, but they serve to outline some of the uses to which radar can be put and are well referenced to assist the interested reader in following up a particular topic. In a plea for airborne radar with the facility for scanning in the vertical plane, the author states that when a bright band is evident ' the pilot can be assured that icing is not serious above the band.' To the reviewer this seems to be a dangerous over-statement for which there is inadequate supporting evidence.

The book, which is beautifully reproduced to the high standard one expects from a university press, concludes with a chapter on ' angels ' and one on special instrumental techniques.

R.F.J.

Obituary Notices

Professor C. T. R. WILSON, C.H., M.A., Hon. Sc.D., F.R.S., Nobel Laureate

Charles Thomas Rees Wilson, Fellow of our Society since 1903, and an Honorary Member from 1952, died on 15 November 1959, at the age of 90. Born in a farmhouse in the Pentland Hills, near Edinburgh, he would have followed the family tradition and become a sheep farmer but for the early death of his father, which caused the family to move to Manchester. After graduating at Owens College, Wilson won an entrance scholarship to Sidney Sussex College, Cambridge, and in Cambridge, except for a brief spell as a schoolmaster, he spent the rest of his working life. He won rapid recognition both as a teacher of physics and as a research worker. He was elected to Fellowship of the Royal Society in 1900 and to the Jacksonian Chair of Natural Philosophy in 1925; in 1927 he shared the Nobel Prize for Physics with A. H. Compton and he was made a Companion of Honour shortly after his retirement to Edinburgh in 1936.

Wilson is best known in the scientific world as the inventor and pioneer user of the cloud chamber, which has become such a universal and essential tool in atomic and nuclear physics research. Amongst a more restricted circle he is known, too, for an equally outstanding contribution to the study of atmospheric electricity and in particular of the electricity of thunderstorms. Both fields had in a sense a meteorological origin, for the Wilson expansion chamber grew almost accidentally out of a fundamental investigation into the physics of the condensation process – the importance of an accidental discovery was, of course, seen in a flash and its consequences pursued with determination, indefatigable patience and great experimental skill. The original investigation had grown out of a desire to understand and imitate, in the laboratory, natural cloud effects which Wilson had himself observed during his rambles on the Scottish mountains and, in particular, during a stay at the old Ben Nevis Observatory in 1894.

Wilson's interest in atmospheric electricity developed from his discovery that ions are being continuously produced in the atmosphere, and was reinforced by certain exciting experiences in thunderstorms on the Scottish hillls. His clear physical insight and experimental skill found here an ideal field for fruitful work and in it he has left a profound mark. His article on atmospheric electricity in the *Dictionary of Applied Physics*, published in 1923, the nearest he came to writing a book on the subject, is a model of clarity and of stimulating and suggestive thinking, and its influence on much subsequent work is obvious and profound. He devised new methods of measurement for various quantities and the literature still abounds with references to Wilson plates and Wilson spheres, but it is perhaps his contribution to the study of thunderstorms and the relation of thunderstorm electricity to the fine-weather effects that is the most noteworthy. To him is due the fundamental suggestion, made as early as 1920 and now after considerable controversy almost universally accepted, that the potential gradient and current of fine weather are maintained by world-wide thunderstorm activity through the intermediary of a conducting upper atmosphere. He devised the experimental methods and made the first observations which yielded reliable estimates of the charges and potential differences involved in a lightning discharge, and which later established the polarity of the cloud, and to him is due the first realisation of the magnitude of the current carried by point discharges below thunderstorms and an attempt to estimate reliably the total vertical current maintained by a storm between the ground and the upper atmosphere.

Wilson published two theories of thunderstorm electrification; both are influence processes, i.e., they will intensify a given initial vertical field whatever its sign. The first, a microphysical process, is an elegant process of selective capture of ions of one sign by precipitation particles, and is supposed to be the predominant process in the early stages of growth of the electric field. The second, which likens the mature thundercloud to a large-scale influence machine, is supposed to take over in the later stages. His account of this second theory was only published in 1956 although he had been thinking along these lines for a long time. His two thunderstorm theories have not found very wide acceptance but this is still a field of lively controversy and a final assessment of their value is hardly possible.

C. T. R. Wilson (C.T.R. to a host of friends and acquaintances) inspired in those who knew him a sense of being in the presence not only of a great physicist but also of a great man. To exceptional physical insight and originality, endless patience and persistence, supreme experimental skill, he added enthusiasm, integrity, humility and gentleness. Admiration was lost in a warm and lasting affection. He retained to the end a remarkable physical and mental vigour and delighted to welcome old friends and students into the intimacy of his family circle in the delightful cottage to which he finally retired near his birth-place; there he would exchange reminiscences or take up some scientific discussion or argument, it seemed at the precise point at which it had been interrupted perhaps twenty or thirty years earlier. There, at the hands of this young man of ninety, one recaptured something of the passionate enthusiasm and excitement of scientific enquiry.

T.W.W.

Mr. J. C. BEECHEY

James Collett Beechey, who had held Fellowship in the Society since 1927, was born on 31 October 1888 at Carrickfergus, Antrim, and died on 10 February 1959 at St. Luke's Hospital, Guildford, Surrey. Having received his education at his father's private academy he entered the printing and publishing trade. He worked on the journalistic side thereof until the 1914-18 war, when he saw service with the Royal Flying Corps and the Royal Air Force. Later he returned to his former occupation, but soon transferred to the

technical side of the trade. In 1922 he was appointed works manager of the *Surrey Advertiser and County Times*, a post which he continued to fill until ill-health compelled his retirement in 1952. Mr. Beechey had taken an enthusiastic interest in both meteorology and astronomy since boyhood and lectured extensively on these subjects in and around Guildford, his home town. During the 1939-45 war he was civilian instructor in meteorology to the Guildford Squadron of the Air Training Corps. For the main details of Mr. Beechey's career given above acknowledgment is made to his son, Mr. Peter J. C. Beechey of Beaminster, Dorset.

E.L.H.

Mr. S. S. ELLIOTT, B.A., B.Sc., A.M.Inst.C.E.

Stanley Steuart Elliott, whose death occurred in Cape Town in September 1959, was the son of the late Sir Charles Bletterman Elliott, formerly general manager of the Cape Government Railways. He received his education in the Cape Colony, graduating from the University of the Cape of Good Hope in 1901. Thereafter he went to England, studying at the City and Guilds Institute and obtaining the B.Sc. degree in engineering from the University of London in 1905. After a brief stay in this country he took an appointment in India with the Madras and South Mahratta Railway. For the rest of his professional life he remained in the service of that organization, eventually rising to the position of Deputy Chief Engineer. On his retirement, in 1937, he took up residence at Sevenoaks, Kent, but after the war he returned to South Africa, where he died at the age of 76. Besides meteorology, a main interest to him from his youth onwards, Mr. Elliott's hobbies were tennis, fishing and motoring. His Fellowship in this Society dated from 15 February 1911.

G.G.E.

Mr. R. F. WESTAWAY

Mr. Reginald Frank Westaway, a Fellow of the Society since 1947, died on 11 August 1959 towards the close of his 62nd year. Except for a period of the 1914-18 war, when he saw duty with the signals section of the Royal Engineers in German East Africa, the whole of Mr. Westaway's working life was passed with the New Barnet firm of S. Mawson & Sons, whom he joined in 1912, serving initially as costing clerk and later as cashier. Out of office hours and during holidays he gave the bulk of his time to the Scout Movement, with which he first became associated in 1913. As scoutmaster since 1921 he built up the Fourth Enfield Troop into a model of discipline, efficiency and enthusiasm. Mr. Westaway's interest in the weather was of long standing. For many years he took daily meteorological observations at Enfield and sought to keep himself in touch with the progress of the science by regular study of its literature. He was thus particularly well qualified to undertake duty on behalf of the local Scouts' association as examiner for the weatherman's badge. We are indebted to his brother, Mr. J. D. Westaway, for most of the information contained in this notice.

E.L.H.

Corrigendum

Vol. **86**, January 1960, p. 119. Lines 12 and 13, should be replaced by the following :

Mr. C. J. Adkins presented his paper on ' The small ion concentration and space charge near the ground ' and Mr. M. Smiddy a paper with Dr. J. A. Chalmers on ' Measurements of space charge in the lower atmosphere using field mills.'

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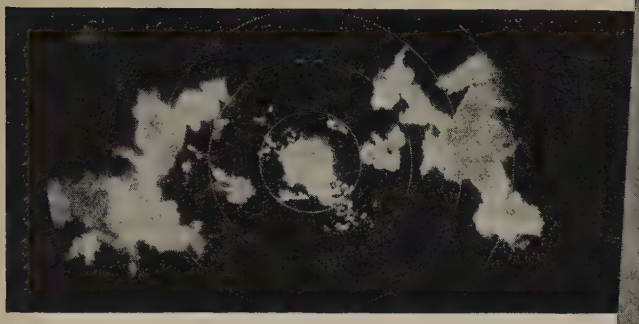

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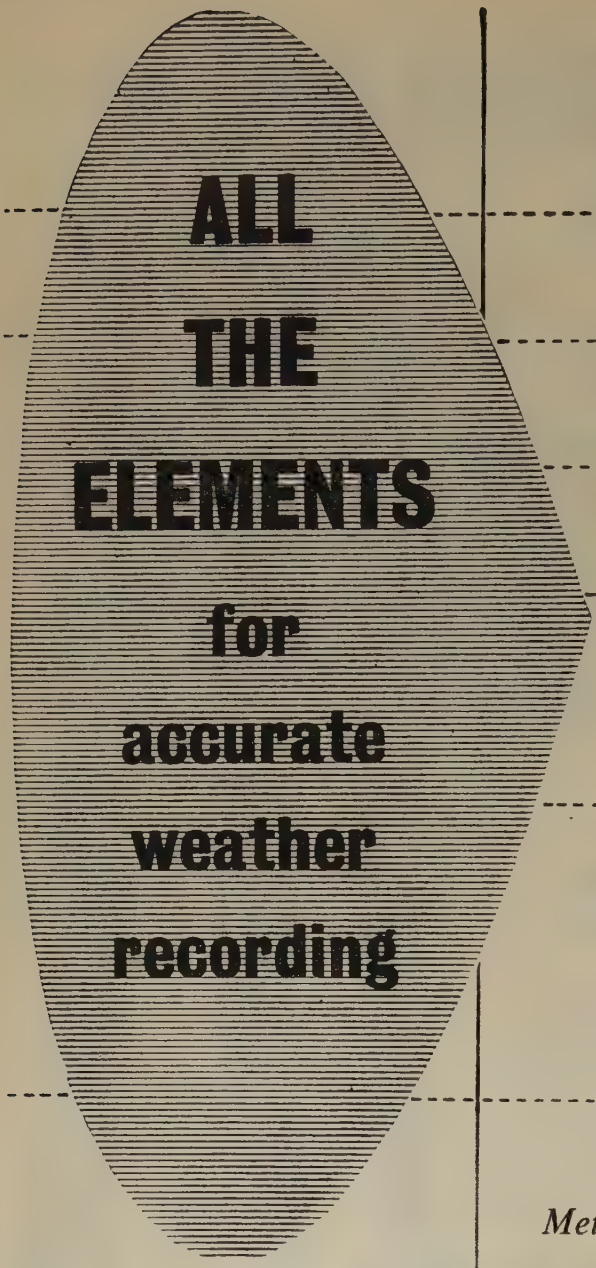

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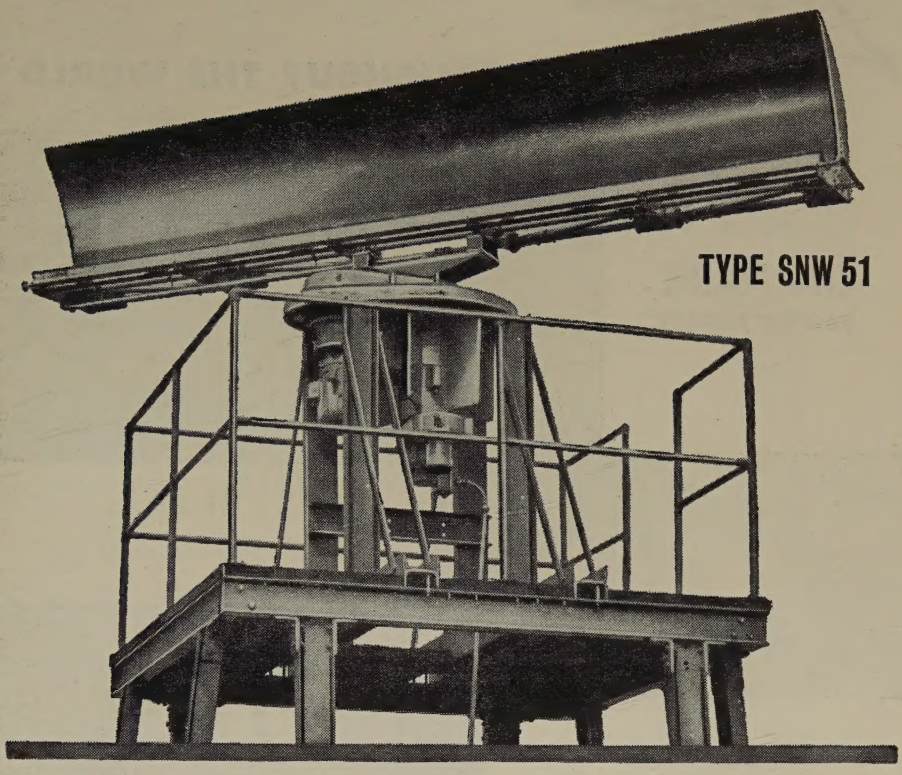

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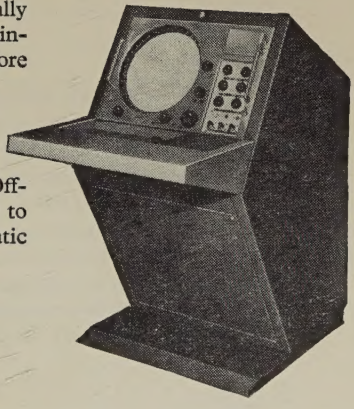

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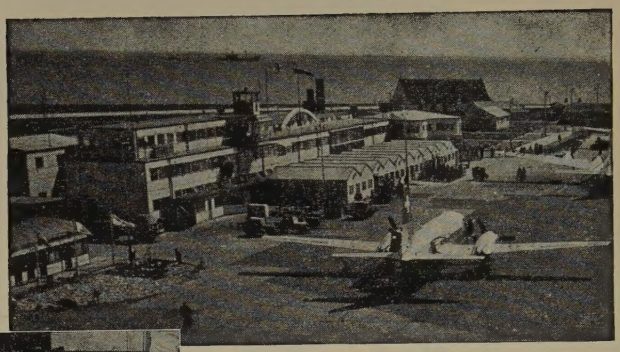

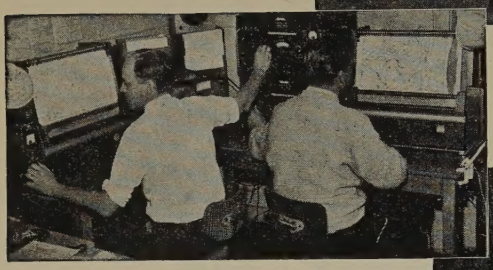

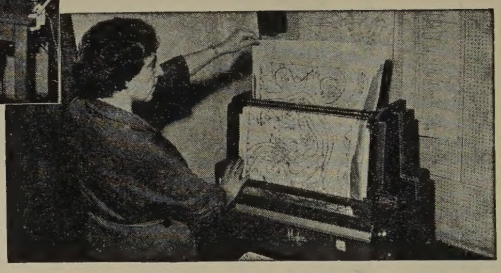

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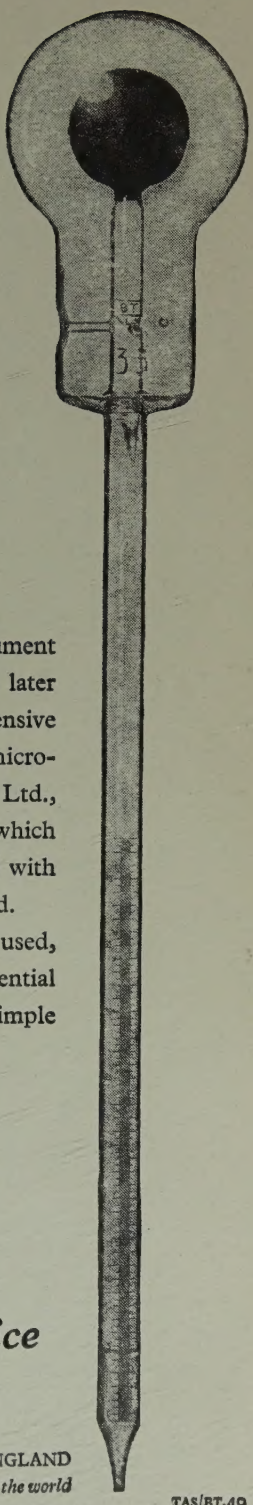

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